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THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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PRESSURE-SHIFT OF SPECTRAL LINES

BY FERNANDO SANFORD

There have been three different explanations proposed to account for the broadening and displacement toward longer wave-lengths of spectral lines when their radiant atoms are under heavy pressure due to a surrounding gas. The first explanation seems to have been proposed by Fitzgerald[1] and later to have been indorsed by Larmor. This explanation attempts to find the cause of the slower luminous oscillations under heavy pressure in the increase of the specific inductive capacity of the medium in which the oscillations take place.

Fitzgerald says:

When a body is a source of electromagnetic radiations the frequency of its vibrations depends in general on the specific inductive capacity of the medium in which it is immersed. An electromagnetic oscillator performs oscillations that can be calculated from a formula of the form $N^{-2} \propto LC$, where L is self-induction and C is capacity. If the medium have a high electric inductive capacity C will be large and consequently N will be small. Now an increase in the pressure of a gas increases its specific inductive capacity and must in consequence alter to some extent the period of vibration of the molecules in it, if their period of vibration depends at all on electric forces due to constant charges. We can consequently conclude that here is a *vera causa* for some shift towards the red in molecules causing light, for in them there can be no doubt that electric forces are at least a part of the forces affecting the periods of vibration.

[1] *Astrophysical Journal*, 5, 210, 1897.

Larmor[1] later undertakes to show that "the dielectric influence of the neighboring molecules is a *vera causa* of the right order of magnitude" to account for the pressure-shift. He says, speaking of the change in the period of oscillation of whatever produces the spectral lines:

> The change must be connected with electric properties of the surrounding gas; mechanical pressure arises merely from the translatory motions of the molecules, and these are so slow as hardly to count in connection with radiation-periods. Thus the phenomenon is probably more strictly describable as a density effect. Electrically, the effect of increase of density is to increase the inductive capacity of the medium, that is, to diminish the effective etherial elasticity which propagates the radiation. This is the averaged result; each molecule individually, through the agency of its plastic field of force or ether-strain, provides a yielding region in the ether in which the effective stiffness is diminished. The elastic energy which maintains the free vibrations of a radiator is located in its field of force in the adjacent ether; and by dynamical principles any loosening of the constraint in that field such as an adjacent molecule would produce, which would itself be somewhat intensified by equality of period, must in general tend toward increasing the free period, involving the displacement of the radiation toward longer wave-length.

I have quoted at length from Fitzgerald and Larmor because the theory proposed by them is the one I wish especially to discuss. There have been, however, two other theories proposed which deserve notice in any discussion of this question. Humphreys[2] concludes that "the pressure-shift shows that specific inductive capacity has but little, if any, influence on the period of whatever it is to which these lines are due," and undertakes to explain the displacement by the magnetic action of the molecules on each other. He adopts the theory that the luminous atom is of the so-called "Saturnian" type, consisting of a large number of electrons revolving in circular or broadly elliptical orbits in a common plane about a central positive subatom, and then argues that these magnetic atoms must act upon each other in the Zeeman manner, producing a separation of individual lines as in a strong magnetic field. In a later paper[3] he calculates the effect which these molecular fields must have upon each other, and concludes that if the

[1] *Astrophysical Journal*, **26**, 120, 1907.

[2] *Ibid.*, **26**, 18, 1907.

[3] *Ibid.*, **27**, 194, 1908.

average magnetic field of a molecule in the region surrounded by its ring of rotating electrons is as great as 45×10^7, the pressure-shift may be accounted for after the manner of the Zeeman separation.

Aside from the improbability that the atoms of all metals are ten thousand times as strongly magnetic as the strongest electromagnet ordinarily used in the laboratory for producing the Zeeman effect, is the further difficulty that King[1] has compared the Zeeman separation of a large number of lines with the pressure-shift of the same lines at 42 atmospheres as given by Humphreys and finds the ratio of the Zeeman displacement to the pressure-shift as not even approximately constant even for lines in the same element. Thus for iron the values of this ratio run from 0.78 to 15.5, while several lines were observed which show pressure-displacements, two of them large, but which show no Zeeman separation even in a field of 20,000 gausses.

A third explanation has been proposed by Richardson,[2] which bases the pressure-shift on the reaction upon the emitting atom of forced vibrations which it has set up by electrostatic induction in neighboring atoms. Richardson's calculations, based upon assumptions regarding the atomic structure and atomic distances which seem to him reasonable, give magnitudes for the pressure-shift from 5 to 200 times as great as the observed values, and require that the displacement shall increase as the third power of the wave-length instead of approximately as the first power, as shown by experiment.

All three of the above-mentioned theories make the change in the period of the vibrating mechanism depend upon the proximity of atoms of its own kind, since the greater the atmospheric pressure around the arc the denser must be the metallic vapor in the interior of the arc. It would accordingly seem to follow from Humphreys' theory that the greatest shift should occur in those metals whose atoms are most strongly magnetic, while from Fitzgerald's theory the greatest shift should occur in those metals whose vapors have the highest specific inductive capacity. Since at the same pressure

[1] *Ibid.*, **31**, 433, 1910; also **33**, 250, 1911.

[2] *Phil. Mag.*, **14**, 557, 1907.

there should be very approximately the same number of molecules in unit volume of the different vapors, it should be possible to decide between these theories if we knew the magnetic and inductive properties of the different metals. The magnetic properties are already known, and a glance at Humphreys' data in *Astrophysical Journal*, **6**, 220 or 224, will show that the pressure-shift is not greater for the magnetic metals than for the others. On the contrary, it is distinctly less. Thus from the data on p. 224 the average shift for *Fe*, *Ni*, and *Co* is 0.026 Å, while for all the lines of all the metals given the average shift is 0.045 Å. Even on the assumption that the magnetic field of the average atom of non-magnetic or diamagnetic material is ten thousand times as great as that of the most magnetic metals when saturated, we might still expect the atoms of the magnetic metals to have stronger fields than the average of all the metals, instead of only half as strong.

The specific inductive capacity of the metals has not been measured, and can only be inferred from their other properties, but any assumption as to the nature of specific inductive capacity must enable one to draw conclusions as to its effect upon other properties of the metal. Thus if we adopt Maxwell's theory, which has been so clearly stated in the quotation from Larmor, that an increase in specific inductive capacity is due to a weakening of the electric elasticity of the ether around the atoms of bodies, we can deduce its effects upon other properties of the body. It seems as clearly established as anything in electrical theory that the forces of affinity and cohesion are attractions between the electropositive and electronegative parts of atoms and molecules. If, for example, the positive subatoms of a metal are held together by their mutual attractions for the same electrons (which seems to be the only conceivable explanation for cohesion in the light of our present knowledge) and if the positive charges of the atoms are of the same order of magnitude, cohesion must be weakest in those metals whose specific inductive capacity is greatest. Such metals should accordingly have greater compressibility, lower melting points, greater expansion coefficients, and should be softer than other metals in which the specific inductive capacity is less. The metals having higher specific inductive capacity should also hold their electrons

with weaker forces than those in which the specific inductive capacity is less, and should accordingly become more electropositive under the action of ultra-violet light. They should also lose electrons when their surfaces are put in contact with metals of lower specific inductive capacity and become the electropositive metals in the voltaic series. For the same reason, when placed in liquids of high specific inductive capacity, thus having their surface cohesion weakened further, they should most easily part with their electropositive subatoms and become the electronegative metals in the electrolytic series. The voltaic series should accordingly correspond with the inductive series.

This fact has been observed for the non-metallic elements. Thus in 1898 Coehn[1] announced the law that for non-metallic elements those having a higher dielectric constant become positively charged when brought into contact with substances having a lower dielectric constant.

In a paper prepared for the meeting of the American Physical Society[2] in February 1908, the present writer exhibited a table

Element	Compressibility $\times 10^6$	Melting Point	Expansion Coefficient	Hardness	Pressure-Shift
Caesium	61	26.5	....	0.2	161
Rubidium	40	38.5	....	0.3	132
Potassium	31.5	58	....	0.5	132
Sodium	15.5	95	0.000072	0.4	108
Lithium	8.8	180	....	0.6	85
Zinc	1.5	419	0.000029	2.5	57
Lead	2.2	330	0.000028	1.5	60
Tin	1.67	230	0.000022	1.8	55
Iron	0.38	1500	0.000012	4.5	25
Silver	0.82	950	0.000019	2.5–3	39
Copper	0.54	1054	0.000017	2.5–3	33
Gold	0.47	1035	0.000015	2.5–3	40
Platinum	0.21	1780	0.000009	4.3	20

including all the metals whose positions in the voltaic series are well known in which the relation of the voltaic series to the cohesion series is shown. This table is reproduced above as then given and a final column is added showing the displacement of the lines of

[1] *Wied. Ann.*, 64, 231, 1898.

[2] *Physical Review*, 26, 410, 1908.

these same metals for a pressure of 12 atmospheres and a wave-length of 4000 Å as given by Humphreys in *Astrophysical Journal*, **6**, 220, 1897.

As will be seen, the agreement of the pressure-shift with the other properties is very close. Iron is the one conspicuous example of misplacement in the voltaic series, the relative positions of zinc, lead, and tin being uncertain. Iron, as is well known, may exist in an active or a passive condition as concerns its avidity for chemical combination with acids. In its passive state it acts very much like gold or platinum and would be placed with these metals in the voltaic series, but its active condition has generally been regarded as the normal one and the passive state as an induced condition. Recently Grave[1] has shown that the passive state is probably the normal condition of iron, and that the active, or electropositive, state is induced, probably by the absorption of hydrogen. If this is correct, iron will take the place in the voltaic series between gold and platinum to which its other properties seem to entitle it.

It will be seen from Humphreys' data on pressure-shift that this is, like many of the other properties of the atoms, a periodic property, the elements of the same periodic group showing a similar pressure-displacement for their spectral lines. In most cases the elements of a well-marked group do not differ widely in their contact electromotive force, and consequently in their specific inductive capacity. The group of alkali metals and the electronegative halogen group at the other extremity of the electrochemical series are exceptions. Since we have no data on the pressure-shift of the latter group, it may be worth while to compare the pressure-shift of the lines of the alkali metals with other properties of these elements.

The variation of the pressure-displacement with the compressibility, the melting point, and the hardness of these metals has already been shown. Fitzgerald, in the paper already cited, says: "In some of the cases I have tried there seems to be some connection between the refractive index of the gas and the amount of the shift." It is plain that if the shift is due to an increase of specific inductive capacity the refractive index of the gas should vary in

[1] *Zeitschrift für physikalische Chemie*, **77**, 513, 1911.

the same direction as the pressure-shift. But little is known of the refractive index of metallic vapors; but Gladstone, Edwards, Eisenlohr, and others have calculated the atomic refraction constants for a number of elements from the refractive indices of their compounds, and while different investigators sometimes reach different values in the case of individual elements, their data all show a general agreement. Only Gladstone's data give the refraction equivalents of all the alkali metals. The column marked "Refraction Constant" in the table which follows is taken from his values of the refraction equivalents of the elements given in the *Philosophical Transactions* of 1870.

There is much other evidence to show that the compounds of the electropositive elements have higher refraction equivalents than the corresponding compounds of the electronegative or less electropositive elements. Thus in a table of the refraction equivalents of salt solutions in water given in Ostwald's *Solutions*, p. 272, the refraction equivalents of twelve pairs of corresponding salts of potassium and sodium are given, and in every case the potassium salts give a higher refraction than the corresponding sodium salts by a quantity which is almost constant, the ratio being on the average 33.11:27.78.

Another source of information as to the inductive capacity of an atom may be found in its atomic volume. T. W. Richards[1] has from time to time called attention to evidence which seems to indicate that the atomic volume of an element may be varied by the cohesion pressure upon it when in a compound. Whether this is true or not, if the atoms are built up of positive and negative subatoms held together by electrical forces (as is certainly the case with the atoms of radioactive elements) the volume of an atom should be greater as its specific inductive capacity is greater. Accordingly, in elements of the same group the atomic volume should give an indication of the inductive capacity of the atom. The relation of the atomic volume to the pressure-shift is also shown in the table which follows.

Still another evidence is available for comparing the pressure-shift with the electric properties of the atoms. In two recent

[1] See his Faraday lecture printed in *Science*, 34, 537, October 27, 1911.

papers[1] I have undertaken to show that the positive subatoms of the elements which enter into the electrolytic process as positive ions have specific charges which may be approximately calculated, and that these charges vary with cohesion and hence with specific inductive capacity; in other words, that the specific inductive capacity of the ether is greater about an atom the greater its positive charge. This is further shown in the fact that in the negative halogen group the compressibility decreases and the melting point and cohesion (as shown by the state of aggregation) increase with the atomic weight. Column 6 of the table below gives values, which are only relative, of these charges as calculated from the velocity of the ions in a water solution under a given electromotive force.

In the papers last mentioned and elsewhere[2] attention has been called to the fact that all these properties are functions of the square root of the atomic weight. Thus the melting points of the alkali metals may be calculated to within the differences of determination of different investigators by the equation $T_a = \frac{531.5}{\sqrt{w}} + 252.5$, where T_a is the absolute temperature of the melting point and w is the atomic weight of the metal.

I have accordingly given the square root of the atomic weight of the metals in the second column of the following table, and for the sake of completeness I have again given the melting point, compressibilities, and hardness which were given in the preceding table for the voltaic series.

	$\sqrt{w}$	Pressure-Shift	Refraction Constant	Atomic Volume	Atomic Charge	Compressibility $\times 10^6$	Hardness	Melting Point
Cs.....	11.5	161	13.8	70.6	157	61	0.2	26.5
Rb.....	9.24	132	14.1	56	101	40	.3	38.5
K....	6.24	132	8.2	45.4	44	31.5	.5	58
Na....	4.8	108	4.8	23.7	18	15.5	.4	95
Li.....	2.6	85	3.8	12.9	5	8.8	.6	180

Taken all together, there accordingly seems to be a considerable amount of evidence to show that the pressure-shift of the spectral

[1] *Physical Review*, 32, 512, 518, 1911.

[2] "A Physical Theory of Electrification," published by the Leland Stanford Jr. University, University Series, No. 6, 1911.

lines is related to just those properties of the atoms which seem to depend upon the specific inductive capacity.

An objection which will at once suggest itself to everyone is the fact that the lines are, though to a less extent, broadened toward the violet. An increase of specific inductive capacity should slow down the oscillations but should not accelerate them, while Humphreys' magnetic theory would displace the lines in both directions. It is well known, however, that the specific inductive capacity of the air and of ordinary gases is very small as compared with that of metallic vapors. Accordingly, when an air molecule enters the field of force of a radiating metallic atom the specific inductive capacity of its field of force should be decreased, and this should accelerate the vibration period of the atom. If this point of view is correct, it would seem that the greatest shift toward the red should be due to those atoms in the center of the arc, while the greatest shift toward the violet should be due to atoms around the outer surface. Perhaps this difference may be capable of observation.

STANFORD UNIVERSITY
November 5, 1911

AN INVESTIGATION OF THE SPECTRA OF IRON AND TITANIUM UNDER MODERATE PRESSURES[1]

By HENRY G. GALE and WALTER S. ADAMS

The results which are contained in this article are the main products of a study of the effect of a gaseous pressure of 9 atmospheres upon the arc spectrum of iron and of pressures of from 3 to 17 atmospheres upon the arc and spark spectra of titanium. The investigation was begun in the spring of 1910 and carried on during a period of three months of that year and a corresponding period of 1911. At first the end in view was to study the effects of pressure upon the spark spectrum alone, but the lack of corresponding results for the arc at moderate pressures required the extension of the investigation to include arc results as well.

Previous studies of the pressure effect have for the most part been at comparatively high pressures. Thus the recent results of Rossi[2] upon the arc spectra of titanium and vanadium are based principally upon pressures between 25 and 100 atmospheres. A part of the earlier work of Humphreys,[3] and Humphreys and Mohler,[4] dealt with pressures as low as 4 atmospheres, but all of the more recent work of Humphreys[5] has been at much higher pressures. Similarly the work of Duffield[6] upon the arc spectra of iron, gold, and silver contains some results obtained at 5, 10, and 15 atmospheres, but the great majority of his values are also for high pressures. In the case of the spark spectrum only two previous investigations are available. The first of these, by Hale and Kent,[7] deals with a small number of iron lines at pressures of from 3 to 53 atmospheres; the second, by W. B. Anderson,[8] gives the displacements of a considerable number of iron lines at 50 atmospheres.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 58.

[2] *Proc. Roy. Soc.*, A **83**, 414, 1910.

[3] *Astrophysical Journal*, **4**, 249, 1896; **6**, 169, 1897.

[4] *Ibid.*, **3**, 114, 1895.

[5] *Ibid.*, **22**, 217, 1905; **26**, 18, 1907.

[6] *Ibid.*, **26**, 375, 1907; *Phil. Trans.*, A **208**, 111, 1908.

[7] *Astrophysical Journal*, **17**, 154, 1903.

[8] *Ibid.*, **24**, 221, 1906.

With the exception of some results by Humphreys[1] for certain lines of iron and chromium in the green and yellow parts of the spectrum all of the values given in these researches are for the more refrangible lines.

In view of these considerations it has seemed desirable to us to carry on our work at relatively low pressures and to include as large an extent of spectrum as possible. A combination of our own results with those obtained at higher pressures by other observers would give a severe test of the law of the proportionality of displacement to pressure, while the values of the displacements for lines of widely different wave-lengths would provide more satisfactory material for a discussion of the variation of displacement with wave-length than could a comparatively limited range of spectrum.

The astronomical applications of pressure results also make these considerations important. So far as our knowledge extends, the effective pressures in the reversing layer of the sun and in stellar atmospheres are moderate in amount. For example, the most recent determination of the pressure in the reversing layer of the sun by Fabry and Buisson[2] gives a value of 5 atmospheres. Accordingly pressures of the same order should be employed in the laboratory work in case a direct comparison of the character and the displacements of the lines is to be made with those in solar and stellar spectra.

The investigation has been carried on in the Pasadena laboratory of the Mount Wilson Solar Observatory and the spectrograph employed is the 9.1 m plane grating Littrow instrument already used by Mr. King and Mr. Babcock in numerous studies of the Zeeman effect. Since this spectrograph has been described in a previous publication,[3] no details are necessary in this place. The grating is a 20 cm plane grating by Michelson with 500 lines to the millimeter, and the second order has been used for all of the photographs taken in connection with this work. The linear scale given by the spectrograph is approximately 1 mm = 0.95 Ångström.

[1] *Ibid.*, **26**, 18, 1907. [2] *Ibid.*, **31**, 97, 1910.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 23; *Astrophysical Journal*, **27**, 204, 1908.

The pressure-chamber consists of a strong brass hood with a flange at its base which is fastened by a series of iron clamps around its rim to a brass base carrying a similar flange. The two portions have been surfaced accurately and at the junction a thin lead washer provides an air-tight union. The top of the hood carries the pressure gauge and the connections for attaching the tanks of compressed gas. The connections for the electrodes are passed through the sides of the box and are very heavily insulated in order to provide for the use of high potential for the spark. The electrodes themselves consist of copper clips placed in a horizontal position, and the arc or spark passes directly between small pieces of metal held in these clips. The distance between the electrodes may be varied by means of a fiber handle on the outside of the box which turns a screw acting through a hard rubber sleeve. At moderate pressures the chamber shows very little leakage.

In the front of the brass hood is the window through which the light from the arc or spark passes to the spectrograph. Since the spectrograph itself contains a glass lens and so is not adapted for work in the extreme ultra-violet part of the spectrum, the window also is made of glass. It is about 2 cm thick and conical in shape, with its base toward the light-source in order to withstand the effect of the pressure from within. The window is mounted in the center of a round brass casting about 12 cm in diameter cut with a quarter thread. This engages with a similar thread in an opening in the pressure-chamber and the block is tightened by means of a powerful wrench, until an air-tight junction is obtained. By taking off this block it is possible to reach the electrodes without unclamping the entire box.

The light passing through the window of the pressure-chamber is focused upon the slit of the spectrograph by means of a glass lens placed midway between window and slit. It has been found possible in all of the work to obtain arc and spark gaps sufficiently wide to avoid the necessity for magnification at the slit of the spectrograph. The diameter of the lens is such that the cross-section of the cone of light at the base of the spectrograph is about six times that of the grating surface, and we have given especial care to the adjustment of the instrument in order to keep the

illuminated area as nearly as possible concentric with the grating throughout all of the observations. To provide for accurate guiding during an exposure when the arc or spark shifts from one point to another upon the electrodes, we have adopted the simple expedient of placing the condensing lens upon a double-slide rack. The observer watches the image upon the slit with a small telescope and corrects for the motion of the arc or spark by a slight vertical or horizontal motion of the lens. It is evident that the effect of such motion is to displace the cone of light slightly with respect to the grating, but the margin of full illumination is so great and the motion of the lens so small that no error is to be feared from this source.

The arc used throughout this investigation was produced by a direct current of 110 volts furnished by a motor generator. A bank of lamps placed in series in the circuit provided the means of varying the amount of current passing through the arc. In general a current of from 4 to 8 amperes has been used for most of the work. By increasing or decreasing the current it is usually possible to vary the amount of reversals of the stronger comparison lines, and this has proved most useful in a part of the investigation.

The alternating current for the spark was furnished by a 5 kilowatt transformer fed by an alternating current of 220 volts. It is capable of giving a potential as great as 64,000 volts. For most of this work either 16,000 or 32,000 volts have been employed. A self-induction coil placed in the secondary circuit has been used for a part of the work for the purpose of reducing the intensities of the enhanced lines. It may be well to state at this point that we have been able to find no distinct evidence of the variation of pressure displacements with change of current, capacity, potential, or arc or spark-gaps for the range throughout which we have worked. Nor does there appear to be any difference between the results for an atmosphere of air or of carbon dioxide. The effect of a hydrogen atmosphere is discussed at a later point.

A considerable number of photographs of an arc between carbon poles was taken, and the displacements of the lines of iron, which occurred as an impurity, were found to be the same as those obtained from an arc with iron poles. Table I contains the results from arcs with iron and with carbon poles.

On all of the photographs the spectrum under pressure is placed in the center with the comparison spectrum of arc or spark at atmospheric pressure on either side, a simple occulting-bar arrangement providing for covering and uncovering the different portions of the slit. The exposure for the comparison spectrum has been divided into two parts, one-half before and one-half after the exposure for the pressure spectrum. In order to test the possible effect upon the photographs of accidental disturbances such as the vibration of the building, we have made a number of exposures upon an arc spectrum separated by considerable intervals of time, but measures of the photographs have shown no appreciable displacements since the spectrograph was definitely adjusted for this work. In general the exposure times, both for the comparison and the pressure spectrum, have been comparatively short. For the arc spectrum in the blue and violet regions they have varied from 1 to 2 minutes for the pressure spectrum to 1 to 6 minutes for the comparison spectrum on Seed "Process" plates. For the less refrangible regions the exposure times are, of course, longer, reaching in the case of the comparison spectrum a maximum of some 40 minutes in the red. The spark spectrum requires considerably more time, and in the red we have been obliged to give as much as $1\frac{1}{2}$ hours to the comparison spectrum of iron.

The great amount of labor involved in the measurement and reduction of the photographs has made it impossible to determine the values of the displacements of all of the lines at all of the pressures employed. Accordingly we have adopted the plan of making as complete a study as possible of all well-measurable lines at a single pressure, and for the other pressures using a selected list sufficient in number to give a mean of high precision for purposes of comparison. The pressure adopted for the principal series was 8 atmospheres above atmospheric pressure, a value sufficiently high to give displacements of considerable amount, but not too high to prevent direct comparison with the results for solar lines. The other pressures used have been 2, 4, 6, 12, and 16 atmospheres above atmospheric pressure and a few photographs have been taken at a partial vacuum. As already stated, no appreciable difference could be detected between the displacements for an atmosphere of air or

carbonic dioxide, and the latter gas has been used throughout the greater part of the investigation.

The extraordinary variety in behavior of different spectrum lines under pressure has, of course, been noted by all observers. In general the lines may be divided roughly into five broad classes:

1. Lines which are symmetrically reversed.
2. Lines which are unsymmetrically reversed.
3. Lines which remain bright and fairly narrow under pressure.
4. Lines which remain bright and symmetrical but become wide and diffuse under pressure.
5. Lines which remain bright and are widened very unsymmetrically toward the red.

The lines in the first class are those which are most readily measurable and for which the degree of precision is highest. They are much more numerous in the arc spectrum of titanium than in that of iron, and in general they are likely to be among the strongest of the lines in the spectrum. The number of lines in the second class is smaller than that in the first and the reversals as a rule are fainter. Most of the enhanced lines in the spark spectrum of titanium belong to this class.

The lines in the third and fourth classes form a majority of all of the lines measured, particularly in the less refrangible part of the spectrum where reversals are comparatively few. The lines in the third class usually are well measurable, those in the fourth less so. The distinction between these two classes is entirely arbitrary as all lines are widened somewhat under pressure.

The fifth class is of great interest. Almost all of the lines belonging to it occur in the yellow and red portions of the spectrum, and all show enormous displacements. The widening of these lines and their lack of symmetry are so great, however, that many of them are practically incapable of measurement, and for the others the degree of accuracy is extremely low. The measures when made are upon the maximum, which is always upon the violet side of the center of the line.

Most of the characteristics of the lines in the arc under pressure apply equally well to the same lines in the spark. All lines in the spark spectrum are much more diffuse, however, and the degree of

accuracy of measurement is distinctly lower. This is especially true of the enhanced lines of titanium, which as a rule show very faint unsymmetrical reversals, which it is extremely difficult to measure with precision.

In the tables which follow are given the results of our measures of the displacements of the principal lines in the spectra of the iron arc and the titanium arc and spark for a total pressure of 9 atmospheres, the comparison spectrum being in each case the same source at atmospheric pressure.

The wave-length of the line according to Rowland's table is given in the first column; its type as regards reversal and symmetry, referred to one of the five classes above, in the second column; its mean displacement in the fourth column; the mean deviation of the separate determinations from the mean in the fifth column, the unit being 0.001 Ångström; and the number of plates in the sixth column. In the discussion of these displacements with relation to their variation with wave-length the lines are treated in different groups. In order to save repetition at a later point the group under which each line is treated is given in the third column. The seventh, eighth, and ninth columns in the table for titanium give the displacements of the corresponding lines in the spark spectrum, the average deviations of the separate determinations from the mean, and the number of plates measured. A few lines not given by Rowland are taken from the lists of Hasselberg[1] and Evans.[2]

In general each plate was measured four times, once in each direction, by two different observers.

Although we shall not attempt in this place to enter into any extended discussion of the characteristics of the various lines under pressure, there are a few general results evident from an inspection of these tables to which reference may be made.

1. Reversal is clearly a function of wave-length for both arc and spark. The number of reversed lines falls off rapidly toward longer wave-lengths, there being extremely few reversals in the red portion of the spectrum, for either iron or titanium. The general law was

[1] *Astrophysical Journal* 4, 212, 1896.

[2] *Ibid.*, 29, 160, 1909.

TABLE I

Iron

λ	Class	Group	Δ	Mean Deviation $\pm$	No. of Plates	λ	Class	Group	Δ	Mean Deviation $\pm$	No. of Plates
3609.008	1	*b*	0.008	1	3	4202.198	1	*b*	0.025	0	6
3618.919	1	*b*	0.011	1	3	4204.101	3	*b*	0.017	1	2
3631.605	1	*b*	0.015	1	3	4210.404	5	*c*	0.053	2	2
3647.988	1	*b*	0.012	4	3	4216.351	3	*b*	0.015	2	2
3680.069	1	*a*	0.007	2	3	4227.606	5	*d*	0.11	10	6
3687.610	1	*b*	0.016	1	3	4233.772	5	*d*	0.09	20	7
3705.708	1	*a*	0.007	2	3	4236.112	5	*d*	0.09	20	8
3709.389	1	*b*	0.013	1	3	4250.287	5	*c*	0.070	4	2
3720.084	1	*a*	0.014	1	3	4250.945	2	*b*	0.022	4	10
3722.729	1	*a*	0.010	2	3	4260.640	2	*c*	0.051	8	8
3727.778	1	*b*	0.014	2	3	4271.934	1	*b*	0.022	3	10
3733.469	1	*a*	0.007	1	3	4282.565	1	*b*	0.021	4	7
3735.014	1	*b*	0.013	2	3	4291.630	3	*a*	0.014	0	1
3737.281	1	*a*	0.016	2	3	4294.301	2	*b*	0.029	5	6
3743.508	1	*b*	0.015	1	3	4299.410	5	*d*	0.09	20	6
3745.717	1	*a*	0.008	1	3	4308.081	1	*b*	0.021	5	7
3746.058	1	*a*	0.008	1	3	4315.262	3	*b*	0.019	3	7
3748.408	1	*a*	0.010	1	3	4325.939	1	*b*	0.020	4	7
3749.631	1	*b*	0.017	2	3	4337.216	3	*b*	0.027	2	7
3758.375	1	*b*	0.022	2	3	4352.908	3	*b*	0.017	3	7
3763.945	1	*b*	0.017	1	3	4369.941	3	*b*	0.023	3	5
3765.689	1	*b*	0.013	2	3	4376.107	3	*a*	0.018	2	7
3767.341	1	*b*	0.017	1	3	4383.720	1	*b*	0.027	4	7
3788.046	1	*b*	0.016	1	3	4404.927	1	*b*	0.021	3	7
3795.147	1	*b*	0.015	2	3	4407.871	4	*c*	0.053	2	2
3815.987	1	*b*	0.024	2	3	4408.582	4	*c*	0.058	2	2
3826.027	1	*b*	0.021	4	4	4415.293	1	*b*	0.018	5	7
3827.980	1	*b*	0.019	2	4	4422.741	3	*b*	0.018	3	2
3834.364	1	*b*	0.016	5	4	4427.482	3	*a*	0.017	3	7
3886.434	1	*a*	0.011	3	10	4430.785	4	*c*	0.048	8	4
3887.196	1	*b*	0.022	4	5	4442.510	4	*c*	0.053	9	7
3888.671	1	*b*	0.020	3	6	4443.365	3	*b*	0.019	4	6
3895.803	1	*a*	0.011	4	7	4447.892	4	*c*	0.051	9	7
3899.850	1	*a*	0.012	5	10	4454.552	3	*b*	0.023	2	2
3903.090	1	*b*	0.022	2	6	4459.301	4	*c*	0.051	8	7
3906.628	1	*a*	0.011	4	7	4461.818	3	*a*	0.015	3	7
3920.410	1	*a*	0.010	3	8	4466.727	4	*b*	0.018	4	7
3923.054	1	*a*	0.011	3	9	4476.185	4	*b*	0.021	3	7
3928.075	1	*a*	0.012	4	9	4489.911	3	*a*	0.015	2	2
3930.450	1	*a*	0.013	3	8	4494.738	4	*c*	0.053	8	6
3956.819	4	*b*	0.014	5	5	4528.798	4	*c*	0.061	4	5
3969.413	1	*b*	0.022	3	8	4531.327	3	*b*	0.029	2	4
3977.891	4	*b*	0.017	4	3	4859.928	5	*c*	0.100	4	2
3997.547	4	*b*	0.015	3	4	4871.512	5	*c*	0.080	5	4
4005.408	1	*b*	0.019	3	9	4872.332	5	*c*	0.094	5	4
4045.975	1	*b*	0.023	4	5	4878.407	5	*c*	0.087	2	2
4063.759	1	*b*	0.020	5	5	4890.948	5	*c*	0.070	8	4
4071.908	1	*b*	0.021	4	5	4891.683	5	*c*	0.052	1	4
4132.235	1	*b*	0.024	5	5	4919.174	5	*c*	0.072	3	4
4134.840	4	*b*	0.027	2	2	4920.685	5	*c*	0.082	5	4
4144.038	1	*b*	0.029	6	6	4957.480	5	*c*	0.083	5	4

TABLE I—*Continued*

λ	Class	Group	Δ	Mean Deviation ±	No. of Plates	λ	Class	Group	Δ	Mean Deviation ±	No. of Plates
4957.785	5	*c*	0.086	3	3	5659.052	5	*d*	0.15	10	2
5227.362	4	*a*	0.031	5	6	5975.575	4	*b*	0.054	7	2
5233.122	5	*d*	0.11	20	4	6027.274	4	*b*	0.062	6	3
5266.738	5	*d*	0.13	30	2	6065.709	4	*b*	0.077	5	4
5269.723	1	*a*	0.027	3	6	6136.829	4	*b*	0.082	7	4
5270.558	4	*a*	0.029	3	5	6137.915	4	*b*	0.078	3	4
5324.373	5	*d*	0.12	20	5	6157.945	4	*b*	0.041	8	3
5328.236	1	*a*	0.029	3	6	6173.553	4	*b*	0.067	5	2
5328.696	4	*a*	0.026	3	3	6191.779	4	*b*	0.086	4	4
5333.089	4	*a*	0.029	3	2	6200.527	4	*b*	0.079	10	2
5340.121	5	*d*	0.14	20	3	6213.444	4	*b*	0.072	4	3
5341.213	4	*a*	0.028	1	3	6219.494	4	*b*	0.073	2	3
5371.734	1	*a*	0.029	2	6	6230.943	4	*b*	0.070	5	4
5393.375	5	*d*	0.14	10	3	6246.535	5	*d*	0.28	10	3
5397.344	4	*a*	0.029	2	6	6252.773	4	*b*	0.077	3	6
5405.989	4	*a*	0.027	3	6	6254.456	4	*b*	0.064	4	5
5429.911	4	*a*	0.029	2	6	6256.572	4	*b*	0.089	5	4
5434.740	4	*a*	0.027	2	6	6265.348	4	*b*	0.070	4	5
5447.130	4	*a*	0.031	3	6	6298.007	4	*b*	0.068	2	1
5455.834	4	*a*	0.029	1	5	6301.718	5	*d*	0.25	30	2
5476.500	4	*a*	0.029	2	3	6318.239	4	*b*	0.080	4	2
5476.778	5	*d*	0.11	10	2	6335.554	4	*b*	0.074	3	2
5497.735	3	*a*	0.030	2	3	6337.048	5	*d*	0.26	10	2
5501.683	3	*a*	0.030	3	3	6393.820	4	*b*	0.072	7	2
5507.000	3	*a*	0.031	4	3	6400.217	5	*d*	0.24	10	2
5535.644	4	*a*	0.034	4	3	6411.865	5	*d*	0.23	10	2
5569.848	5	*d*	0.14	10	5	6421.570	4	*b*	0.068	2	2
5573.075	5	*d*	0.14	10	5	6431.066	4	*b*	0.068	4	2
5576.320	5	*d*	0.16	10	2	6495.213	4	*b*	0.065	4	2
5586.991	5	*d*	0.12	2	2	6546.479	4	*b*	0.073	3	2
5603.186	5	*d*	0.15	10	2	6593.161	4	*b*	0.076	6	2
5615.877	5	*d*	0.13	20	2	6594.121	4	*b*	0.070	2	2
5624.769	5	*d*	0.16	10	2	6678.235	4	*b*	0.086	1	3
5638.488	5	*d*	0.15	10	2						

stated by Hale in 1902[1] for the spark under pressure, but Duffield's[2] observations upon the iron arc between λ 4000 and λ 4500 did not appear to indicate the same effect for this source. Apparently the discrepancy is due to the comparatively limited region of spectrum investigated by Duffield.

2. The number of unsymmetrical bright lines increases toward longer wave-lengths. This is true particularly of the high-temperature lines of iron, the low-temperature lines (those relatively

[1] *Astrophysical Journal*, **15**, 227, 1902.

[2] *Phil. Trans.*, A **208**, 111, 1908.

TABLE II

TITANIUM

λ	Class	Group	Arc Δ	Arc Mean Deviation ±	Arc No. of Plates	Spark Δ	Spark Mean Deviation ±	Spark No. of Plates	
3729.952	1	*a*	0.009	1	3	.....	..	..	
3741.205	1	..	0.011	1	3	.....	..	..	
3753.003	1	*a*	0.014	4	3	.....	..	..	
3753.732	1	*a*	0.006	2	2	.....	..	..	
3759.447	1	..	0.011	4	4	.....	..	..	Enhanced line
3761.464	1	..	0.008	1	4	.....	..	..	Enhanced line
3771.798	1	*a*	0.007	3	3	.....	..	..	
3898.645	3	*a*	0.014	3	4	0.019	1	3	
3900.681	3	..	0.034	12	5	0.041	5	4	Enhanced line
3901.114	2	*a*	0.010	4	5	0.019	4	2	
3904.926	1	*a*	0.019	4	7	0.023	3	4	
3913.609	3	..	0.036	4	6	0.037	3	4	Enhanced line
3914.477	1	*a*	0.006	2	6	0.017	2	4	
3921.563	1	*a*	0.005	2	6	0.013	4	4	
3924.673	1	*a*	0.010	2	6	0.020	4	4	
3926.465	4	..	0.048	5	5	0.042	2	2	
3930.022	1	*a*	0.008	3	6	0.018	3	4	
3947.918	1	*a*	0.004	2	6	0.014	4	4	
3948.818	1	*a*	0.013	2	7	0.021	6	4	
3956.476	1	*a*	0.010	2	6	0.021	4	4	
3958.355	1	*a*	0.015	3	7	0.021	1	3	
3962.995	1	*a*	0.010	4	6	0.021	1	4	
3964.416	1	*a*	0.010	3	6	0.022	7	4	
3981.917	1	*a*	0.016	4	8	0.022	3	3	
3982.630	1	*a*	0.006	2	6	0.020	2	4	
3989.912	1	*a*	0.016	3	9	0.016	7	3	
3998.790	1	*a*	0.016	5	9	0.015	3	3	
4009.079	1	*a*	0.008	2	5	0.019	1	4	
4009.807	2	*a*	0.009	2	5	0.020	2	4	
4021.893	4	..	0.051	2	3	.....	..	..	
4024.726	1	*a*	0.008	1	4	0.021	1	4	
4028.497	3	..	0.018	2	2	.....	..	..	Enhanced line
4060.415	1	*a*	0.015	2	4	0.025	4	4	
4065.239	1	*a*	0.011	2	4	0.023	2	4	
4078.631	1	*a*	0.005	1	4	0.017	2	4	
4082.589	1	*a*	0.019	4	4	0.025	3	4	
4099.327	4	*a*	0.028	3	4	0.025	2	2	
4112.869	1	*a*	0.014	3	4	0.015	1	4	
4151.129	4	*a*	0.034	7	4	0.031	2	3	
4159.805	4	*a*	0.035	4	4	0.025	1	2	
4163.818	4	..	0.041	2	4	.....	..	..	Enhanced line
4164.27	4	*a*	0.031	9	4	.....	..	..	
4166.45	4	*a*	0.030	3	3	.....	..	..	
4169.46	4	*a*	0.034	1	3	.....	..	..	
4171.213	4	*a*	0.035	3	4	0.044	0	2	
4172.066	4	..	0.042	2	3	.....	..	..	Enhanced line
4174.61	4	*a*	0.035	7	3	.....	..	..	
4183.45	4	*a*	0.040	1	3	.....	..	..	
4186.280	1	*a*	0.016	3	5	0.030	1	4	

TABLE II—*Continued*

λ	Class	Group	Arc Δ	Arc Mean Deviation ±	Arc No. of Plates	Spark Δ	Spark Mean Deviation ±	Spark No. of Plates	
4188.84	4	*a*	0.044	2	4	.....	..	..	
4200.946	4	*a*	0.039	4	3	.....	..	..	
4203.620	4	*a*	0.038	2	4	0.030	1	2	
4224.792	4	..	0.064	2	3	.....	..	..	
4227.822	4	..	0.073	1	3	.....	..	..	
4238.00	4	*a*	0.014	1	4	.....	..	..	
4263.290	4	*a*	0.026	2	2	.....	..	..	
4265.832	4	*a*	0.037	1	2	.....	..	..	
4272.701	3	*a*	0.016	2	4	.....	..	..	
4276.587	4	*a*	0.030	3	4	.....	..	..	
4278.390	4	*a*	0.043	3	4	.....	..	..	
4281.530	4	*a*	0.014	3	4	.....	..	..	
4282.860	4	*a*	0.027	2	4	.....	..	..	
4285.164	4	*a*	0.036	3	4	.....	..	..	
4286.168	1	*a*	0.021	2	4	.....	..	..	
4287.566	1	*a*	0.024	2	4	.....	..	..	
4288.310	3	*a*	0.019	1	4	.....	..	..	
4289.237	1	*a*	0.025	3	4	.....	..	..	
4290.080	3	*a*	0.033	1	4	.....	..	..	
4290.377	3	..	0.046	2	5	0.048	9	6	Enhanced line
4291.114	1	*a*	0.022	2	5	0.016	2	6	
4294.204	3	..	0.029	1	5	0.039	6	6	Enhanced line
4295.914	1	*a*	0.022	2	5	0.015	3	5	
4298.828	1	*a*	0.025	2	4	0.019	2	6	
4299.410	1	*a*	0.023	3	4	0.034	4	6	
4299.803	1	*a*	0.023	2	4	0.024	3	6	
4300.211	3	..	0.029	4	4	0.051	4	6	Enhanced line
4300.732	1	*a*	0.021	2	4	0.018	3	6	
4301.158	1	*a*	0.024	1	4	0.025	2	6	
4302.085	3	..	0.033	2	2	.....	..	..	Enhanced line
4306.078	1	*a*	0.024	2	4	0.027	4	6	
4308.64	3	*a*	0.019	6	3	.....	..	..	
4313.034	3	..	0.047	4	3	.....	..	..	Enhanced line
4314.479	3	*a*	0.018	3	3	.....	..	..	
4314.964	1	..	0.032	4	3	.....	..	..	
4326.520	3	*a*	0.029	2	4	0.025	3	6	
4338.084	3		0.017	3	4	0.040	8	6	Enhanced line
4346.26	3	*a*	0.006	2	3	.....	..	..	
4360.644	3	*a*	0.042	4	3	.....	..	..	
4394.093	4	*a*	0.015	3	3	.....	..	..	
4395.201	3	..	0.025	2	3	.....	..	..	Enhanced line
4399.935	3		.....	..	..	.....	..	..	
4417.450	3	*a*	0.027	4	4	0.033	3	6	
4421.928	4	..	0.038	8	3	.....	..	..	Enhanced line
4422.985	4	*a*	0.028	4	3	.....	..	..	
4426.201	4	*a*	0.026	5	3	.....	..	..	
4427.266	1	*a*	0.017	3	7	0.026	1	7	
4434.168	4	*a*	0.035	4	3	.....	..	..	
4440.515	4	*a*	0.029	5	4	0.027	4	6	
4443.976	4	..	0.021	2	4	0.040	4	6	Enhanced line
4449.313	1	*a*	0.029	4	8	0.033	3	9	

TABLE II—*Continued*

λ	Class	Group	Arc			Spark			
			Δ	Mean Deviation ±	No. of Plates	Δ	Mean Deviation ±	No. of Plates	
4451.087	1	*a*	0.029	2	9	0.037	3	3	
4453.486	1	*a*	0.040	4	10	0.045	3	9	
4453.876	1	*a*	0.026	5	5	0.042	2	2	
4455.485	1	*a*	0.041	2	9	0.048	2	3	
4457.600	1	*a*	0.039	3	10	0.044	4	9	
4465.975	1	*a*	0.025	2	10	0.033	4	9	
4468.663	4	..	0.046	4	6	0.059	7	8	Enhanced line
4471.408	1	*a*	0.024	4	10	0.029	4	9	
4475.026	4	..	0.074	4	5	.....	..	..	
4479.879	4	*a*	0.027	2	5	0.025	1	2	
4480.752	3	*a*	0.027	4	5	0.032	4	2	
4481.438	1	*a*	0.023	2	10	0.031	3	9	
4488.493	..	..	.....	..	..	0.054	7	2	Enhanced line
4489.262	1	*a*	0.029	5	6	0.033	4	8	
4501.445	4	..	0.046	4	6	0.063	6	8	Enhanced line
4512.906	1	*a*	0.029	2	10	0.039	4	9	
4518.198	1	*a*	0.029	2	10	0.039	3	9	
4518.866	3	*a*	0.025	5	5	0.029	1	2	
4522.974	1	*a*	0.031	3	10	0.040	4	9	
4527.490	1	*a*	0.029	3	10	0.041	4	9	
4533.419	1	*a*	0.031	2	10	0.036	3	9	
4534.139	3	..	0.044	5	6	0.063	3	7	Enhanced line
4534.953	1	*a*	0.034	3	10	0.036	5	9	
4535.741	1	*a*	0.029	3	9	0.031	3	3	
4536.094	1	*a*	0.023	2	4	0.036	2	2	
4536.222	1	*a*	0.031	1	2	.....	.	.	
4544.864	1	*a*	0.031	2	10	0.035	2	9	
4548.938	1	*a*	0.031	2	10	0.039	6	9	
4549.808	4	..	0.048	2	6	0.062	8	8	Enhanced line
4552.632	1	*a*	0.029	3	10	0.041	3	9	
4555.662	1	*a*	0.029	2	8	0.041	3	9	
4562.814	3	*a*	0.008	1	5	0.026	5	2	
4563.939	4	..	0.034	4	6	0.053	6	7	Enhanced line
4572.156	4	..	0.051	5	6	0.065	4	8	Enhanced line
4590.126	..	..	.....	..	..	0.052	5	2	Enhanced line
4617.452	1	*a*	0.029	2	5	0.038	1	2	
4623.279	1	*a*	0.027	3	5	0.040	4	2	
4629.521	1	*a*	0.037	4	5	0.042	4	2	
4638.050	4	*a*	0.043	3	3	.			
4645.368	1	*a*	0.039	6	3	0.043	2	2	
4650.193	4	*a*	0.037	4	3	0.040	4	2	
4656.644	1	*a*	0.017	2	3	0.027	2	2	
4667.768	1	*a*	0.020	4	3	0.023	1	2	
4675.294	4	*a*	0.030	1	3	0.051	2	2	
4682.088	1	*a*	0.018	2	3	0.018	1	2	
4691.523	1	*a*	0.038	2	3	.			
4698.946	2	*a*	0.037	2	3	0.036	2	2	
4710.368	1	*a*	0.041	4	3	0.046	7	2	
4722.797	4	*a*	0.036	4	3	..			
4723.359	4	*a*	0.040	2	3	..			
4742.979	1	*a*	0.037	4	3	0.040	2	2	

TABLE II—*Continued*

λ	Class	Group	Arc			Spark			
			Δ	Mean Deviation ±	No. of Plates	Δ	Mean Deviation ±	No. of Plates	
4758.308	1	*a*	0.027	7	5	0.030	0	2	
4759.463	1	*a*	0.031	4	5	0.036	2	2	
4766.48	3	*a*	0.036	2	3	.....	..	..	
4778.441	4	*a*	0.049	4	3	.....	..	..	
4781.91	3	*a*	0.034	1	3	.....	..	..	
4799.984	4	*a*	0.038	6	3	0.044	4	2	
4805.285	3	..	0.048	2	3	.....	..	..	Enhanced line
4805.606	4	*a*	0.074	4	6	.....	..	..	
4820.593	1	*a*	0.033	6	9	0.041	1	3	
4836.313	4	*a*	0.051	7	7	.....	..	..	
4841.074	1	*a*	0.007	2	10	0.018	2	3	
4848.605	4	*a*	0.029	1	3	.....	..	..	
4856.203	1	*a*	0.029	2	6	.....	..	..	
4868.451	1	*a*	0.026	4	4	.....	..	..	
4870.323	1	*a*	0.029	4	5	.....	..	..	
4885.264	1	*a*	0.026	4	7	.....	..	..	
4900.095	1	*a*	0.018	4	4	.....	..	..	
4913.803	1	*a*	0.022	6	4	.....	..	..	
4915.414	3	*a*	0.021	4	4	.....	..	..	
4919.99	4	*a*	0.029	4	4	.....	..	..	
4921.963	4	*a*	0.029	2	4	.....	..	..	
4928.511	4	*a*	0.032	4	4	.....	..	..	
4975.530	4	*a*	0.028	1	4	.....	..	..	
4981.912	1	*a*	0.025	5	7	.....	..	..	
4991.247	1	*a*	0.029	1	7	.....	..	..	
4997.283	3	*a*	0.013	1	4	.....	...	..	
4999.689	1	*a*	0.028	2	7	.....	..	..	
5007.398	1	*a*	0.027	2	7	.....	..	..	
5009.829	3	*a*	0.017	1	4	.....	..	..	
5014.369	2	*a*	0.021	6	4	.....	..	..	
5016.340	1	*a*	0.027	4	4	.....	..	..	
5020.208	1	*a*	0.029	5	4	.....	..	..	
5023.052	1	*a*	0.028	2	5	.....	..	..	
5025.027	1	*a*	0.027	2	4	.....	..	..	
5036.089	1	*a*	0.045	4	4	.....	..	..	
5036.645	1	*a*	0.042	3	4	.....	..	..	
5038.579	1	*a*	0.048	5	4	.....	..	..	
5040.138	1	*a*	0.005	2	4	.....	..	..	
5040.787	3	*a*	0.034	2	4	.....	..	..	
5043.761	3	*a*	0.032	4	4	.....	..	..	
5045.582	3	*a*	0.029	3	4	.....	..	..	
5053.056	4	*a*	0.035	4	4	.....	..	..	
5062.285	4	*a*	0.034	5	4	.....	..	..	
5064.836	1	*a*	0.012	1	4	.....	..	..	
5066.12	4	..	0.069	5	3	.....	..	..	
5071.666	4	*a*	0.080	5	4	.....	..	..	
5087.239	4	*a*	0.032	5	4	.....	..	..	
5100.601	3	*a*	0.032	2	2	.....	..	..	
5113.617	4	*a*	0.032	8	4	0.026	7	2	
5120.592	4	*a*	0.007	1	3	.....	..	..	
5145.636	4	*a*	0.029	7	4	.....	..	..	

TABLE II—*Continued*

λ	Class	Group	Arc Δ	Arc Mean Deviation ±	Arc No. of Plates	Spark Δ	Spark Mean Deviation ±	Spark No. of Plates	
5147.652	4	*a*	0.017	1	4	0.022	2	2	
5152.361	4	*a*	0.018	2	4	0.024	2	2	
5173.917	1	*a*	0.023	2	2	0.021	1	2	
5193.139	1	*a*	0.019	4	7	0.022	3	5	
5210.555	1	*a*	0.015	3	6	0.016	2	4	
5212.503	4	*a*	0.047	2	3	.....	..	..	
5219.875	3	*a*	0.025	4	3	0.037	2	2	
5238.742	3	*a*	0.048	2	3	0.041	2	3	
5246.30	4	*a*	0.018	2	3	.....	..	..	
5246.733	4	*a*	0.033	2	2	.....	..	..	
5251.085	4	*a*	0.027	6	3	.....	..	..	
5252.276	3	*a*	0.023	2	3	0.031	2	3	
5255.973	5	..	0.29	50	3	.....	..	..	
5260.142	5	..	0.33	30	2	.....	..	..	
5263.669	5	..	0.26	110	3	.....	..	..	
5266.141	5	..	0.17	20	2	.....	..	..	
5282.576	3	*a*	0.028	5	3	0.042	1	3	
5283.613	5	..	0.15	20	3	.....	..	..	
5284.601	3	*a*	0.035	9	3	.....	..	..	
5289.02	4	*a*	0.042	5	2	.....	..	..	
5295.955	4	*a*	0.027	3	3	0.048	2	3	
5297.407	5	..	0.13	5	3	.....	..	..	
5298.672	5	..	0.088	5	3	.....	..	..	
5300.152	3	*a*	0.030	6	3	.....	..	..	
5313.422	3	*a*	0.035	1	2	.....	..	..	Probably *Ti*
5338.517	3	*a*	0.043	5	2	.....	..	..	
5341.77	4	*a*	0.068	4	2	.....	..	..	
5351.261	5	..	0.27	20	3	.....	..	..	
5366.827	3	*a*	0.032	1	3	0.043	2	3	
5369.782	4	*a*	0.027	2	3	0.030	3	3	
5389.371	3	*a*	0.029	3	3	.....	..	..	
5390.203	4	*a*	0.029	2	3	0.052	2	3	
5396.778	3	*a*	0.018	2	3	0.053	..	3	
5404.25	4	*a*	0.057	6	3	.....	..	..	
5409.81	4	*a*	0.015	1	2	0.042	1	3	
5419.42	4	*a*	0.047	6	3	.....	..	..	Probably *Ti*
5426.474	3	*a*	0.029	9	3	0.044	1	3	
5429.349	4	*a*	0.044	1	3	.....	..	..	
5436.938	3	*a*	0.046	11	3	0.043	2	3	
5438.507	3	*a*	0.053	6	2	.....	..	..	
5446.797	3	*a*	0.027	7	3	.....	..	..	
5453.860	4	*a*	0.051	4	3	0.046	3	3	
5460.721	3	*a*	0.019	5	3	0.035	1	3	
5471.414	4	*a*	0.037	2	3	0.045	1	3	
5472.90	4	*a*	0.030	1	3	.....	..	..	
5474.436	3	*a*	0.029	2	3	.....	..	..	
5482.078	3	*a*	0.028	3	3	....	..	..	
5490.367	4	*a*	0.038	5	3	.....	..	..	
5504.117	4	*a*	0.074	4	3	.....	..	..	
5512.013	3	*a*	0.021	1	3	.....	..	..	
5512.741	1	*a*	0.044	6	3	.....	..	..	

TABLE II—*Continued*

λ	Class	Group	Arc			Spark			
			Δ	Mean Deviation ±	No. of Plates	Δ	Mean Deviation ±	No. of Plates	
5514.563	1	*a*	0.048	2	2	.....	..	..	
5514.753	1	*a*	0.043	2	2	.....	..	..	
5565.700	4	*a*	0.049	6	8	.....	..	..	
5597.90	3	..	0.000	0	5	.....	..	..	Fluting line
5644.365	4	*a*	0.033	1	5	.....	..	..	
5648.796	5	..	0.29	40	3	.....	..	..	
5662.374	5	..	0.23	3	3	.....	..	..	
5675.647	5	..	0.25	20	3	.....	..	..	
5689.694	5	..	0.25	10	3	.....	..	..	
5702.876	5	..	0.25	20	3	.....	..	..	
5708.46	5	..	0.24	10	4	.....	..	..	
5712.07	5	..	0.25	20	4	.....	..	..	
5715.308	4	*a*	0.048	3	5	.....	..	..	
5727.271	3	*a*	0.058	4	3	.....	..	..	
5739.698	4	*a*	0.042	2	5	.....	..	..	
5740.195	3	*a*	0.029	2	5	.....	..	..	
5781.130	3	*a*	0.046	5	5	.....	..	..	
5823.910	3	*a*	0.032	1	4	.....	..	..	
5866.675	1	*a*	0.034	2	5	.....	..	..	
5880.55	3	*a*	0.029	3	5	.....	..	..	
5899.518	4	*a*	0.034	1	5	.....	..	..	
5903.555	3	*a*	0.025	2	5	.....	..	..	
5918.773	3	*a*	0.022	4	6	.....	..	..	
5922.334	4	*a*	0.048	4	3	.....	..	..	
5938.035	3	*a*	0.038	2	3	.....	..	..	
5941.985	4	*a*	0.050	4	3	.....	..	..	
5953.386	4	*a*	0.020	3	3	.....	..	..	
5966.055	4	*a*	0.036	4	3	.....	..	..	
5978.768	4	*a*	0.018	3	3	.....	..	..	
5996.11	3	*a*	0.028	1	3	.....	..	..	
5999.920	3	*a*	0.050	1	3	.....		.	
6064.853	3	*a*	0.024	4	3	.....		..	
6085.490	4	*a*	0.020	4	3	.....		..	
6091.395	4	*a*	0.048	2	3	.....	..	..	
6093.00	5		0.14	2	2	.	.	.	
6098.92	5	.	0.33	20	3	.....	.	.	
6121.24	5		0.16	2	2	.			
6126.435	4	*a*	0.018	4	3	.....	.	..	
6146.48	5	.	0.15	5	2	.....	..	..	
6150.02	3	*a*	0.049	4	3	.....	.	..	
6186.65	3		0.000	0	3	.....	..	..	Fluting line
6258.322	1	*a*	0.071	7	6	0.065	8	3	
6258.927	1	*a*	0.066	8	5	0.061	1	3	
6261.316	1	*a*	0.070	10	5	0.065	6	3	
6303.985	4	*a*	0.048	3	2	0.076	4	3	
6336.329	3	*a*	0.029	7	2	0.042	3	3	
6366.564	3	*a*	0.032	6	2	0.038	2	3	
6497.92	3	*a*	0.056	5	2	0.061	5	3	
6508.37	3	*a*	0.057	2	2	0.049	6	3	
6546.470	4	*a*	0.049	6	2	0.061	1	3	
6554.470	4	*a*	0.034	6	2	0.060	14	3	

TABLE II—*Continued*

λ	Class	Group	Arc Δ	Arc Mean Deviation ±	Arc No. of Plates	Spark Δ	Spark Mean Deviation ±	Spark No. of Plates
6556.308	4	*a*	0.049	0	2	0.059	2	3
6599.353	3	*a*	0.035	15	2	0.052	1	3
6719.90	4	*a*	0.058	3	2	.....		..
6861.770	4	*a*	0.084	2	2	.....	..	..

strengthened at low temperatures) being symmetrical and, though broad, fairly well defined as a rule.

3. Except in the ultra-violet the enhanced lines of titanium are always bright in the arc under a pressure of 9 atmospheres. In the titanium spark at the same pressure they are usually faintly reversed. This agrees with Duffield's hypothesis that conditions in the electric spark are more favorable to the production of reversals than they are in the arc.

4. The number of reversed lines at a pressure of 9 atmospheres is somewhat less than that found by Duffield[1] for pressures of 20 to 25 atmospheres. This agrees with the conclusion of Humphreys that in the case of iron, for moderate pressures at least, the number of reversals increases with the pressure.

Some additional characteristics of certain of the lines will be referred to in the course of the discussion of the displacements.

Since our work upon the iron spectrum has been limited to the arc, and all of the photographs have been taken at a pressure of 9 atmospheres, while the results for titanium include both arc and spark, and for some of the lines a variety of pressures as well, we shall discuss the displacements for the two elements separately.

IRON

In the following table we have compared our results with the most recent values obtained by Humphreys and by Duffield[2] for such lines as are common to all three sets of observers. To reduce to a value of 9 atmospheres total pressure, Duffield's values for

[1] *Phil. Trans.*, A **208**, 111, 1908.

[2] *Jahrbuch der Radioaktivität und Elektronik*, **5**, 324, 1908.

11 atmospheres are reduced by one-fifth, while Humphreys' values for 42 atmospheres are divided by the quantity 5.1. Series B in Duffield's investigation has been used, as this is considered preferable by the author.

λ	Humphreys	Duffield	Gale and Adams
4132.235	0.020	0.021	0.024
4144.038	.023	.021	.029
4202.198	.014	.017	.025
4233.772	.047	.144	.090
4250.945	.017	.018	.022
4260.640	.048	.051	.051
4271.934	.016	.017	.022
4282.565	.009	.018	.021
4294.301	.017	.019	.029
4308.081	.018	.019	.021
4315.262	.007	.013	.019
4325.939	.019	.026	.020
4337.216	.018	.034	.027
4352.908	.010	.020	.017
4369.941	.011	.026	.023
4376.107	.008	.017	.018
4404.927	.022	.043	.021
4415.293	.017	.033	.018
4422.741	.013	.022	.018
4427.482	.011	.018	.017
4430.785	.037	.057	.048
4442.510	.037	.057	.053
4443.365	.012	.022	.019
4447.892	.035	.062	.051
4454.552	.016	.018	.023
4459.301	.031	.063	.051
4461.818	.012	.018	.015
4466.727	.011	.021	.018
4476.185	.014	.021	.021
4494.738	.040	.058	.053
4531.327	0.015	0.026	0.029

If we omit the line λ 4233.772, which is unsymmetrical and extremely difficult of measurement, a comparison of the remaining 30 lines gives the following mean values:

Humphreys	0.019
Duffield	0.029
Gale and Adams	0.027

The agreement of our results with those of Duffield is close, but the average value found for the observations of Humphreys is rather surprisingly small. This may be due in part to the fact that the linear relationship between displacement and pressure fails to

hold at high pressures, although this seems improbable in view of the very close accordance found by other observers for severa elements and shown by our own measures upon the titanium spectrum. A few of these lines measured by Humphreys in a previous investigation at lower pressures give distinctly larger values for the displacements, and these agree closely with those obtained by Duffield and ourselves.

In any discussion of the effect of pressure upon the lines of the spectrum there is a natural tendency to institute a comparison with the other two causes which are known to affect different lines of a spectrum differently. The first of these is variation of temperature; the second is the presence of a magnetic field.

The lines of iron which are strengthened relatively at low temperatures and which are usually referred to as "flame" lines are well known.[1] The most prominent of these lines for which we have determined the pressure displacements are the following:

λ	Δ	λ	Δ
3680.069	0.007	4427.482	0.017
3705.708	.007	4461.818	.015
3720.084	.014	4489.911	.015
3722.600	.010	5227.362	.031
3733.469	.007	5269.723	.027
3737.281	.016	5328.236	.029
3745.717	.008	5333.089	.029
3746.058	.008	5341.213	.028
3748.408	.010	5371.734	.029
3886.434	.011	5397.344	.029
3895.803	.011	5405.989	.027
3899.850	.012	5429.911	.029
3906.628	.011	5434.740	.027
3920.410	.010	5447.130	.031
3923.054	.011	5455.834	.029
3928.075	.012	5476.778	029
3930.450	.013	5497.735	.030
4291.630	.014	5501.685	.030
4376.107	0.018	5507.000	0.031

With the exception of the two lines λ 4291.630 and λ 4489.911, which are faint, all of these lines are suitable for measurement, being fairly sharp under pressure, and the precision of the results

[1] De Watteville, *Phil. Trans.*, A **204**, 139, 1904; Adams, *Contributions from the Mount Wilson Solar Observatory*, No. 40; *Astrophysical Journal*, **30**, 86, 1909.

is high, especially for the more refrangible lines. Except in the violet region there are no reversed lines among them.

It is apparent at a glance that the displacements given by these lines are decidedly smaller than those of the other lines in the same region. The average displacement of the 17 flame lines in the ultra-violet region is about 0.010 Ångström, while the average for 27 other lines in the same region is 0.015. In the green region the difference is even more striking, although here it is not possible to compare the flame lines directly with the other lines in the same region owing to the fact that the latter appear to belong to a totally distinct group as regards displacements, giving values about four times as great as the flame lines. We can, however, compute a value for this region from the value in the blue region, assuming the law of variation according to the cube of the wave-length which seems to hold closely for these lines. In this way we obtain 0.044 as against 0.029 for the flame lines. An interesting characteristic of the flame lines is the close agreement of the displacements of the different lines in the same part of the spectrum. This fact taken in connection with their behavior at reduced temperatures makes it very probable that they form members of a single group. An examination of the results obtained by King[1] for the electric-furnace spectra under pressure shows that the flame lines give distinctly lower displacements than the other lines in this source as well. It is worthy of note that in an investigation of the displacement of the spectrum lines at the sun's limb[2] it was found that the flame lines in general gave smaller values than the other lines of the same element.

A study of the relationship between displacements under pressure and separations in a magnetic field has been made by King[3] for a large number of iron lines and a considerable number of titanium and chromium lines. For the purpose of comparing the displacements and separations he has formed the quotients of all the lines for

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 53; *Astrophysical Journal*, 34, 37, 1911.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 43; *Astrophysical Journal*, 31, 30, 1910.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 46; *Astrophysical Journal*, 31, 433, 1910.

which the data are available, and from a comparison of these quotients has concluded that no direct relationship exists between the two quantities, although for the average of large groups of lines the effects are in the same direction.

It is hardly possible in this place to repeat the calculations with the use of the values of the displacements given in this investigation. A few results, however, which can readily be confirmed are as follows:

1. The use of the pressure displacements given here reduces greatly a considerable number of the largest differences found by King. As examples we may refer to λ 3920.410, λ 3923.054, λ 3928.075, λ 3977.891, λ 4315.262, and λ 4376.107, all of which are brought into much closer agreement with the other lines.

2. A majority of the serious discordances between pressure displacements and magnetic separations are in the cases of lines which are found to be complex in the magnetic field. Apart from the uncertainty of measurement of the separations of many of these lines, it is improbable on the basis of any of the theories of the Zeeman effect that there would be a direct relationship between separations and pressure displacements for such lines except perhaps for those of exactly the same type. We should expect rather that these lines would be greatly widened under pressure. It is interesting to note in this connection that nearly all of the strong flame lines in the yellow portion of the spectrum are complex in the magnetic field, while many of the other lines are triplets. The former lines show some of the most serious discrepancies between separation and displacement that are found in the entire list. Similarly in the violet part of the spectrum we may refer to the complex lines λ 4191.595 and λ 4233.722, which show large differences.

3. Since the pressure displacements for iron vary as the cube of the wave-length and the magnetic separations as the square, it is necessary in comparing the results to multiply the separations by the ratio of wave-lengths. This is found to bring the results in the less refrangible region of the spectrum into much closer agreement with those in the violet.

4. It seems probable from the fact that the lines of iron appear to divide themselves into groups, both as regards pressure displace-

ments and magnetic separations, that the ratios of the two quantities may by no means be the same for different groups, although constant within any given group for the same types of lines.

5. The significance of the agreement of very large and very small displacements and separations for certain lines seems to us of great importance. Of the former type we may mention the triplets λ 4210.494, λ 4407.871, λ 4430.785, and λ 4878.407. In the red we may refer to the triplet λ 6302.709, which is not measurable under pressure but has an immense displacement. The evidence from the behavior of such a group of lines as that between λ 4430 and λ 4461, for which displacements and separations unmistakably rise and fall together, is especially important.

6. The number of very marked discrepancies between size of displacement and separation among the triplets is not large, but a few of them are very important. A single most striking case is λ 6173.553 which gives an average pressure displacement but an exceedingly wide separation. It is a wide triplet in sun-spots. Another case in which the discordance is not so great is λ 4071.908.

7. Three lines which show no measurable separation in a magnetic field have been measured under pressure. Two of them, λ 3746.058 and λ 5434.740, give small displacements under pressure, while the third line, λ 3767.341, gives an average value. All three are rather narrow under pressure.

8. Attention should be called to the extreme sensitiveness of these comparisons to errors of observation. To illustrate: if we assume a magnetic separation of 0.350 Ångström for a given line and a pressure displacement of 0.020 Ångström, which is not far from an average value for the violet, we obtain the quotient 17.5. For the very best lines under pressure it is difficult to obtain probable errors of less than ± 0.002, which would produce a variation in the quotient of from 16 to 19. For many lines the probable error is several times this amount. Furthermore, our experience with titanium at different pressures indicates that the probable error increases almost in proportion to the pressure, so that the use of high pressures brings little gain on account of the deterioration of the lines. On the side of the magnetic separations errors of measurement also influence the comparisons, so that a very con-

siderable range in the results is to be expected from these sources. The type of separation also is very often uncertain, as is shown by the numerous changes in classification made by the same observer when different analyzing apparatus is used.

In view of these results it seems to us that the balance of evidence is somewhat in favor of a direct relationship between pressure displacements and separations in the magnetic field for lines which are of the same type. For lines of complex type and for lines which under pressure become extremely unsymmetrical the values are discordant, a result which is not surprising.

VARIATION OF DISPLACEMENT WITH WAVE-LENGTH

As soon as the measures of the displacements for all the iron lines were completed we plotted the results upon a large scale, using wave-lengths as abscissas and displacements as ordinates. A similar chart is shown in Fig. 1. The chart at once shows that certain groups of lines for which the average displacements differ widely apparently belong together. Furthermore, a comparison of the displacements of the flame lines with those of the other lines indicates clearly that such lines are to be considered as a separate group. In this way four groups have been found, and the lines included in each group are indicated in Table I by the letters *a*, *b*, *c*, and *d*. Group *a* includes all of the flame lines and two or three additional lines in the yellow portion of the spectrum, which may be flame lines but for which observations are lacking. Group *b* is a large one and includes all lines of small displacements which are not included in group *a*. It may well be complex in nature but in the absence of criteria for separating it into smaller groups we are obliged to consider it as a unit. Group *c* consists of lines showing much larger displacements than those of group *b*. It contains two fairly distinct clusters of lines, one in the violet and one in the blue green. The lines are all bright under pressure and especially in the green region unsymmetrical and difficult of measurement. Group *d* is made up of a very few lines in the violet, a fair-sized group in the greenish yellow, and a small number in the red, all of which show immense displacements. The lines are bright and widened enormously to the red, the wings extending sometimes from 5 to 10

Ångström units. The precision of measurement upon these lines necessarily is very low, in some cases the determination amounting to little more than an estimate of the maximum within the broad band.

The average values of the displacements and of the corresponding wave-lengths for the lines forming these groups provide us the

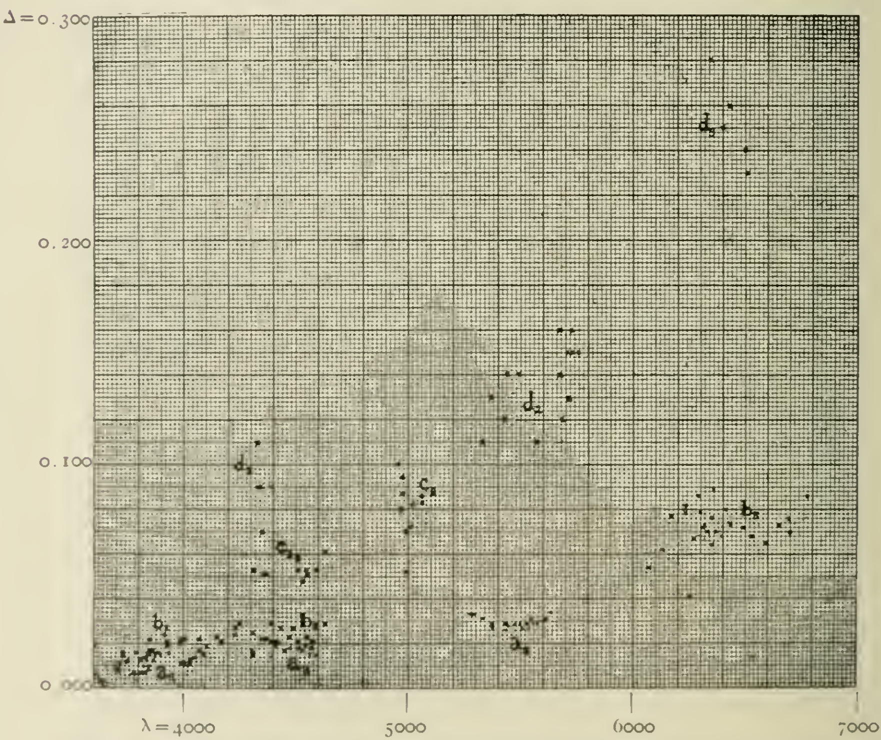

FIG. 1.—Displacement of lines of the iron arc at 8 atmospheres pressure.

material for a discussion of the variation of displacement with wave-length. In the following table the results are collected for the four groups. The second column gives the mean of the wave-lengths of the lines for which the mean displacement is formed. The third column contains the mean displacements expressed in Ångström units. The fourth column gives the number of lines for each portion of the group. It is evident from a simple inspection of the

results that the increase of displacement toward longer wave-lengths is much more than would be given by a law based on the first power of the wave-length. In the last two columns of the table are given the residuals for two hypotheses: first, that the displacement varies as the square of the wave-length; second, that it varies as the cube of the wave-length.[1] The reductions have been made by least squares, with weights assigned according to the number of lines. The fourth group is of course of very low weight compared with the others.

Group	Mean λ	Mean Δ	No. Lines	Residuals (Obs.—Comp.)	
				$\Delta=\left(\frac{\lambda}{\lambda_0}\right)^2 k$	$\Delta=\left(\frac{\lambda}{\lambda_0}\right)^3 k$
a.	3813	0.0105	17	−0.0032	+0.0002
	4409	.0158	5	−0.0025	−0.0001
	5398	.0292	19	+0.0017	−0.0001
b. . .	3791	.0164	27	−0.0072	+0.0007
	4287	.0219	29	−0.0083	−0.0008
	6292	.0719	27	+0.0069	+0.0001
c. . .	4395	.0547	11	−0.0058	−0.0020
	4902	.0803	10	+0.0051	+0.0016
d.	4249	.09	4	−0.01	+0.02
	5498	.14	15	−0.02	−0.01
	6339	0.25	5	+0.04	+0.01

These results are also shown graphically in Fig. 2. The points correspond to the observed, and the curves to the computed values, assuming the law of the cube of the wave-length.

It is clear from the results that the displacements for all of the groups of lines are represented with a surprisingly high degree of accuracy by a law which involves the third power of the wave-length, and in view of the great range of wave-length covered by the observations this may probably with safety be assumed to be true of the entire iron spectrum. In some recent work by Rossi upon the arc spectrum of vanadium,[2] he concludes from his observa-

[1] Strictly speaking the means of the second and third powers of the individual wave-lengths should be used instead of the second and third powers of the means of the wave-lengths, but the difference is negligible for the purposes of this comparison.

[2] *Astrophysical Journal*, 34, 21, 1911.

tions that "the displacement seems to be roughly proportional to the square or a higher power of the wave-length," and Duffield in his investigation of the iron spectrum found that the linear relation-

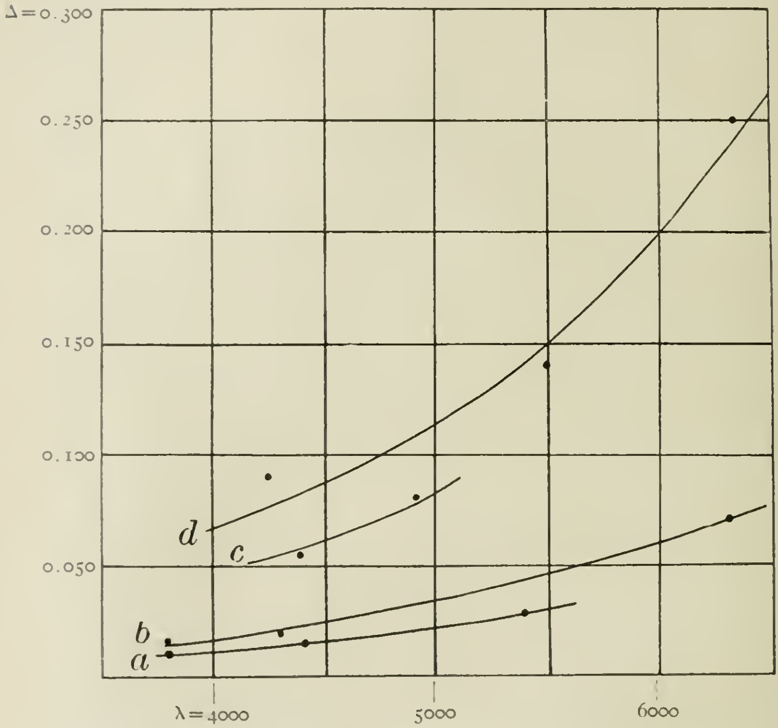

FIG. 2.—Mean displacement and mean wave-length of iron groups. Curves represent computed, points observed, values.

ship would not hold. Similarly Duffield in his discussion of the effect of pressure upon the arc spectra of silver and gold writes:[1]

It seems at least certain that the displacement varies with a higher power of the wave-length than the first upon which it was previously believed to depend. The results of the present investigation favor a dependence upon the third power of the wave-length, and this agrees with some theoretical deductions made by Dr. O. W. Richardson.[2]

From the results of his investigation of the iron spectrum Duffield concluded that there are three groups of lines for which

[1] *Phil. Trans.*, A **211**, 66, 1911. [2] *Phil. Mag.*, **14**, 557, 1907.

the displacements are approximately in the ratio of 1:2:4. A comparison with his results shows that so far as the same lines have been measured these groups are identical with our groups *b*, *c*, and *d*, except for the fact that we have included the flame lines in a separate group. In addition, the line λ 4260.640 which Duffield places in group 3 is placed by us in group *b*. It is a very unsymmetrically reversed line and subject to great variations in appearance under pressure. If we compare the violet portions of these groups, reducing to wave-length λ 4287, we obtain the ratios:

b	*c*	*d*
1	2.3	4.5

Similarly Duffield has for his groups:

I	II	III
1	2.0	4.5

The agreement between the two sets of results is close and in view of the accuracy of the results the departure from the whole numbers seems to be sufficiently great to make the integer relationship improbable. If we include the group of flame lines we obtain the ratio 0.67 between groups *b* and *a*; or, if we reduce all values to group *a* we find the relationship:

a	*b*	*c*	*d*
1	1.5	3.4	6.6

If the first two groups are united we have the ratios

1	2.5	4.9

It seems to us therefore improbable that the displacements of the different groups of iron lines bear ratios to one another which are proportional to small integers.

TITANIUM

Two previous investigations of the arc spectrum of titanium under pressure are available. The first of these, by Humphreys,[1] was carried on at pressures between 42 and 101 atmospheres, and the second, by Rossi,[2] at pressures between 16 and 101 atmospheres. Unfortunately very few lines are common to all three series of

[1] *Astrophysical Journal*, **26**, 18, 1907.

[2] *Proc. Roy. Soc.*, A **83**, 414, 1910.

observations. A comparison with the results of Humphreys shows that our values are larger by about the same amount as in the case of iron. The average of the lines agrees well with that of Rossi, although there are some large individual discrepancies.

One of the features which stands out most prominently in these results is the behavior of the enhanced lines. A large majority of them show displacements decidedly larger than those of the great proportion of arc lines in the vicinity, in some parts of the spectrum the average difference amounting to as much as 50 per cent. The effect, however, is not true of all enhanced lines, such lines, for example, as λ 4338.084 and λ 4443.976 showing small values as compared with neighboring arc lines. Nearly all of the enhanced lines present in the arc spectrum are bright at a pressure of 9 atmospheres, and being rather broad and diffuse are difficult of measurement. We have for this reason given especial attention to them, particularly in the region λ 4300 to λ 4600, and few of the values are based upon the measurement of less than four plates. The exceptional behavior of most of these lines is fully confirmed by the spark results, and is of marked interest in view of their importance in solar and stellar spectroscopy.

An inspection of the results given in Table II shows that the intermixture of the groups in the spectrum of titanium, if such exist, must be most complicated, and the simple expedient of plotting the displacements graphically, which proved so useful in separating the groups of lines in the iron spectrum, has been of little value. In the violet part of the spectrum nearly all of the lines as far as λ 4080 are reversed and show small displacements. At this point, however, begins a series of unreversed lines extending to λ 4285, which show relatively large displacements. Beyond λ 4290 both reversed and unreversed lines occur and the average displacement is less than in the region λ 4080–λ 4290. In the green, yellow, and red portions of the spectrum the number of reversed lines falls off steadily but the displacements are found to range between very small and very large values, although showing a marked increase on the average. There is probably little doubt that the collection of lines showing displacements of some 0.2 Ångström in the green and yellow parts of the spectrum form parts of a group

similar to group *d* in the iron spectrum. The violet members of this group may perhaps be λ 4318 and λ 4321, which have displacements of about 0.1 Ångström. These lines are all unreversed and immensely widened toward the red, and very similar in appearance to the iron lines of group *d*.

A comparison of the titanium lines of group *a* with those which are affected in sun-spots shows that the lines which are most strengthened in spots as a rule show small displacements under pressure, although the agreement is by no means perfect. The fact that the number of low-temperature lines increases very rapidly in the less refrangible parts of the spectrum may account, perhaps, for the lower average for the displacements of the titanium lines in these regions.

In view of the impossibility of separating the groups in the titanium spectrum it is perhaps not surprising that a general comparison of our pressure displacements with the separations in a magnetic field gives little evidence of any direct relationship between these quantities. As in the case of iron, the use of these displacements instead of those of Humphreys reduces in a marked degree the number of large discordances for lines which are simple triplets in the magnetic field. Such lines, however, as λ 4841.074, λ 5040.138, and λ 5120.592, which show very small pressure displacements but average separations, must necessarily form part of a separate group and a much larger number of groups than in the case of iron must be assumed in order to account for the wide variations observed. The question must be considered an open one.

The relationship between displacement and wave-length in the case of titanium clearly is complicated by the presence of the different groups of lines. A rather surprising result is given if we form simple means of the displacements for definite regions of the spectrum. Omitting the enhanced lines and some lines with very large displacements already referred to, the remainder (indicated by *a* in Table II) give the average values on the following page.

In the last two columns are given the residuals, assuming: first, that the displacement varies as the square of the wave-length; second, that it varies as the third power of the wave-length. The results show clearly that the law of the square of the wave-length

satisfies the observations the more closely. In the iron spectrum, on the other hand, the law was found to be the third power of the wave-length. As already indicated, the relatively large value of the

Region	Mean λ	Mean Δ	No. Lines	Residuals (Obs. − Comp.)	
				$\Delta=\left(\frac{\lambda}{\lambda_0}\right)^2 k$	$\Delta=\left(\frac{\lambda}{\lambda_0}\right)^3 k$
3700–4700...	4288	0.0250	115	+0.0016	+0.0064
4700–5700...	5142	0.0344	100	+0.0008	+0.0023
5700–6900...	6127	0.0439	38	−0.0038	−0.0102

average displacement in the violet is due to a considerable extent to the group of unreversed lines of large shifts between λ 4080 and λ 4285. If these were omitted, however, and only reversed lines considered throughout the entire spectrum, the difficulty would remain that the reversed lines in the region λ 4400 to λ 4600 give a value of the displacement quite as large on the average as do the reversed lines in the region λ 4800 to λ 5200. Accordingly it seems impossible without separating the lines in a very arbitrary way to obtain a series of average displacements which will harmonize with a law based upon the third power of the wave-length. It is sufficient to state that for the general averages of large numbers of lines taken throughout the spectrum the law of the square of the wave-length seems to hold closely, and to leave further discussion to a time when it is possible to distinguish groups in the titanium spectrum.

It may be of interest in this connection to call attention to some other results which suggest the possibility that the law of variation of displacement with wave-length may differ between titanium and iron. At the sun's limb the lines of nearly all the elements are shifted slightly toward the red relative to their positions in the spectrum of the center of the sun,[1] a result due probably to the greater influence of the lower strata of gas in the production of the lines in the limb spectrum. A study of the displacements of a large number of lines of different elements by Adams[2] shows that in the case of iron the change of displacement with wave-length is not

[1] Halm, *Astronomische Nachrichten*, **173**, 273, 1907.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 43; *Astrophysical Journal*, **31**, 30, 1910.

far from proportional to the square of the wave-length, while in the case of titanium it is much less, being more nearly proportional to the first power. In other words, if two groups of iron and titanium lines had the same average displacement in one part of the spectrum λ_0, in any other part λ the average displacements would bear the relationship

$$\Delta_{Fe} = \left(\frac{\lambda}{\lambda_0}\right)\Delta_{Ti}.$$

This is the same result as that which we have found for iron and titanium under pressure. Conditions in the sun are, of course, very complicated, and the displacement of each line depends upon its level, the amount of scattering and absorption of light in different portions of the spectrum, the thickness of the absorbing vapor, and many other factors as well. For these reasons we might expect that the law of variation of displacement with wave-length would differ from that found in laboratory spectra. Unless we assume a radically different level for iron and titanium in the solar atmosphere, however, we should expect the two elements to bear the same relationship to one another in the sun as that which they bear in the laboratory, and this appears to be the case.

Another result which may have a bearing upon the question of a difference in the laws of variation of displacement with wave-length for different elements is contained in a few measurements we have made upon some series lines in the spectrum of calcium at a pressure of 9 atmospheres (total). The second subordinate series of calcium contains a triplet in the violet beginning at λ 3949 and a triplet in the red at λ 6102. The red lines are fairly well measurable, but the violet lines are extremely poor in quality. We have, however, succeeded in obtaining a few measures upon two of them. The results follow:

λ	Δ	No. Plates
3957.177	0.081	3
3973.864	.085	3
6102.937	.139	2
6122.434	.136	2
6162.390	0.137	2

The close agreement of the separate lines is largely accidental, as their quality is not such as to warrant it. Averaging the two sets of lines we find:

λ	Δ	Obs. – Comp.
3965	0.083	−0.003
6129	0.137	+0.004

It is clear that the relationship between displacement and wave-length for these results is very nearly a linear one. The last column gives the residuals on such a hypothesis. Since these are series lines there can be no doubt of their connection with one another, and although the accuracy of measurement is low, it is hardly probable that the results can be in error sufficiently to admit of a law containing the second power of the wave-length, much less the third.

Although it is not possible to consider differences in the law of change of displacement with wave-length for different elements as established by these results, there is perhaps sufficient evidence to warrant a reference to the chemical relationship of the three elements. Calcium, titanium, and iron appear in the second, fourth, and eighth groups of the Mendeléjeff table, with atomic weights of 40, 48, and 56, and atomic volumes of 25, 13, and 7, respectively. It is perhaps conceivable that the law which connects pressure displacement with wave-length may be different for the different groups of the Mendeléjeff table.

The principal question remaining for consideration in these results is that of a possible difference between the displacements of arc and spark lines at the same pressure. The quality of the spark lines under pressure unfortunately is as a rule by no means equal to that of the corresponding arc lines, the reversals being much wider and more diffuse and the unreversed lines often less symmetrical. An inspection of the tables shows a marked tendency toward higher values for the spark displacements. Probably the most accurate results may be obtained from a comparison of a list of 20 lines, none of which is enhanced, between λ 4427 and λ 4555, employed in an investigation of the variation of displacement with amount of pressure and given in Table III. These lines have been measured upon an exceptionally large number of spark photographs

so that the results obtained should be reasonably comparable in accuracy with the arc values. Taking the means of the displacements of these 20 lines we find the values:

$$\Delta_{spark}=0.037,\quad \Delta_{arc}=0.030;\quad \text{or } \Delta_{spark}-\Delta_{arc}=0.007 \text{ Ångström}.$$

Very nearly the same value is given by a comparison of the lines throughout the entire spectrum. It seems hardly probable that a difference of this magnitude can be due to uncertainty of measurement, and the fact that arc and spark photographs were taken under exactly the same conditions, sometimes alternating with one another, reduces the probability of instrumental source of error. Since the reversals of the spark lines are broader than those of the arc it is possible that the larger displacements in the spark are due to an average widening of the reversals toward the red more than toward the violet, and as the amount of this effect would no doubt be peculiar to each line it might well account for the large variations between separate lines. The reversals in most cases, however, appear symmetrical upon the photographs.

In the case of the enhanced lines the differences between arc and spark displacements are even more marked. *The average difference for all of the enhanced lines which occur in both lists is* 0.014 *Ångström*, or about twice that found for the other lines. The values are

$$\Delta_{spark}=0.051;\quad \Delta_{arc}=0.037.$$

The enhanced lines in the spark spectrum are exceedingly difficult of measurement, but there probably can be little question of the existence of a difference of considerable size. The percentage of increase of the spark values over those of the arc also is greater than for the other lines.

The important question whether the relationship between displacement and amount of pressure is strictly linear has been considered by all who have carried on pressure investigations. Humphreys, Duffield, and Rossi have all shown that over a wide range of pressures, for the most part above 15 atmospheres, the displacements are directly proportional to pressure within the accuracy of the observations, although Duffield found some contradictory results between 15 and 25 atmospheres. In order to make an accurate test of the law at low pressures we have taken a special

series of photographs of the titanium arc in the region λ 4300–λ 4600 at pressures ranging from 2 atmospheres to 16 atmospheres above atmospheric pressure. A few photographs in a partial vacuum have been obtained as well. We have selected a list of 20 lines, all of which are reversed and well adapted for measurement on these photographs, and have used the averages of the displacements of these 20 lines at the various pressures for purposes of comparison. The results for each line together with the mean deviations are given in Table III.

TABLE III

λ	2 Atmos. 6 Plates		4 Atmos. 4 Plates		6 Atmos. 4 Plates		8 Atmos. 10 Plates		12 Atmos. 4 Plates		16 Atmos. 4 Plates	
	Δ	Mean Dev.	Δ	Mean Dev.	Δ	Mean Dev.	Δ	Mean Dev.	Δ	Mean Dev.	Δ	Mean Dev.
4427.266	4	1	9	2	15	1	17	3	30	2	41	2
4449.313	8	2	15	1	24	2	29	4	49	1	69	2
4451.087	8	1	16	3	25	3	28	2	48	3	73	3
4453.486	12	1	23	2	33	1	40	4	62	6	87	7
4455.485	12	1	24	3	34	3	41	2	64	3	92	6
4457.600	10	1	23	3	33	2	39	3	63	1	86	3
4465.975	8	2	15	1	21	2	25	2	42	1	51	5
4471.408	7	1	12	1	19	2	23	3	44	1	53	6
4481.438	6	1	13	1	19	2	23	2	43	2	59	3
4512.906	9	1	15	2	25	2	30	2	50	3	68	3
4518.198	8	1	15	2	25	1	30	2	45	2	68	5
4522.974	9	1	16	3	25	1	31	3	52	2	71	6
4527.490	9	1	17	2	26	1	32	4	48	1	71	2
4533.419	8	1	17	2	24	1	31	2	49	4	66	1
4534.953	12	3	13	1	25	1	34	3	49	1	70	2
4535.741	8	1	15	1	24	2	29	3	51	1	66	3
4544.864	9	1	16	4	26	1	31	2	50	3	68	2
4548.938	8	1	17	2	25	3	31	2	51	3	70	5
4552.632	8	1	17	2	24	2	30	3	50	1	70	6
4555.662	10	1	17	3	28	2	31	2	55	3	75	6
Mean	8.6	1	16.2	2	25.0	2	30.3	3	49.8	2	68.7	4

If we assume the linear relationship and reduce the mean values by a least-squares solution we obtain the following residuals:

	Obs.−Comp.	Δ per Atmos.
2 atmos.	+0.0005	0.0043
4 atmos.	−0.0003	.0040
6 atmos.	+0.0003	.0042
8 atmos.	−0.0026	.0038
12 atmos.	+0.0004	.0042
16 atmos.	+0.0028	0.0043

The largest deviation from the linear law, accordingly, at any of these pressures is found to be only 0.0028 Ångström. The displacement per atmosphere is given in the last column of the table. The average of these values is 0.0041 Ångström. Similarly for the average displacement between 15 and 100 atmospheres for 16 of the same lines Rossi finds 0.0045. The close agreement of these results makes it almost certain that the linear law of the variation of displacement with pressure holds perfectly between atmospheric pres-

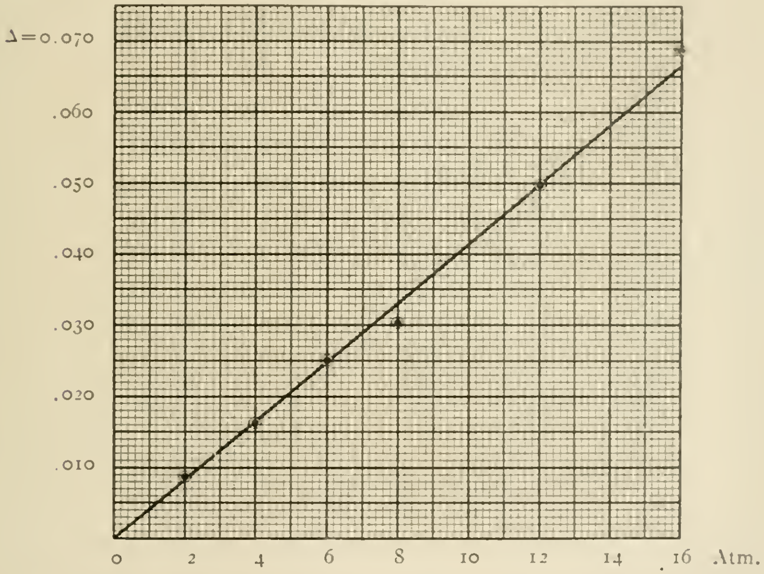

Fig. 3.—Mean displacement of 20 selected titanium lines at different pressures.

sure and 101 atmospheres. The results are shown graphically in Fig. 3.

In the course of the investigation a number of photographs were made of the titanium arc in an atmosphere of illuminating gas. The arc under such conditions burns very poorly, and it is necessary to employ a very short gap and secure the exposure from a series of intermittent flashes rather than a continuous flame as in air or carbon dioxide. The photographs were all made at a pressure of 4 atmospheres. It is well known that an atmosphere of hydrogen has the effect of strengthening the enhanced lines at atmospheric

pressure and the same is found to be true at a pressure of 4 atmospheres. We have measured on these photographs most of the arc lines in the selected list of 20 lines and also all well-measurable enhanced lines in the same region. The appearance of the lines does not differ greatly from that in an atmosphere of carbon dioxide except for the greater relative strength of the enhanced lines. The arc lines measured are all reversed; the enhanced lines unreversed. The results are shown in Table IV, compared with the corresponding values for the same pressure in an atmosphere of carbon dioxide. The displacements are in thousandths of an Ångström.

TABLE IV

λ	4 Atmos. H	No. Plates	4 Atmos. CO_2	No. Plates	Difference $\Delta_H - \Delta_{CO_2}$	
4427.266	11	5	9	4	+ 2	
4443.976	9	6	7	5	+ 2	Enhanced line
4449.313	15	2	15	4	0	
4451.087	13	2	16	4	− 3	
4453.486	23	2	24	4	− 1	
4455.485	22	5	24	4	− 2	
4457.600	24	5	23	4	+ 1	
4465.975	15	5	14	4	+ 1	
4468.663	31	6	26	5	+ 5	Enhanced line
4471.408	10	2	11	4	− 1	
4481.438	10	2	13	4	− 3	
4501.445	34	6	27	5	+ 7	Enhanced line
4512.906	14	5	15	4	− 1	
4518.198	17	2	15	4	+ 2	
4522.974	14	2	16	4	− 2	
4527.490	17	5	17	4	0	
4534.139	24	6	20	5	+ 4	Enhanced line
4544.864	17	5	16	4	+ 1	
4548.938	14	2	17	4	− 3	
4549.808	41	6	31	5	+10	Enhanced line
4555.062	15	4	17	4	− 2	
4563.939	29	6	22	5	+ 7	Enhanced line
4572.156	41	5	33	5	+ 8	Enhanced line

The interesting result is brought out by these measures that while the 16 arc lines in the list give practically identical values in hydrogen and carbon dioxide, the enhanced lines show distinctly larger values in hydrogen. The average values for $\Delta_H - \Delta_{CO_2}$ are:

16 arc lines..................	−0.0007
7 enhanced lines............	+0.0061

Moreover, the increase of displacement in hydrogen for an enhanced

line seems to be a direct function of the normal displacement of the line, amounting to about 25 per cent at 4 atmospheres. Thus the line λ 4443.976 which is displaced only 0.007 in CO_2 is displaced 0.009 in H, while λ 4501.445 which is displaced 0.027 in CO_2 is displaced 0.034 in H, the same ratio holding for both. In view of the quality of the lines and the accuracy of the measures it seems to us that this result is almost certainly geniune and that the nature of the surrounding gas exerts an influence upon the displacements of the enhanced lines. It appears probable, therefore, that a change from a carbon dioxide to a hydrogen (illuminating gas) atmosphere affects not only the intensities but the displacements of the enhanced lines in the same way as does a change from arc to spark.

We have already referred to some of the more important applications of the results of this investigation to solar spectroscopy. The fact that the enhanced lines show materially larger displacements both at the sun's limb and under pressure than do the other lines strengthens greatly the view that pressure is the effective agent in producing the solar displacements. As was pointed out previously, evidence in the same direction is afforded by the two facts that both in the arc under pressure and at the sun's limb the low-temperature lines give small displacements, and that both in arc and sun the ratio of the laws of change of displacement with wave-length for iron and titanium is the same. We have referred briefly in an earlier communication[1] to the possible bearing upon the character of the spectrum of the solar chromosphere of the fact that at moderate pressures the enhanced lines remain bright while a majority of the other lines are reversed. A result which probably has an application in the same direction but especially to the spectrum of the upper chromosphere and of prominences is furnished by our photographs of the titanium arc at reduced pressure. These show a marked increase of relative intensity for the enhanced lines as compared with that which they have at atmospheric pressure.[2] At the very low pressures of the upper portions of the solar atmosphere this fact may well account for the prominence of the enhanced

[1] *Science*, **32**, 881, 1910.

[2] A similar result has been found by Barnes for *Al*, *Mg*, and *Cu*, *Astrophysical Journal*, **34**, 159, 1911.

lines in the flash spectrum. The change of intensities of the enhanced lines at reduced pressure is well shown in Plate III.

SUMMARY

An investigation of the arc spectrum of iron at a pressure of 9 atmospheres and of the arc and spark spectra of titanium at pressures of from a partial vacuum to 17 atmospheres over a range of wave-length from λ 3600 to λ 6800 furnishes the following results:

1. Reversal is a function of wave-length, being most frequent in the more refrangible part of the spectrum and becoming less so toward longer wave-lengths.

2. With reduction of pressure below one atmosphere the enhanced lines in the spectrum of titanium become relatively stronger.

3. The low-temperature lines of iron appear to form a distinct group and have small displacements under pressure.

4. The other lines of iron may be divided into three groups for which the displacements bear the approximate ratios 1 : 2.3 : 4.5. If the flame lines are taken as a separate group the ratios are 1 : 1.5 : 3.4 : 6.6.

5. There appears to be some evidence in favor of a direct relationship between pressure displacement and magnetic separation for iron when lines of the same group and of the same type of separation are considered. In the case of titanium, for which it has not been possible to distinguish well-marked groups of lines, no evidence of a connection with magnetic separation is found.

6. The values of the average displacement for the four iron groups at different wave-lengths are well represented by a law of variation of displacement with the third power of the wave-length. If we form simple means of the displacements of the titanium lines for considerable portions of the spectrum the values are well represented by a law of variation with the second power of the wave-length. The difference from iron may be due to the intermixture of various groups in the titanium spectrum. Measures of some calcium lines belonging to the second subordinate series indicate a variation of displacement according to the first power of the wave-length. The measures, however, are of low weight.

7. The displacements of the titanium arc lines are found to be accurately proportional to pressure for a range of from 2 to 16 atmospheres above atmospheric pressure.

8. The enhanced lines as a rule show much larger displacements under pressure than do the other lines in the titanium arc spectrum and are almost always unreversed. The amount of displacement, however, depends upon each individual line and a few enhanced lines give very small values.

9. In an atmosphere of hydrogen (illuminating gas) the displacements of the enhanced lines are appreciably larger than in an atmosphere of air or carbon dioxide at the same pressure. The other lines show the same displacements.

10. The displacements of the lines in the titanium spark appear to be larger on the average than in the arc, the largest differences being for the enhanced lines.

In the difficult work involved in the measurement of the photographs we have so far as possible followed the plan of having each plate measured by two observers. We are greatly indebted to Miss Lasby for her skilful treatment of the photographs and her active interest throughout the progress of the entire investigation.

Mount Wilson Solar Observatory
November 1911

THE SELECTIVE REFLECTION OF SALTS OF CHROMIUM AND CERTAIN OTHER OXYGEN ACIDS[1]

By HERBERT A. CLARK

As long ago as 1868, Mendeléjeff and Lothar Mayer announced the periodic system of the elements, according to which the physical and chemical properties of the elements are, apparently, periodic functions of their respective atomic weights. Two years earlier, R. Bunsen[2] found that solutions of different salts of didymium show absorption bands which are shifted toward the red end of the spectrum with increase in the molecular weight of the salt.

From then until now, a bewildering mass of data has been obtained by W. N. Hartley, E. C. C. Baly, H. C. Jones, W. W. Coblentz, H. Ley, and others, to get at the relations between the absorption spectra of solutions, principally in the visible and photographic regions, and their chemical constitution. Hartley,[3] using solutions of simple metallic nitrates; H. C. Jones and J. A. Anderson,[4] using solutions of the bromide and of the nitrate of cobalt; and E. C. C. Baly and C. H. Desch,[5] comparing solutions of simple metallic nitrates with the corresponding nitrites; all found the same kind of shift of absorption bands that Bunsen had found for didymium salts. The (emission) spectral series equations of Kayser and Runge for elements belonging to the same Mendeléjeff group, indicate the same sort of shift.

Much of the experimental evidence, however, is either negative or contradictory. Since nearly all of the absorption data are for solutions, many of these being of organic compounds, this is not to be wondered at—especially in view of the enormous complications introduced by variations in temperature, concentration, nature of the solvent, thickness of the absorbing layer of the solution, state of ionization, and in what the chemist tries to account for

[1] *Phoenix Physical Laboratory Contributions*, No. 25.

[2] *Poggendorff's Annalen*, **128**, 100, 1866.

[3] *Jour. Chem. Soc.*, **81**, 571, 1902.

[4] "Absorption Spectra of Solutions," *Carnegie Publication No. 110*, p. 30.

[5] *Jour. Chem. Soc.*, **93**, 1757, 1908.

under the name of stereochemistry. Consequently it would seem that fundamental relations between molecular and atomic weights and optical properties might well be looked for first in the reflection spectra, which show only the presence of such absorption bands as are very intense, and hence are the more characteristic.

Drude[1] has shown that in substances with normal dispersion ultra-violet absorption bands are probably due to resonance of negative electrons, having the same value of e/m as have cathode rays; and the infra-red bands, to resonance of masses of the size of a positively charged atom or molecule. Consequently reflection bands in the infra-red spectrum are more likely to be characteristic of a substance than are the ultra-violet maxima. The enormous extent of the infra-red region relative to that of the shorter wave-lengths is also in favor of the former region, in a search for fundamental relations.

A. H. Pfund[2] concluded from his data on the infra-red reflection from certain simple inorganic salts that the mechanism giving rise to these maxima is localized in the acid radical. Soon after, W. W. Coblentz[3] found that the infra-red reflection maxima of certain sulphates and carbonates are shifted toward the long waves with increasing atomic weight of the base. Next, L. B. Morse[4] found that, in certain simple inorganic salts of oxygen acids having a common base, the infra-red reflection maxima are shifted toward the long waves, as the weight of the element in the acid radical which is combined with a *constant* amount of oxygen increases. If it is assumed that a strong reflection maximum is a resonance effect, that the oxygen atom in the molecule is the resonator, and that the resonator is closely loaded with other oxygen atoms and with the acid-forming element in the molecule, but is loosely loaded with the base, the above results are consistent with one another, as well as with the chemists' views regarding the strength of the different "bonds" in the molecule. The present work was undertaken to test the conclusions of Pfund, of Coblentz, and of Morse, by the infra-red reflection from the salts of the chromium acids.

[1] *Annalen der Physik* (4) **14**, 958, 1904.

[2] *Astrophysical Journal*, **24**, 40, 1906.

[3] *Phys. Rev.*, **25**, 136, 1907.

[4] *Astrophysical Journal*, **26**, 242, 1907.

APPARATUS

A common arrangement for infra-red work, a Nichols radiometer with a reflection spectrometer having a rock-salt prism and a Wadsworth[1] mirror, was used. The apparatus was, in the main, the same as was used by L. B. Morse[2] in his work on the carbonates, and hence will be described somewhat briefly.

The optical system.—An image of the source, a Nernst glower, *A*, Fig. 1, was cast by the silvered concave glass mirror, *B*, upon the (plane) surface under investigation at *D*, at an angle of incidence of about 6°. An image of this image, in turn, was cast upon

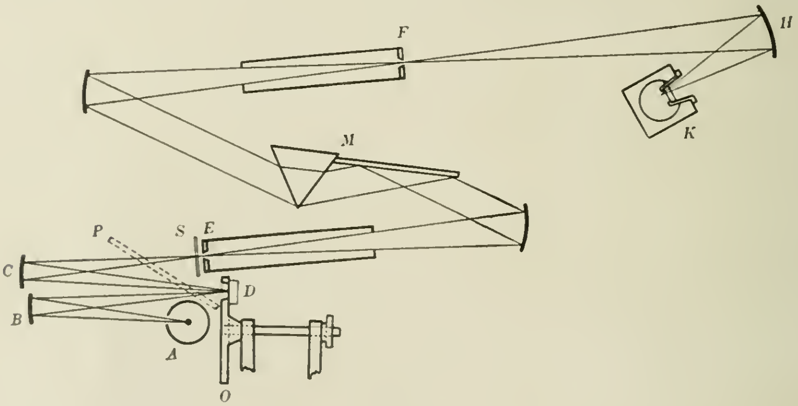

Fig. 1

the collimator slit, *E*, of the reflection spectrometer, by a second mirror, *C*. After resolution by the spectrometer, an image of the telescope slit, *F*, was cast by the mirror, *H*, upon one vane of the Nichols radiometer, *K*.

The spectrometer was a Schmidt and Haensch instrument with mirrors of 4 cm aperture and 35 cm focal length. The prism was of rock-salt, with faces 5 cm×8 cm and a refracting angle of 59° 57′ 20″. The Wadsworth mirror-prism, *M*, enabled the spectrometer arms to remain fixed. The two slits, always of equal width, were as narrow as possible, consistent with sufficient energy. The usual range of slit-widths (varying from 0.30 to 1.00 mm for wave-lengths longer than 9.0 μ) is indicated in Fig. 4. No change of

[1] *Phil. Mag.* (5), 38, 337, 1894. [2] *Loc. cit.*

slit-width was ever made, however, while passing over a reflection maximum. The same figure also shows the slit-width at one point (11.0 μ) in terms of the wave-length interval entering the radiometer when the spectrometer is set for that particular wave-length, an interval of 0.16 μ at this point for slit-widths of 0.30 mm. This range increases to 0.21 μ for slit-widths of 0.50 mm at a wave-length of 12.5 μ; to 0.30 μ for slit-widths of 0.80 mm at 14.0 μ; and to 0.36 μ for slit-widths of 1.00 mm at 15.0 μ.

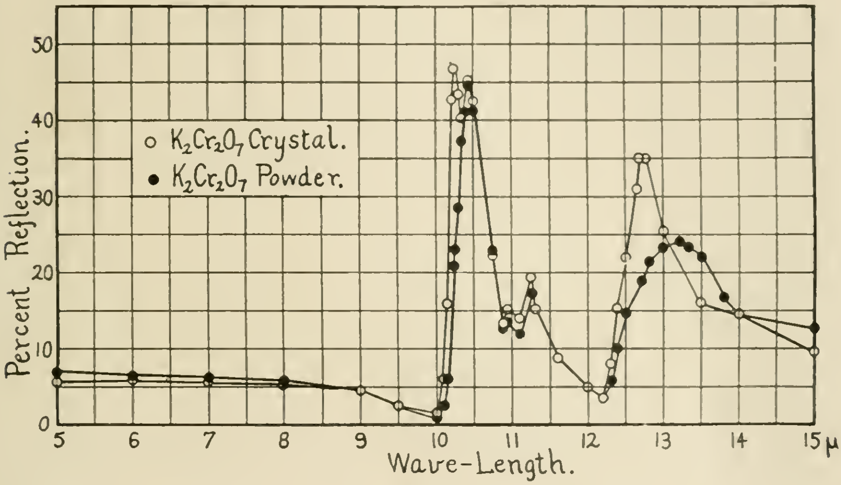

FIG. 2

Wave-lengths.—The deviations necessary to obtain representative wave-lengths were calculated in the usual way, from the indices of refraction for rock-salt as given by H. Rubens[1] and by H. Rubens and A. Trowbridge,[2] and a calibration-curve was plotted. The indices as given in the latter paper were corrected to agree with corrections found in a reprint. The calibration-curve was tested by means of the two sylvite absorption bands and the long wave-length reflection band of calcite. An air-tight metal-and-glass hood with engine-oil seal was always kept over the prism when it was not in use. A vessel of P_2O_5 was kept near the prism, under the hood, as a drying agent.

[1] *Annalen der Physik*, 54, 482, 1895.

[2] *Ibid.*, 60, 733, 1897.

Source.—A direct-current Nernst glower, consuming 132 watts at 110 volts, in series with a low-reading ammeter and a rheostat for close adjustment, was operated from a 70-cell storage battery. This arrangement enabled variations in the energy of the glower to be made inappreciable. The largest obtainable glower was used, in order that the greatest slit-width (1.0 mm) should be well covered by the image of the glower.

The Nichols radiometer had mica vanes, about 0.75 mm by 5.0 mm (a little larger than the image of the slit as cast upon the vane), placed about 5.0 mm apart, blackened with platinum black and shellac. The rock-salt window was protected by a P_2O_5 dryer when not in use. The period (time of swing of the vanes one way) varied from 40 to 60 seconds, according to the air-pressure, which was adjusted according to the speed or sensibility desired. The radiometer was wrapped in a thick layer of felt and covered with a box of heavy sheet copper, to protect it from sudden changes of room temperature. This proved so effective that the radiometer zero seldom drifted, due to varying room temperature, more than 1.0 cm during a half-day's "run." What drift there was, was very slow and steady. The reflection from silver, using a 1.0 mm slit, gave a scale reading of 54 mm at 15.0 μ and of 18 mm at 16.3 μ; while the zero drift was so slight that an accumulation of observations would determine deflections of 3 mm to a few per cent. To indicate deflections (which could be estimated to 0.1 mm), the image of a fine wire in front of an 88-watt Nernst glower was projected on a millimeter scale (not shown in Fig. 1) at a meter's distance, by a small, platinum-plated, concave mirror[1] attached to the radiometer suspension. This glower, being in parallel with the source, was so steady that it did not affect the radiometer zero perceptibly, after the first large shift due to lighting the glowers. Numerous diaphragms were used, to protect all parts of the entire optical system from any stray light, except for a faint, diffuse glow.

PURITY OF THE SPECTRUM

A very serious source of error in infra-red spectrum work is the unavoidable impurity of the spectrum. The present work deals principally with wave-lengths from 10.0 μ to 16.0 μ, for which the

[1] See A. H. Pfund, *Astrophysical Journal*, 24, 22, 1906.

energy of a Nernst glower is extremely small as compared with that at 2.0 μ to 4.0 μ, for instance. Consequently the presence of a very small part of the energy of the shorter wave-lengths occurring as an impurity in the spectrum of the longer, would constitute a very large part of the total (apparent) energy of the latter. This can readily cause a very large error, not only in the intensity of a reflection maximum, as measured, but also in its position, shifting it toward the shorter waves. To reduce this effect, screens

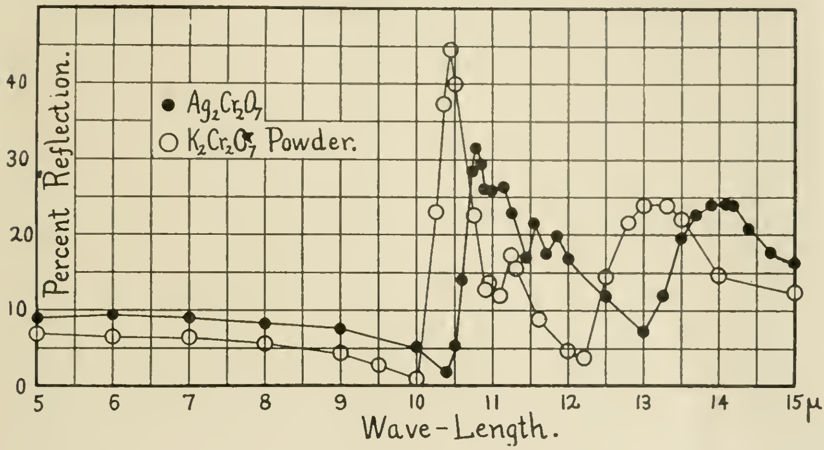

Fig. 3

were used (S, Fig. 1) which are opaque to the spectral region considered, but transparent for shorter wave-lengths.[1] They were arranged to slide smoothly in vertical ways in front of the collimator slit. A glass screen was used for wave-lengths from 5.5 μ to 10.0 μ or 10.5 μ, according to the position of the reflection band; and one of fluorite for the longer wave-lengths (see Fig. 4). Even with this arrangement, however, there probably is enough stray energy at the longest wave-lengths to affect the results considerably. The blackened wood shutter, P, Fig. 1, was used for wave-lengths shorter than 5.5 μ.

MOUNTING THE SPECIMENS

Each specimen (D, Fig. 1) was mounted on the rear of a wheel, O, to cover a hole in the wheel. A plane-glass mirror, silvered on

[1] See Rubens and Hollnagel, *Phil. Mag.* (6), **19**, 765, 1910.

the front face, was mounted in another hole 60° away. Reflection from this silver comparison mirror was assumed to be 100 per cent, probably a trifle too large.[1] This assumption introduces a very slight error in the height of the reflection curve, but not in its shape. The assumption is thus justifiable, especially in view of the fact

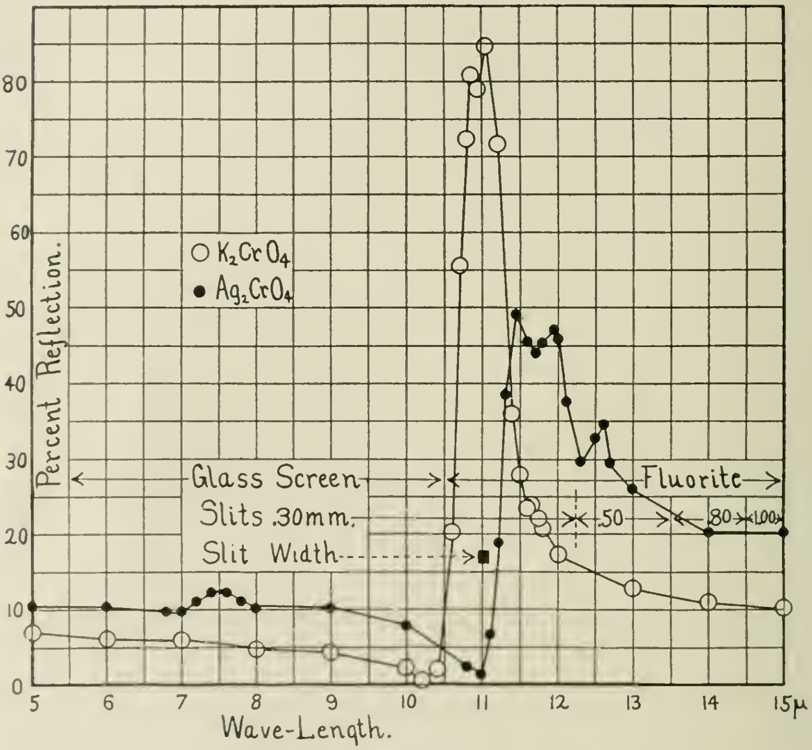

FIG. 4

that the polish of the different reflecting surfaces was unavoidably different. Rotation of the wheel between carefully adjusted stops served to place either surface, at will, in the optical path, in closely identical positions. The two surfaces were adjusted to the same plane by leveling screws, as determined by the reflection of an incandescent lamp filament from each surface, in turn, into a telescope having ocular cross-wires. This arrangement, which

[1] See Hagen and Rubens, *Annalen der Physik* (4), 11, 873, 1903.

worked very smoothly, with neither jar nor lost motion, proved entirely satisfactory. Cords served conveniently to manipulate the wheel and the screens, from the viewer's seat at the reading scale.

SURFACES

Substances.—All of the salts of all the chromium acids available were ordered from the dealer. As many as possible were obtained from Kahlbaum; the rest were made to order, at considerable expense, by Theodor Schuchardt, of Görlitz. See the list in Table I. All were anhydrous as obtained from the makers, except the chromates of lithium, calcium, and magnesium. The second one of these was made anhydrous by heating in a drying oven; the others could not be entirely dried. This is mentioned again, under the discussion of the curves.

Preparation of the surfaces.—Many of the salts were obtainable only in the form of a fine powder; consequently, all were powdered, sifted through a very fine geologist's sieve, and compressed into cakes in a hydraulic press under about 30 tons per square inch (a method used previously by Miss Langford[1] for the phosphates, in the Phoenix Physical Laboratory), then polished, if possible, to give a plane reflecting surface. These are designated hereafter as "polished powder" surfaces. Some were too friable to be polished at all, so they were pressed against a piece of plate glass under about 12 tons per square inch. The removal of the glass left a reflecting surface that was more or less brilliant in every case but one, $BaCr_2O_4$. The obtainable polish varied greatly with the substance. A number of the surfaces reflected poorly in the visible region. The image of an Edison lamp filament at nearly perpendicular incidence, however, could be seen when adjusting, in all except two, $CaCrO_4$ and $BaCr_2O_4$. It was necessary to use a Nernst glower at perpendicular incidence in adjusting the former, and at an angle of incidence of about 45° for the latter. The lack of a perfect polish, however, affects the magnitude, but not the position, of a resonance maximum. These surfaces are hereafter designated as "glass" surfaces. One salt, $CaCr_2O_4$, resisted both these methods; hence a thin plate of rock-salt was pressed lightly

[1] *Physical Review*, **33**, 138, 1911.

against the powdered chromite contained in a cavity in a piece of cork, and the reflection was measured from the chromite-rock-salt surface. Such surfaces are hereafter designated as "rock-salt" surfaces. Two other salts (Li_2CrO_4 and $MgCrO_4$) are so very deliquescent that sufficiently permanent surfaces could be prepared only in this way. The nature of each surface is indicated in Table I. All curves are for "air" surfaces (either "polished powder" or "glass" surfaces) except as otherwise designated on the curves. Surfaces of several of the less troublesome salts, already prepared

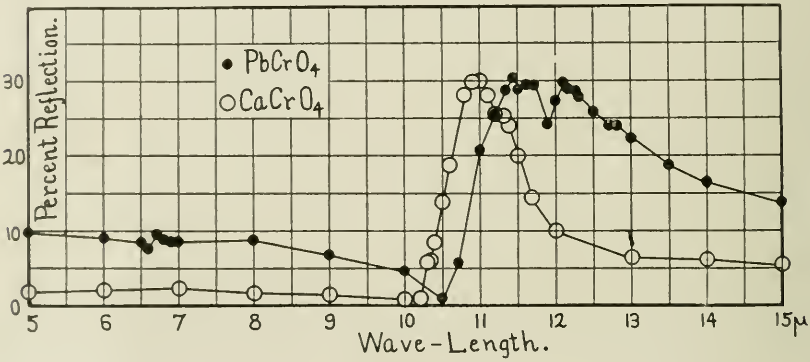

FIG. 5

in one of the first two ways and measured, were made by the third method also, to serve as a check on this method. (See Table I.)

The selective absorption of rock-salt in the long wave-length region of the spectrum would have a tendency to shift the apparent location of resonance regions toward the shorter wave-lengths, when measurements are made on a "rock-salt" surface. An attempt to avoid this difficulty was made by placing a similar plate of rock-salt over the silver comparison mirror. To do this, two large, slightly prismatic plates of rock-salt were polished on both sides; these each had a mean thickness of 2.60 ($\pm$0.005) mm and a refracting angle of 53′ ($\pm$5′). Each large plate was then sawed up into smaller ones of the requisite size. The small plates were paired, one of a pair being chosen from each large plate to give two of the same thickness. One of a pair was then put on a powder surface, as mentioned before, and the other was put about

TABLE I

Typical Formula	Salt	Surface	Weight of Base in Molecule	Weight in Acid Radical with O_3	Reflection Maxima					
					Probably Impurities					
$M'_2Cr_2O_7$	$Ag_2Cr_2O_7$	"polished powder". . .	215.8	44.6	. .	10.78	11.15	11.54	11.88	14.05
	"	"rock-salt". . .	"	"	. . . .	. . .	. . . . .			
	$K_2Cr_2O_7$	crystal.	78.2	"	. . .	10.25	10.45	10.96	11.23	12.75
	"	"polished powder". .	"	"	. . . .	"	10.45	10.96	11.23	13.15
	"	"rock-salt". .	"	"	. . . .	. . . . .	10.4			12.6
M'_2CrO_4	Ag_2CrO_4	"polished powder". .	215.8	39.0	7.5	11.45	11.95	12.60		
	K_2CrO_4	"	78.2	"	. .	10.86	11.05	11.65		
	" . . .	"rock-salt". .	"	"	. . .	. . . .	11.0			
	*Li_2CrO_4	" .	14.0	"	. . . .	.	10.75			
$M''CrO_4$	$PbCrO_4$. .	"glass"	207.1	"	6.7	11.45	11.65	12.10	12.20(?)	12.80(?)
	$BaCrO_4$.	"	137.4	"	. . . .	10.90	11.08	11.58	12.35	12.60
	$SrCrO_4$	"polished powder". . .	87.6	"		10.48(?)	10.65	11.35	11.80	12.25(?)
	†$CaCrO_4$.	"glass"	40.1	"			10.32(?)	10.96	11.30	
	*$MgCrO_4$.	"rock salt". .	24.3	"	12.1			10.90		
$M''Cr_2O_4$ . . .	$BaCr_2O_4$	"glass" . .	137.4	78.0	11.5	15.0	15.95			
	$CaCr_2O_4$ (observed)	"rock salt".	40.1	"		14.6	15.4			
	$CaCr_2O_4$. (corrected)	(calculated for air)	"	"		15.2	16.0			
	$MgCr_2O_4$. . . .	"glass"	24.3	"		15.0	15.8			
	"	"rock-salt". .	"	"		14.4	15.2			

* Not anhydrous. † Made anhydrous by heating.

1.0 cm in front of the silver comparison mirror, far enough away to avoid interference due to multiple reflection. The energy reflected from the *front* surface of each of the rock-salt plates was sent out of the optical system, due to the angle between the two surfaces. That reflected from the *rear* surface of that plate which was over the silver comparison mirror, was sent off similarly, since the whole plate was tilted slightly with respect to the mirror. Calculation shows that the beam is shifted at the collimator slit, by the thin prism, a distance of 0.6 mm more for a wave-length of 15 μ than for the sodium line, for which the mirrors B and C, Fig. 1,

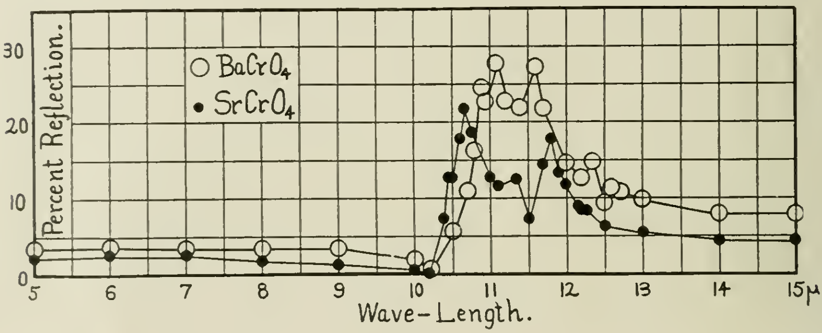

Fig. 6

were adjusted. Unfortunately, this was not determined until it was too late to repeat the observations. The refracting edges of both prisms of a pair, however, were turned the same way; thus the shift of the beam would diminish the *absolute* amount of energy entering the spectrometer, but not the *relative* (i.e., the percentage reflection). Further, if the reflection maxima are due to true resonance, it ought not to be necessary to focus the "reflected" beam exactly on the spectrometer slit.[1] It would seem, therefore, that the only effect of the rock-salt plate upon the reflection maxima would be due to the fact that the reflection takes place from a chromate-rock-salt, instead of from a chromate-air surface. This would merely reduce the amount of the reflected energy, due to the smaller variation in refractive index at the reflecting surface; but should not, supposedly, alter the *position* of the maximum,

[1] See J. A. Anderson, *Astrophysical Journal*, 26, 73, 1907.

since the refractive index of rock-salt has not yet begun to change very rapidly. Why this method does not eliminate all shift in the positions of the maxima, is not at present clear. That it does not do so is evident from the shift in the 13 μ and the 15 μ bands, Fig. 10. This will be considered further in the discussion of the results from "rock-salt" surfaces.

Permanence of surfaces.—Measurements on the surfaces were taken over a period of from one day to three months each, varying with the surface. In no case was any variation in the reflecting surface detected, either visually or by means of the measurements, except in the cases of $BaCrO_4$, of $PbCrO_4$, and of $Ag_2Cr_2O_7$ when under rock-salt. The first showed a slight darkening over the entire surface exposed to the air, after a period of three months. An unavoidable interruption in the work scattered observations on this surface over that length of time. The observations indicated no change in the surface during that period. The $PbCrO_4$, after three weeks, showed a streak, slightly darker than the background, over the small part of the surface which was covered by the image of the Nernst glower (the source). Again the data gave no evidence of any change in the surface. The $Ag_2Cr_2O_7$ when put under rock-salt, changed immediately to $AgCl$, judging from the immediate change in color of the powdered salt. This is mentioned later, in the discusssion of the results from the "rock-salt" surfaces.

METHOD OF OBSERVING

A zero reading was taken before and after each deflection, to detect any zero drift. Two or more readings were taken for each wave-length from the surface under investigation, then from the silver comparison surface, before disturbing the wave-length setting. Many of the observation points are omitted from the published curves where the trend of the curve is regular, to avoid crowding the points. The observation points were usually 0.5 μ apart for the flat, short wave-length portion, and 0.1 μ to 0.2 μ apart for the longer wave-lengths, except when going over the sharp reflection peaks from 10 μ to 12 μ. Here they were from 0.02 μ to 0.05 μ apart. At least two observations were taken at every point. When these differed appreciably, they were repeated

until a consistent series was obtained. Observations began, in every case, at the wave-length 1.5 μ, except for a few of the "rock-salt" surfaces. However, since there was nothing of interest in the early infra-red, the curves are all plotted to begin at longer wave-lengths.

THE REFLECTION CURVES

"*Air*" *surfaces.*—Fig. 2 shows the results from a "polished powder" surface of $K_2Cr_2O_7$ and from the surface of an excellent crystal of the same, polished parallel to one of the cleavage planes.

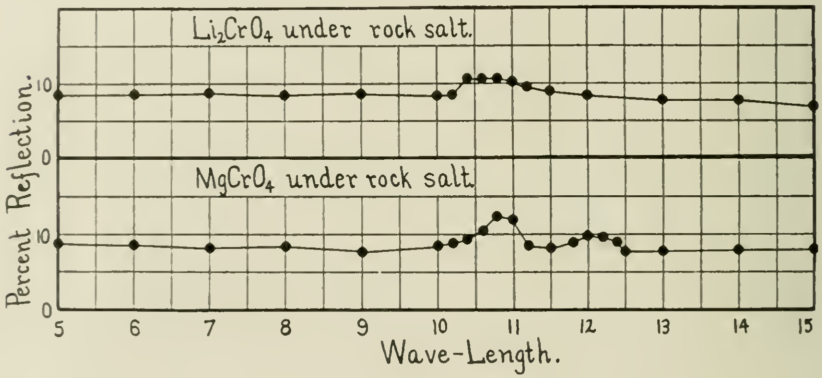

FIG. 7

This is the only crystal measured. The absence of the first (short-wave) peak from the "polished powder" surface, and the shifting of the last, will probably require polarized energy to explain.[1] J. T. Porter[2] had located a maximum for a polished crystal of $K_2Cr_2O_7$ at 10.3 μ by the method of *Reststrahlen*. The "polished powder" curve is repeated in Fig. 3, for comparison with that of $Ag_2Cr_2O_7$, also a "polished powder" surface. The chromates of silver and potassium are shown in Fig. 4. (The nature of each surface is indicated in Table I.) Each has a single complex maximum, with three peaks. One peak in the latter, that at 11.65 μ, is faint, but its presence is not in doubt. The potassium chromate curve is notable for its great intensity and sharpness; consequently

[1] See R. E. Nyswander, *Physical Review*, 28, 291, 1909.

[2] *Astrophysical Journal*, 22, 229, 1905.

it ought to prove a good source of *Reststrahlen*. The faint maximum in Ag_2CrO_4 at 7.5 μ is assumed to be due to a trace of an impurity, possibly $AgNO_3$ (from which the chromate was probably made), which has a sharp maximum at 7.45 μ[1]. Arc spectrum photographs taken with a ten-foot concave Rowland grating, of each salt having a suspected impurity, did not, in any case, identify the impurity.

Fig. 5 shows the results from the chromates of lead and calcium. The calcium salt in the stock bottle was not anhydrous, but was made so by heating in an air-bath at 150° C. for eight hours. Since

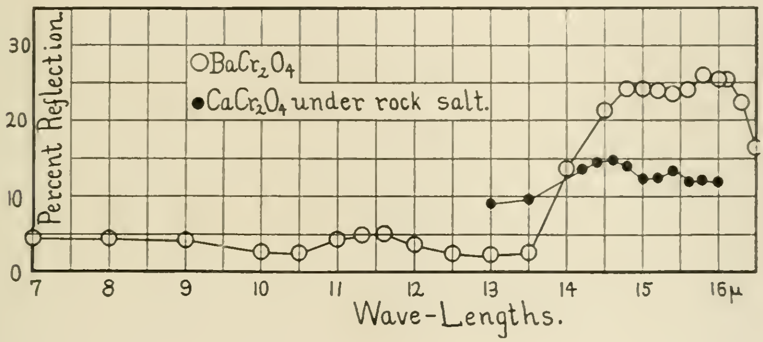

FIG. 8

no change could be detected in the surface after standing in the dry research-room during the period of observation, two days, it is assumed that it remained anhydrous. Each curve has a single complex maximum. The lead curve seems to have five peaks, two being so faint as to be somewhat in doubt (indicated by ? in Table I). The calcium curve has three maxima, one being somewhat in doubt. $PbCrO_4$ (Fig 5), like Ag_2CrO_4 (Fig. 4), has a low, short wave-length maximum, possibly due also to some unknown impurity. This chromate was probably made from the acetate; but the reflection curve of the acetate is unknown. L. B. Morse[2] had already roughly located a maximum in crocoite (natural crystal of $PbCrO_4$) at 11.5 μ. For the chromates of barium and strontium,

[1] A. H. Pfund, *Ibid.*, **24**, 35, 1906.

[2] *Physical Review*, **26**, 525, 1908; see also W. W. Coblentz, "Investigations of Infra-Red Spectra," *Carnegie Publication No. 97*, p. 181.

see Fig. 6. Each has a single complex maximum with five peaks, two in the latter being somewhat in doubt. (See Table I.)

Magnesium chromite is shown in Fig. 9, and the chromites of barium and calcium in Fig. 8. The low maximum at 11.5 μ in $BaCr_2O_4$ was suspected to be due to a trace of $BaCrO_4$ as an impurity (compare Fig. 6). A letter of inquiry to the maker, Theodor Schuchardt, confirmed this suspicion. The rapidly increasing absorption of the rock-salt prism and radiometer window, as well as the decreasing intensity of the source, with increasing wave-lengths, made observations to 16.0 μ or 16.5 μ (on the chromites)

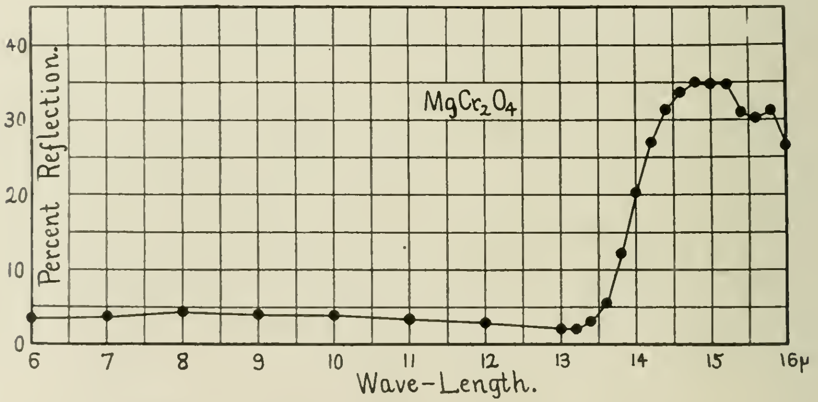

FIG. 9

extremely difficult. Probably the spectrum is decidedly impure here also, although a fluorite screen was used. Repeated observations were made to locate the maxima as definitely as possible. The order of accuracy has been indicated previously, under the discussion of the radiometer.

"*Rock-salt" surfaces.*—The chromates of lithium and magnesium are shown in Fig. 7. Both of these, as obtained from the stock bottle, contained water of crystallization. Attempts to dry them in an air-bath at 110° C. for twenty-four hours, still left about one molecule of water in the former, and two in the latter, as determined by analyses kindly made by Mr. R. F. McCracken, of the chemistry department at Columbia University. They were then so extremely hygroscopic, absorbing moisture readily even in the

artificially dried atmosphere of the research-room (kept below 60 per cent humidity), that they were mounted under rock-salt, as mentioned before, in holes lined with sealing wax, and hermetically sealed in.

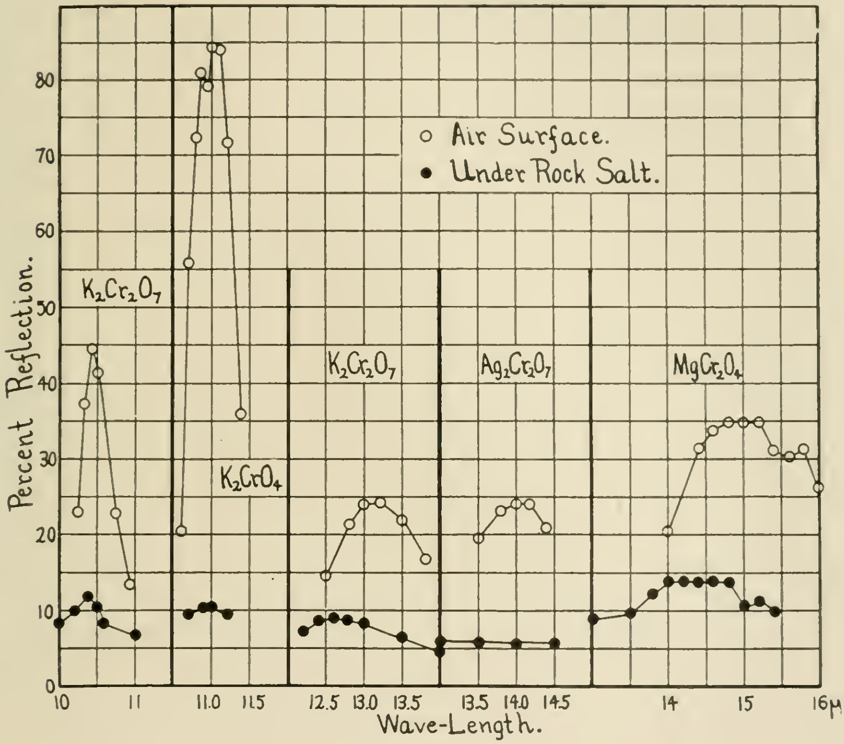

FIG. 10

The positions of the maxima in $CaCr_2O_4$ ("rock-salt" surface, Fig. 8) suggested a shift due to rock-salt, although an attempt has been made to eliminate the effect of the rock-salt, as has been described under the heading *Preparation of the surfaces*. To test this, measurements were made on a "rock-salt" surface of $MgCr_2O_4$ and compared with the "glass" surface curve (Fig. 9) of the same substance, which was in contact with the air. (See Fig. 10.) The shift toward the short waves (0.6 μ for each peak), from the "air" surface to the "rock-salt" surface, is more evident

than is the cause. It is probably caused, however, by the selective absorption of rock-salt in this region of the spectrum. An attempt to determine this shift as a function of the wave-length resulted in the other curves in Fig. 10. At 10.5 μ ($K_2Cr_2O_7$) and at 11.0 μ (K_2CrO_4), there is no observable shift. At 13.0 μ ($K_2Cr_2O_7$), the shift is practically the same as at 15.0 μ ($MgCr_2O_4$). The shift

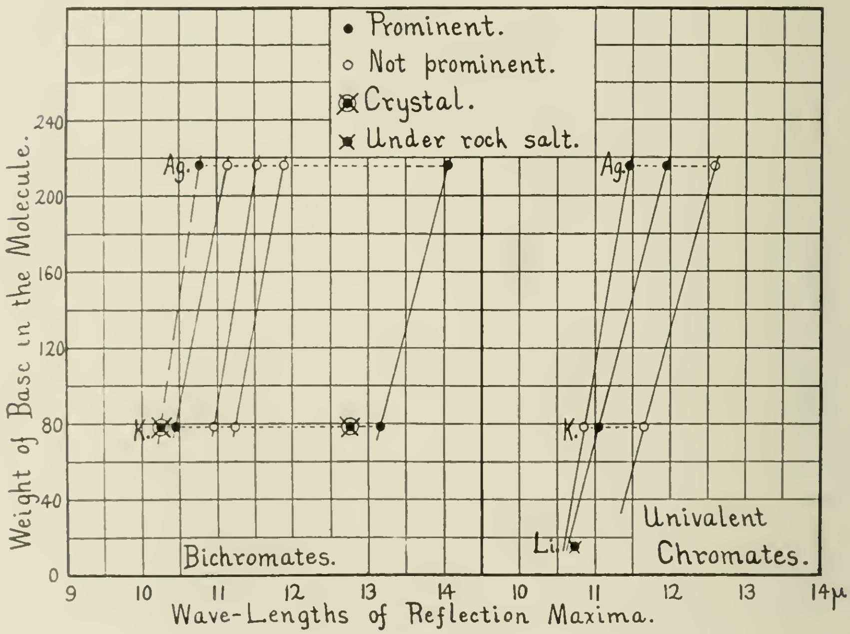

FIG. 11

at 14.0 μ could not be determined. The $Ag_2Cr_2O_7$ changed color immediately on contact wth the rock-salt surface, probably due to formation of $AgCl$. The absence of a reflection peak confirms this view. These are the only surfaces whose maxima seemed sharp and prominent enough for such a test. These data are not sufficient to locate the curve desired, especially as the exact location of the centers of such broad, low maxima is very much in doubt. However, all the maxima located under rock-salt except one (the 12.1 μ band in $MgCrO_4$) are in one of two regions; between 10.5 μ and 11.0 μ where the shift is zero, and near 15.0 μ where

the shift is 0.6 μ. These shifts are assumed to be characteristic of their respective wave-lengths, only. Consequently Table I shows both "observed" and "corrected" locations of the resonance maxima for the only substance concerned, $CaCr_2O_4$, the corrected wave-length being calculated by assuming that the shift for this surface is the same as for $MgCr_2O_4$ under rock-salt (Fig. 10). It seems likely that the 12.1 μ band in $MgCrO_4$ (Fig. 7) may be shifted a little toward the short waves, as are bands in other substances at 13 μ and at 15 μ (Fig. 10) by the rock-salt cover plate.

SHIFT OF MAXIMA WITH WEIGHT OF BASE

Attempts to plot the shift of position of reflection maxima, with atomic weight of base or with total weight of base in the molecule, proved fruitless until the salts considered were limited, in any one comparison, to those having a single typical formula. This, probably, is to be expected. Consequently the bichromates, the univalent chromates, the bivalent chromates, and the chromites are tabulated and plotted separately in Table I and in Figs. 11 and 12. In the typical formulae, M' represents any univalent base; and M'', any bivalent base. Shift lines are drawn for each separate peak in the complex maxima, since many of the maxima are so broad and complicated that their centers of gravity are indeterminable. To compare these results with those of other observers, corresponding data for sulphates, nitrates, silicates, and carbon-

TABLE II

Typical Formula	Salt	Weight of Base in Molecule	Reflection Maxima			Authority
$M''SO_4$	$BaSO_4$	137.4	8.35	8.9	9.1	Coblentz
	$SrSO_4$	87.6	8.2	8.76	9.1	"
	$CaSO_4$	40.1	..	8.6	9.1	"
$M'NO_3$	$AgNO_3$	107.9	7.45		..	Pfund
	KNO_3	39.1	7.05–7.15			Pfund and Coblentz
$M''SiO_3$	$CaSiO_3$	40.1	9.2			Morse
	$MgSiO_3$	24.3	9.1			Coblentz
$M''CO_3$	$PbCO_3$	207.1	7.2	11.94	14.8	Morse
	$BaCO_3$	137.4	6.86	11.60	14.5	"
	$SrCO_3$	87.6	6.76	11.56	14.37	"
	$CaCO_3$	40.1	6.6	11.31	14.2	"

ates are plotted in Fig. 13. (See Table II.) These include all the data obtainable for salts of simple inorganic acids, except those containing water of crystallization, and those which are the sole representatives of their own typical formulae. For the sake of brevity, merely a part of the data on one, only, of the carbonate bands, is given; the abbreviation does not affect the following conclusions.

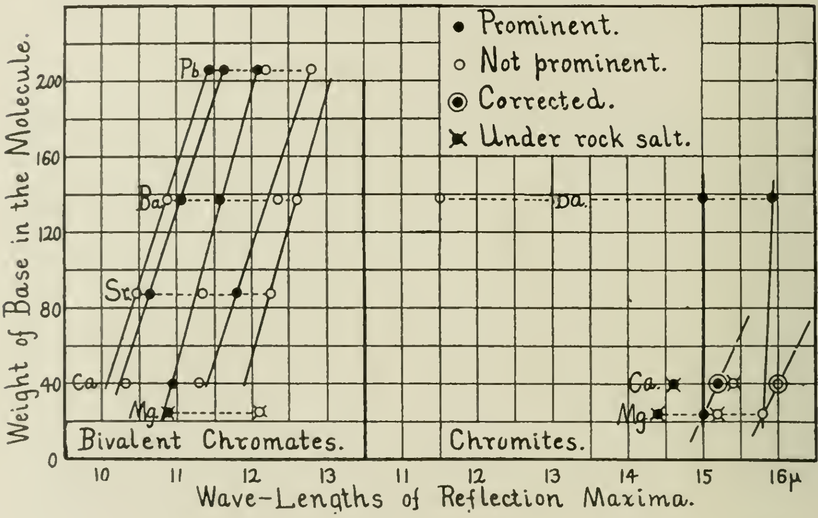

FIG. 12

The chromite shift (Fig. 12) seems so erratic that it will not be considered further in this connection. This may be due, at least in part, to the difficulty with which measurements are made in this region; also to the fact that the maxima are not very definitely located, being broad and low, as well as to the effect of the rock-salt cover plate; all of these chances for error are discussed above. The stray maximum at 11.5 μ in $BaCr_2O_4$ is due to $BaCrO_4$, as mentioned before. W. W. Coblentz[1] gives the reflection of iron chromite from 1.0 μ to 11.0 μ, practically constant at 4 per cent. This result is consistent with the present work on the chromites of barium, calcium, and magnesium.

It is unfortunate that many of the shifts are determined by

[1] "Investigations of Infra-Red Spectra," *Carnegie Publication No. 97*, p. 15.

only two points, especially when those points are as close together as in the case of the silicates (Fig. 13). They are given merely for what they are worth, only for the sake of completeness, without attaching much value to the silicate shift lines; or, possibly, even to the nitrate. The Li_2CrO_4 (Fig. 11) point is so low and faint that it may conceivably belong exactly on the second shift line. Similarly, the 10.9 μ band in $MgCrO_4$ (Fig. 12) may readily belong on the third shift line. The shift lines suggest that the 12.1 μ

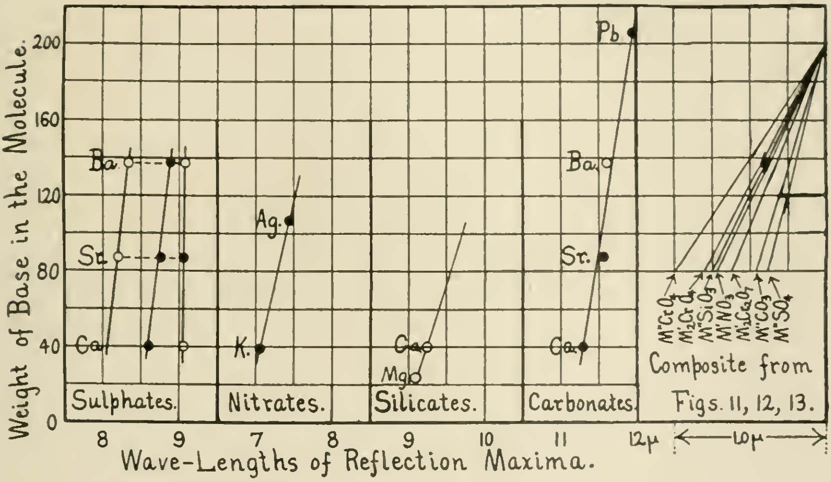

FIG. 13

band in this last salt is due to an impurity. It is possible also that it really belongs at a longer wave-length, as was suggested above. This salt contains about two molecules of water. Rubens and Ladenburg's[1] work on the reflection from water does not suggest the presence of anything in this region of the spectrum due to water. However, their work was on pure water, not on a salt whose molecules may be loaded down with water. Li_2CrO_4 (Fig. 7) also contains water (about one molecule), but its curve gives no indication of it. It is conceivable that a sufficiently high resolving power and amount of energy would show that $CaCrO_4$ and $MgCrO_4$ (Figs. 5, 7, and 12) have five peaks, like their neighbors; and,

[1] *Verhandlungen der Deutschen Physikalischen Gesellschaft*, 11, 16, 1909.

correspondingly, Li_2CrO_4 (Figs. 7 and 11), three peaks. The bivalent sulphates[1] show a similar simplification of a complex band as the atomic weight of the base is decreased.

One shift line from each of these types of salts (except the chromite) is collected in the "composite" in Fig. 13, all plotted from a common point, to show the relative rate of shift. The second (from the short wave-length side) shift line is chosen, some-

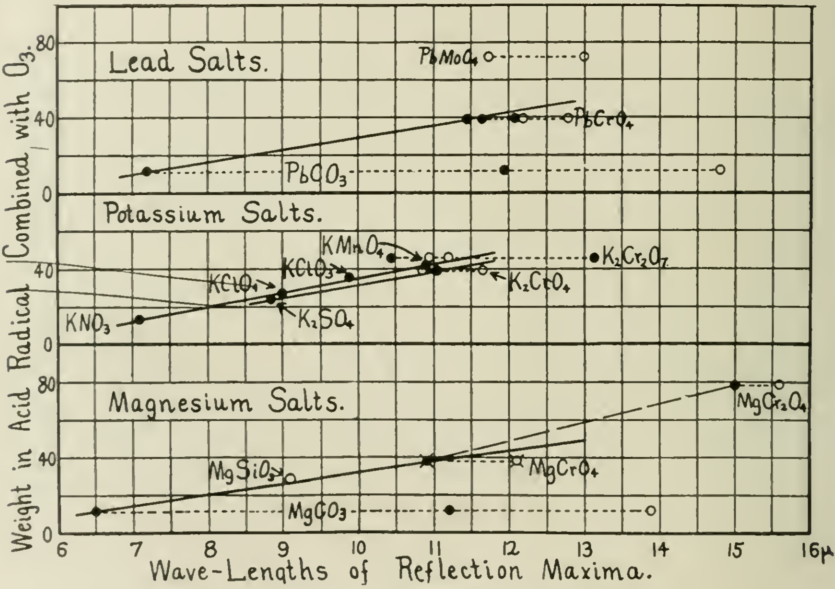

FIG. 14

what arbitrarily, from the bichromates, the univalent chromates, and the sulphates; and the third, from the bivalent chromates. The "composite" has the same scale of ordinates as the others but the scale of abscissas is doubled, to separate the lines somewhat more. The chance for error in determining the exact direction of some of these lines, as mentioned above, is so great that little or no dependence can be placed on their exact order. There seems to be no question, however, that the rate of shift varies with the type of salt.

[1] W. W. Coblentz, "Investigations of Infra-Red Spectra," *Carnegie Publication No. 65*, pp. 77–79.

SHIFT OF MAXIMA WITH WEIGHT IN ACID RADICAL

Figs. 14, 15, and 16 show the positions of resonance maxima as plotted against that fractional part of the total weight of the characteristic element in the acid radical, which is combined with an arbitrarily fixed amount of oxygen, O_3.[1] For instance, in any carbonate, as $MgCO_3$, the atomic weight of carbon, 12, is the plotting value; but in a sulphate, as $MgSO_4$, three-fourths of the atomic weight of sulphur ($\frac{3}{4}\times 32=24$), which is the amount of sulphur combined with three parts of oxygen, O_3, is the plotting value; similarly for the salts of other acids. In order that the observed shift may be due to the acid radical alone, salts of any one base, only, are compared with one another. The data for these curves (see Table III) as for Figs. 11 to 13, inclusive, contain all the data obtainable from all observers, for salts of simple inorganic acids, except those containing water of crystallization, and those individual salts which have the exclusive use of their respective bases. The uncertainty as to the exact direction of these shift lines is even greater than in Figs. 11 to 12, since the different salts of any one base do not always have the same number of peaks, nor are their peaks arranged in the same order of intensity. Here, again, some of the shift lines are, unfortunately, defined by two points, only; this is particularly undesirable in the case of the silver salts, for which the points are very close together. The attempt was made to locate the shift lines by the points which are the most prominent, and hence, presumably, the most nearly representative.

In Fig. 14, $PbMoO_4$ seems entirely out of harmony with the other lead salts. The potassium salts show two shift lines; one for the univalent acids and one for the bivalent. It is hardly to be expected that the two should coincide. The lone dichromate is out of harmony with both of these shift lines. However, it seems just a little surprising that salts in which *both* parts of the acid radical change (as KNO_3 and $KClO_4$) should line up together as well as they do. On the other hand, the relation between these two salts seems much simpler than that between $K_2Cr_2O_7$ and any of the other potassium salts, unless we assume the impossible hypothesis that all the atoms of one kind in a molecule are rigidly locked

[1] L. B. Morse, *Astrophysical Journal*, **26**, 241, 1907.

TABLE III

Base Element	Salt	Weight in Acid Radical with O_3	Reflection Maxima						Authority
Lead	$PbMoO_4$	71.0	11.75	13.0	.....	.....	.....	.....	Coblentz
	$PbCrO_4$	39.0	11.45	11.65	12.10	12.20(?)	12.80(?)	.....	Clark
	$PbCO_3$	12.0	7.2	11.94	14.8	.....	.....	.....	Morse
Potassium	$K_2Cr_2O_7$	44.6	10.25	10.45	10.96	11.23	12.75	13.15	Clark
	K_2CrO_4	39.0	10.86	11.05	11.65	.....	.....	.....	"
	K_2SO_4	24.0	8.85	.....	.....	.....	.....	.....	Pfund
	$KMnO_4$	41.3	10.9	.....	.....	.....	.....	.....	Morse, Coblentz
	$KClO_3$	35.5	9.9	.....	.....	.....	.....	.....	" "
	$KClO_4$	26.6	9.0	.....	.....	.....	.....	.....	" "
	KNO_3	14.0	7.05–7.15	.....	.....	.....	.....	.....	Coblentz, Pfund
Magnesium	$MgCr_2O_4$	78.0	15.0	15.8	.....	.....	.....	.....	Clark
	$MgCrO_4$	39.0	10.9	12.1	.....	.....	.....	.....	"
	$MgSiO_3$	28.3	9.1	.....	.....	.....	.....	.....	Coblentz
	$MgCO_3$	12.0	6.5	11.2	13.9	.....	.....	.....	Morse
Strontium	$SrCrO_4$	39.0	10.48(?)	10.65	11.35	11.80	12.25(?)	.....	Clark
	$SrSO_4$	24.0	8.2	8.76	9.1	.....	.....	.....	Coblentz
	$SrCO_3$	12.0	6.76	11.56	14.37	.....	.....	.....	Morse
Barium	$BaCr_2O_4$	78.0	15.0	15.95	.....	.....	.....	.....	Clark
	$BaCrO_4$	39.0	10.9	11.08	11.58	12.35	12.60	.....	"
	$BaSO_4$	24.0	8.35	8.9	9.1	.....	.....	.....	Coblentz
	$BaCO_3$	12.0	6.86	11.6	14.5	.....	.....	.....	Morse
Silver	$Ag_2Cr_2O_7$	44.6	10.78	11.15	11.54	11.88	14.05	.....	Clark
	Ag_2CrO_4	39.0	11.45	11.95	12.60	.....	.....	.....	"
	$AgNO_2$	21.0	8.0	.....	.....	.....	.....	.....	Morse, Coblentz
	$AgNO_3$	14.0	7.45	.....	.....	.....	.....	.....	Pfund
Calcium	$CaWO_4$	138.0	11.3	11.8	12.4	.....	.....	.....	Coblentz
	$CaCr_2O_4$	78.0	15.2	16.0	.....	.....	.....	.....	Clark
	$CaTiO_3$	48.1	14.2	.....	.....	.....	.....	.....	Morse, Coblentz
	$CaCrO_4$	39.0	10.32(?)	10.96	11.30	.....	.....	.....	Clark
	$CaSiO_3$	28.3	9.2	.....	.....	.....	.....	.....	Morse, Coblentz
	$CaSO_4$	24.0	8.6	9.1	.....	.....	.....	.....	Coblentz
	$CaCO_3$	12.0	6.6	11.31	14.2	.....	.....	.....	Morse

together. These curves would suggest, therefore, what is not inconceivable, that the shift lines of the O_3 and the O_4 acids are so close together as to be indistinguishable, while that of the O_7 acids is observably different from each of the others.

The magnesium salts (Fig. 14) show another variation, the line connecting $MgCrO_4$ and $MgCr_2O_4$ (broken line) having less slant than the main (solid) shift line. However, it would be rather surprising if this were not the case. It is hardly conceivable that *two*

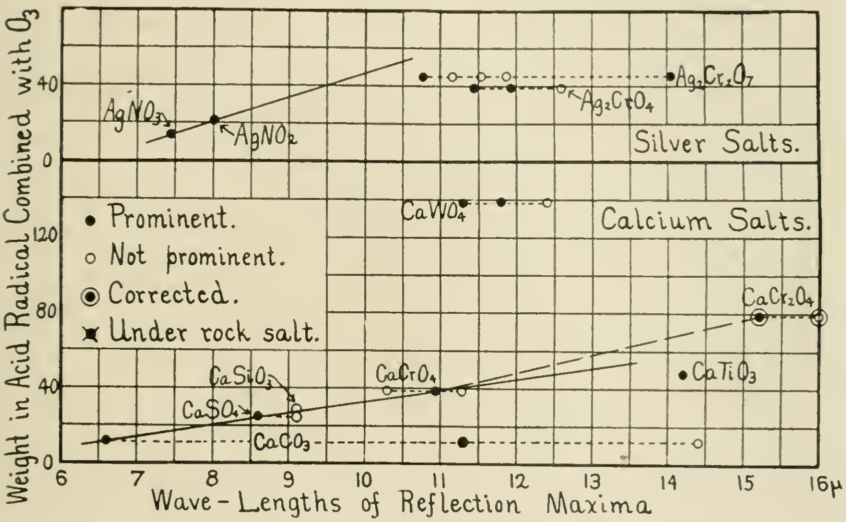

FIG. 15

atoms of chromium would have the same effect upon the shift as a *single* atom having *twice* the atomic weight of chromium. Fig. 12 suggests that the 12.1 μ band in $MgCrO_4$ is due to an impurity; Fig. 14 suggests the same. The short wave-length peak is chosen, somewhat arbitrarily, in all the chromites, in locating the shift line. Choosing the other point would, however, affect the above conclusions in degree only, not in kind.

Ag_2CrO_4 and $Ag_2Cr_2O_7$ (Fig. 15) do not line up with one another, nor with the other silver salts; this is to be expected, as is suggested for the potassium salts. $CaWO_4$ and $CaTiO_3$ seem, like $PbMoO_4$ in Fig. 14, unaccountably out of harmony with their neighbors. There is the same break in the direction of the shift line when

going from $CaCrO_4$ to $CaCr_2O_4$ (Fig. 15), and also from $BaCrO_4$ to $BaCr_2O_4$ (Fig. 16), as in the corresponding magnesium salts. The fact that the *first* (short wave) $MgCrO_4$ maximum, the *second* $CaCrO_4$, and the *third* $BaCrO_4$ are on the shift lines in Figs. 14, 15, and 16, confirms the suggestion that these are corresponding peaks. (See the third shift line for the bivalent chromates, Fig. 12.) The "corrected" values, only, are shown in Fig. 15 for $CaCr_2O_4$, obtained from the values for the "rock-salt" surface as

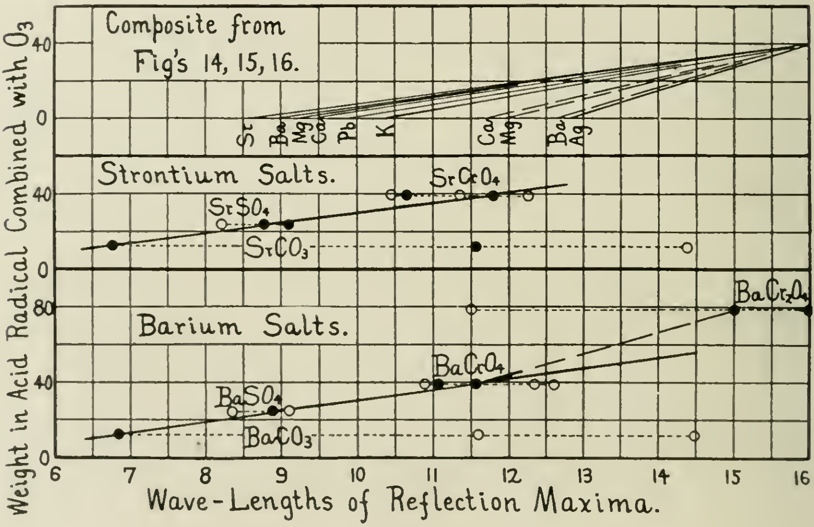

FIG. 16

indicated above. The 11.5 μ point in $BaCr_2O_4$ is due to the presence of $BaCrO_4$, as mentioned above. The explanations of the significance of the different kinds of plotting points, as given on Figs. 12 and 15, apply to Figs. 12 to 16, inclusive. The strontium salt shift is also shown on Fig. 16. At the top of this figure are plotted all the shift lines from Figs. 14 to 16, inclusive, from a common point, on the same scale as the original lines. Solid lines correspond to solid, and broken to broken.

As in the case of the composite of shift lines in Fig. 13, not much dependence can be placed on the relative location of two adjacent shift lines. There is no question, however, that the

broken lines are considerably steeper than the solid; this, as indicated above, is to be expected.

SUMMARY

1. Reflection curves for the available salts of the chromium acids have been determined in the region 1.5 μ to 15.0 μ or 16.0 μ. The short wave-length portions of these curves are omitted here, because the reflection is constant in that region.

2. One surface was a polished crystal, which should be examined further with polarized energy. Some surfaces were made from the powdered salt, compressed against a steel plate, and polished; others were pressed against glass, to avoid polishing. Still others (including those strongly hygroscopic) were pressed against rock-salt cover plates, and the reflection measured without removing the cover plates. This form of surface may prove valuable for very hygroscopic salts, but its errors must be investigated further than has been done in the present work.

3. The dichromates show two complex maxima, in the regions 11.0 μ and 13.5 μ, respectively. Each of the other salts shows only a single complex maximum; these maxima are in the region 11.0 μ to 12.0 μ for all but the chromites, in which they are from 15 μ to 16 μ. In general, the salts of any one type have the same number of peaks in each complex maximum.

4. One salt, K_2CrO_4, ought to prove a good source of *Reststrahlen*, with a mean wave-length of about 11.0 μ.

5. When salts of one type, only, are compared with one another, all (with the exception of the chromites) show an approximately linear shift of the resonance region toward the long waves, with increasing atomic weight in the base. This shift holds, not only for resonance regions as a whole, but, in most cases, for individual peaks in complex maxima.

6. Representative shift lines for all of these types of salts, as well as for other types taken from the data of other observers, suggest that each type of salt has its own characteristic rate of shift.

7. A rate of shift enormously greater than the above is shown for all the salts of any one base, when the shift is plotted against that fractional part of the weight of the characteristic element in the acid radical, which is united with three atoms of oxygen.

8. Although the chance for error makes the exact location of shift lines of this sort much less determinate than that of shift lines of the first sort, it seems likely that the rate of shift is characteristic of salts of any one base. It is evident, however, in the case of the chromates and chromites concerned, that the shift due to replacing one chromium atom by two is much less rapid than if the one atom were replaced by another of twice its atomic weight.

9. The shifts of reflection maxima, as pointed out by Pfund, Coblentz, and Morse, are therefore confirmed for salts of the chromium acids, and further relations are pointed out.

The author is indebted to nearly everyone in the department of physics of Columbia University, where this work was done, also to several members of the departments of chemistry and mineralogy, for suggestions or help of various kinds. He is especially indebted to President E. F. Nichols of Dartmouth College, who set the problem and suggested many of the methods employed in its solution, and who has so kindly kept in touch with it as far as possible, since leaving Columbia for Dartmouth.

Syracuse University
October 12, 1911

NOTES ON BAND SPECTRA BY W. RITZ[1]

By PIERRE WEISS

I. MECHANISM OF EMISSION OF BAND SPECTRA

In the paper[2] in which Ritz studies a simple electromagnetic mechanism emitting series spectra, he states on p. 673 (*Œuvres*, p. 112):

> These theories are not applicable to band spectra. I will only say in this connection that they might perhaps be ascribed to closed rings or polygons consisting of the elementary magnets under consideration, on the assumption that such formations play an important rôle in the construction of the atom, and that they must first be touched off by the electrical or chemical processes producing the light before a series spectrum can come into existence.

Among the papers left by Ritz there has been found a small sheet bearing some equations and a rough draft of the theory of this mechanism of emission of band spectra, which I shall attempt to expand.

Let us recall that the organ of emission of *series spectra*, conceived first by him as formed of magnetic and non-magnetic rods, juxtaposed in a straight line, may be realized in different ways. Ritz preferred to regard[3] the small magnetic rods as produced by solids of revolution charged with electricity at their surface and having a rapid rotational motion around their axis. He had taken into account particularly that for any solids of revolution there can be found a superficial distribution of electricity which renders them equivalent to systems of two magnetic poles situated on the axis. When the magnetic poles approach the surface, the electric density increases indefinitely in their vicinity and the surfaces carrying the electricity become practically equivalent to point charges. These solids are alternately positive and negative, and endowed

[1] Note added to the *Œuvres de W. Ritz*, published by the Société suisse de physique, (Gauthier-Villars, 1911). Translated from the MS from M. Weiss.

[2] "Magnetische Atomfelder und Serienspektren," *Annalen der Physik*, **25**, 660, 1908; *Œuvres de W. Ritz*, chap. vii, p. 98.

[3] *Op. cit.*, p. 670.

with rotations in opposite directions. They are fixed each to the other, in the form of a linear chain, by their electrostatic attraction. Ritz had thought that the non-magnetic rods, required also by the theory of series spectra, might be bodies similarly charged but deprived of rotation, and then, going a step farther, that the vibrating electron and the free electric pole at the extremity of the file of rods are one and the same thing.

Disregarding now the non-magnetic rods, let us consider a file of magnetic rods. We may assume that when it is subjected to a tension a^2 it is capable of vibrating in a manner analogous to that of a cord, or rather that of a chain.

There prevails along this cord a magnetic field H directed lengthwise, and it carries equidistant electric charges. Let us assume that these charges set up circular vibrations around the axis under the combined influence of the tension a^2 and the field H. (Ritz's notes say nothing in regard to the reason why the effect of the field on the adjacent positive and negative charges does not neutralize them. We may perhaps invoke for this purpose a difference of configuration of the positive and negative charges, which has been assumed elsewhere. Those occupying a more extended place would be, for example, partially outside the field.)

The vibratory state is then expressed by

$$\left.\begin{aligned} y &= A \sin \frac{\mu \pi x}{a} \sin \nu t \\ z &= A \sin \frac{\mu \pi x}{a} \cos \nu t \end{aligned}\right\} \qquad (1)$$

where a is the distance between two consecutive knots for the fundamental vibration, and $\nu : 2\pi$ the frequency. The equations of motion of an element of cord, dx, of mass μdx, and of charge ϵdx, will contain the term of inertia and the forces proceeding from the magnetic field and from the tension of the cord:

$$\left.\begin{aligned} \mu \frac{\delta^2 y}{\delta t^2} + \epsilon H \frac{\delta z}{\delta t} - a^2 \frac{\delta^2 y}{\delta x^2} &= 0 \\ \mu \frac{\delta^2 z}{\delta t^2} - \epsilon H \frac{\delta y}{\delta t} - a^2 \frac{\delta^2 y}{\delta x^2} &= 0 \end{aligned}\right\} \qquad (2)$$

from which, substituting (1), twice the same equation:

$$\mu \cdot \nu^2 + \epsilon \cdot \nu \cdot H - \frac{\mu^2\pi^2}{a^2} a^2 = 0 \qquad (3)$$

in which H can also be replaced by $-H$, then

$$\nu = \pm \frac{\epsilon H}{2\mu} \pm \frac{\epsilon H}{2\mu} \sqrt{1 + \frac{4m^2\pi^2\mu}{\epsilon^2 H^2 a^2} a^2}\,. \qquad (4)$$

The solutions corresponding to the two positive signs and to the two negative signs are alone acceptable. In placing $\frac{4\pi^2 a^2 \mu}{\epsilon^2 H^2 a^2} = k^2$, a number which is small when the tension of the cord has a subordinate rôle with respect to the field, we have

$$\nu = \frac{\epsilon H}{\mu}\left[1 + \frac{m^2k^2}{4} - \frac{m^4k^4}{16} + \dots\right]. \qquad (5)$$

For $m = 0$, we find the frequency of a charge describing a circle in the field H. If we stop with the second term, we have Deslandres' law, with $\nu_0 = \frac{\epsilon H}{\mu}$ for the head of the band. If we retain the third term, ν increases less rapidly, as experiment requires.

Ritz is here brought to choose between a rectilinear file and a closed ring. He gives the preference to the latter in the following terms: "If there are two extremities, the lines would have to be first simple ($m = 1, 2 \dots$), for as m increases, the different vibrations will correspond accordingly as we are at the extremities or the center. Consequently, circular ring."

Since ν increases with m, the band has the head on the red side. To obtain decreasing values, it is necessary to assume a^2 negative. The ring, instead of being extended, is compressed in the direction of the periphery.

The separation between two consecutive lines is given from the complete formula (4) by

$$\frac{d\nu}{dm} = \frac{\nu_0}{2} \frac{mk^2}{\sqrt{1 + m^2k^2}}\,.$$

It increases more slowly than Deslandres' law indicates, and that is in accord with the experiment. But the separation does not

cease increasing. The formula does not give then the maximum of separation of the experiments of Kayser and Runge[1] on the spectrum of cyanogen.[2]

Among some other notes of Ritz there is the trace of numerous attempts to find the best empirical formula with three constants representing these experiments of Kayser and Runge. These notes seem to be prior to his ideas on the electromagnetic origin of spectra and are accordingly only indirectly related to the above. He tries particularly

$$\nu = a + bm^2\sqrt{1+cm^4}$$

and the first three terms of its development

$$\nu = a + bm^2 + cm^6,$$

and finds that the term in m^6 varies too rapidly. He tries

$$\nu^2 = a + bm^2 + cm^4$$

and

$$\nu = a + bm^2 + cm^4.$$

He finds this last formula preferable to the others, and notes in this connection that "*from the 160th line the functions* $\nu = f(m)$ *and* $\nu^2 = f(m)$ *behave in a manner not regular.*"

We shall revert to this point. He tries further

$$\nu^2 = \frac{a + bm^2}{1 + cm^2},$$

and the error is a little larger than formerly.

In a conversation Ritz expressed an idea which relates to the mechanism of emission what he calls "the irregular character of the function ν for the lines of high order." He expressed himself about as follows:

There are in certain bands a considerable number of lines whose position is determined with exactness; but, whatever the empirical law by which we

[1] *Wiedemanns Annalen*, 38, 80, 1889.

[2] The same notebook also contains the following information, referring to another possible solution of the problem, in which the tension a^2 is not involved: "Beyond the constant magnetic field which it produces for its whole length, a ring can still be submitted to exterior magnetic fields, variable from point to point, and weak with respect to the first."

seek to represent the distribution of the lines in these bands, there comes a time when for a number of high order of lines this law fails. If we have recourse to a graphical representation, the curve turns short with an abruptness which the usual formulae do not give.

Let us assume that the part of the atom whose vibrations emit band spectra has a structure analogous to that of a chain composed of links of known length. One would understand then very well that the vibrations are produced for the greatest part of the phenomenon as if the chain were a continuous structure, while for wave-lengths of nearly the same length as the link (or for certain particular values in relation with it) the numbers of vibrations are influenced by the finite length of the element.

II. STRUCTURE OF THE BANDS

To the question, "Is it not established that the bands have sometimes two 'heads,' one on the side of large wave-lengths and the other on the side of small wave-lengths?" (hypothesis of Thiele[1]), Ritz replied merely, "That idea is not tenable."

The tables of numbers found in his notes show that this conviction is based also on the study of the bands of cyanogen observed first by Kayser and Runge,[2] and later by Jungbluth,[3] with heads at λ 3883.56, 3871.53, 3861.85, and 3854.85.

In a few words the status of the matter is this: King,[4] having discovered new heads directed from the side of short wave-lengths, has believed it possible to consider them as "tails" corresponding to the "heads" previously known, and has associated them by making the bands overlap each other. As a proof of this coordination, he gives numerical relations between the wave-lengths of the heads and tails. They are contained in the following table:

T_n	Q_n	T_n/Q_n	T_n	Q_n	T_n/Q_n
3590.52	3203.84	1.12069	3883.60	3465.69	1.12059
3585.99	3180.58	1.12746	3871.59	3433.17	1.12770
3584.10	3160.32	1.13409	3861.91	3405.04	1.13417

The value of this table as a demonstration seems to me small. According to Deslandres' law, which is applicable to the heads of

[1] *Astrophysical Journal*, **6**, 65, 1897.

[2] *Wiedemanns Annalen*, **38**, 80, 1889.

[3] *Astrophysical Journal*, **20**, 237, 1904.

[4] *Ibid.*, **14**, 323, 1901.

bands of a series, as well as to the lines of a band, the distances between the successive heads, measured on a scale of frequencies, form an arithmetical progression. Suppose that we associate two series of bands turned in opposite directions, and both obeying this law, but entirely independent as to their origin. If the ratios of the two arithmetical progressions are close, as frequently happens,[1] the distances between the heads and tails will also form an arithmetical progression (criterion of dependence quoted by Jungbluth). It will be the same whatever two bands are taken to start with; and this will happen according as the ratios will be of the same or of contrary sign, when the first bands are displaced in the same or in contrary sense.

In the first approximation, the quotient of the frequencies will vary, in the same cases, in arithmetical progression. This is what King finds. In regard to the great similarity of the series of successive bands, we should not know how to attach any importance to the fact that this quotient passes twice approximately through the same three values.

This argument appears, nevertheless, to have had sufficient weight in the conviction of Kayser,[2] who considers it certain that King has found the tails corresponding to the heads, and that consequently the hypothesis of Thiele is correct.

Jungbluth proposes to test this hypothesis by making new measures on a part of the bands formerly known. To discuss them (Fig. 1),[3] he uses as abscissas the wave-lengths, and as ordinates the difference of wave-lengths of two successive lines. The curves which he thus obtains for four of the bands of cyanogen proceed from their heads T_1, T_2, T_3, and T_4 in an approximately parabolic manner, which corresponds to Deslandres' law $\nu = A + (Bm+C)^2$, where ν is the frequency and m a whole number; but for the lines of high orders the curve is distinctly below the parabola and the separation of the consecutive lines passes even through a maximum. These experimental curves are continued in dots and

[1] Ch. Fabry, *Journal de physique*, 4, 245, 1905.

[2] *Handbuch der Spectroscopie*, 2, 487.

[3] The figure given here is Jungbluth's redrawn on the basis of the tabular values contained in his paper.

seem to end naturally, in the original drawing by Jungbluth, for the last three bands with tails Q_2, Q_3, Q_4 indicated by King. For the first, there being no head according to King in the region where Jungbluth expects it, he carries the curve on and thus determines

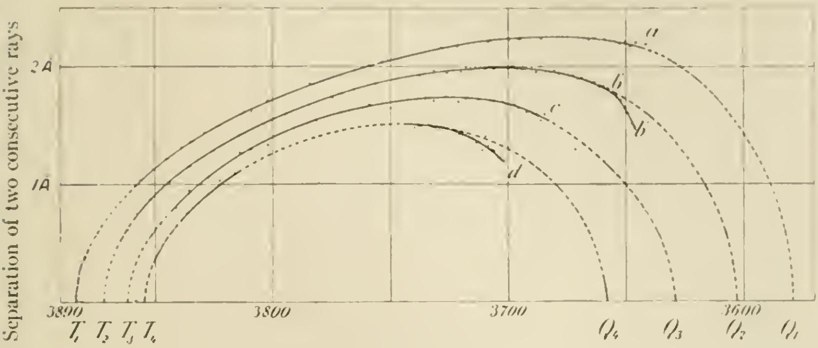

FIG. 1

the position of the tail Q_1 in a region where it is not observable on account of the presence of an intense band.

Omitting this, we have:

Heads according to Jungbluth	Heads according to King	Assumed Tails
T_2 3871.53	4152.93	Q_2 3603.12
T_3 3861.85	4158.22	Q_3 3628.98
T_4 3854.85	4165.54	Q_4 3658.27

The co-ordination of the heads and tails according to Jungbluth is therefore in direct contradiction to that of King. Moreover, according to Jungbluth, the complete bands of King overlapping each other are replaced by bands fitting one within the other (Fig. 2).

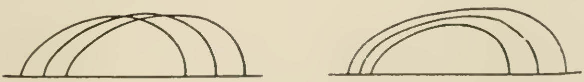

FIG. 2

Jungbluth, who expressly notes this circumstance, does not fear to add that, taken in connection with the numerical relations of King, it brings a new confirmation to King's views.

To discover exactly what is the import of the experiments of Jungbluth, I have marked on the drawing (Fig. 1) the points observed. The experimental part, in heavy line, ends for the four bands in *a*, *b*, *c*, *d*. For two of the bands, at *b*, and at *d*, it is separated from the part extrapolated by Jungbluth, represented by a dotted line, by a greater curvature, seeming to confirm the idea of Ritz, and rendering impossible the assignment of the tails made by Jungbluth.

But if we compare the experiments of Jungbluth with that of Kayser and Runge, the agreement which is good as far as b' ceases. It is easily seen that Jungbluth has at b' passed inadvertently to the lines of a neighboring band, which are placed in respect to those that he has followed to that point somewhat as the lines of a vernier are to those of the principal scale. Moreover, Jungbluth suppressed in his drawing the portion $b'b$. Definitively for the four bands, the correlation between the heads and tails is admissible rigorously for two of them (T_2-Q_2; T_3-Q_3).

Ritz's idea of the irregular nature of the function ν for the lines of high order which rests on the sharpness of the turn at b' has in part as its origin an error of Jungbluth.

It seems, nevertheless, that as a result of the paper by Jungbluth the conviction has become general that the hypothesis of Thiele is correct. In 1905 A. Hagenbach[1] expressed this in a monograph on band spectra. It has not been remarked that in reality the *conclusions* of Jungbluth at the end of his paper are much less affirmative than his *curves*.

We find in Ritz's notes the following: "The tails according to King, particularly the one at λ 3603, are impossible, because they are composed of lines relatively intense with almost *constant differences*, while the differences ought to increase very rapidly toward the head of the band."

This remark is very probably suggested by an examination of the plate of Kayser and Runge[2] on which it is easy to recognize the appearance described by Ritz. It is possible to measure the distance of the lines to about 0.5 Å, which carries the arc of the

[1] *Wullner-Festschrift*, 133, 1905.

[2] *Akad. Berlin. Phys. Abh. nicht zur Akad. gehör. Gelehrter*, 1, 44, 1889.

corresponding curve well outside the limits of the figure. The same is visible on the plate of Jungbluth (*op. cit.*).

This argument seems definitively to destroy what remains of probability in the assumptions made by Jungbluth. We have already done justice incidentally to the argument which Jungbluth deduces on the ground that the *lengths* of the bands vary in arithmetical progression. Let us mention that, on the contrary, Ritz notes carefully, as an important fact, the arithmetical progression, pointed out by Jungbluth, of the maxima separations of the lines of the four bands (2.25; 2.00; 1.75; 1.5 Å). This fact retains its value independently of every hypothesis as to the existence of one head or of two heads.

We may then conclude that the hypothesis of Thiele is strengthened by:

1. The existence of heads directed in the two senses.
2. The existence of a maximum in the separation of the lines.

But we have not been able to continue the demonstration of this hypothesis up to the present time, either by the possibility of co-ordinating without confusing the heads and tails, or by pursuing the decrease of the distance of the lines in an interval sufficiently extended beyond the maximum. The separation of the lines in the region of the tail at λ 3603 disproves it strictly.

Ritz's idea is not in contradiction with the facts. But the indications in his favor which remain with regard to the four bands of cyanogen are slightly diminished after suppressing the faulty part, $b'b$, of Jungbluth's.

It would be of great interest to make new determinations on bands composed of a large number of lines and, perhaps, to resume the discussion of the data already found.

ZURICH, SWITZERLAND
March, 1911

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The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language may be accepted for publication, but unless a wish to the contrary is expressed by the author, they usually will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts typewritten, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

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THE SPECTRUM OF COMET MOREHOUSE

By A. FOWLER

In a recent description and discussion of their admirable photographs of the spectrum of Comet Morehouse,[1] Messrs. Pluvinel and Baldet have confirmed the presence of the "low pressure" spectrum of carbon monoxide to which I had previously called attention.[2] The results of their work in this connection are summarized in the following words:

> The number of our doublets is 21, while Fowler's are only 12. Hence, there remain 9 doublets which have not been observed in the laboratory. Furthermore, of the 12 doublets, 2 do not agree very satisfactorily, as already explained. But the accordance of the two spectra in their brightest and most conspicuous parts is too close to admit of any doubt that the doublets in the spectrum of Comet Morehouse are practically identical with those of carbon monoxide at very low pressure.

Pending a more complete investigation of the laboratory spectrum, I should like to explain that in preparing my list of bands I was careful to include only such doublets as could be well established on the photograpbs available; some were doubtless overlooked on account of their faintness and admixture with other bands characteristic of carbon monoxide at higher pressures. The less refrangible parts of the spectrum in particular were incompletely recorded on this account, and most of the cometary doublets

[1] *Astrophysical Journal* **34**, 89, 1911.

[2] *Monthly Notices R. A. S.*, **70**, 176, 484, 1910.

not shown in my tables were situated in this region. The cometary spectrum, however, may be expected to aid in the proper identification of the bands in the terrestrial spectrum, as it is evident that in the tails of comets the peculiar spectrum in question is more completely isolated than in any laboratory spectrum which I have hitherto obtained. An increase in the relative intensity of the tail bands as the density of the gas is diminished is very definitely indicated by the observations which have been made, and the present difficulty is to produce sufficient luminosity when the gas is rarefied to such an extent that all other carbon monoxide bands might be expected to disappear. Further experiments, however, will be undertaken with special reference to the missing bands.

As to the discrepancies in the tabulated wave-lengths of the terrestrial and cometary bands, no claim for great accuracy could be made in my own work, as the spectrum was always feeble, and only small dispersion could be employed. Nevertheless, the errors in the main were unlikely to amount to more than an Ångström unit, except perhaps for the bands in the green, and I have gone over the measurements again without finding any substantial changes in the published wave-lengths to be necessary. In particular, I have confirmed the position of the band λ 3415, 3429, which Pluvinel and Baldet identify with their comet doublet 3436, 3446, and the discordance must therefore be attributed to the faintness of this part of the cometary spectrum, and the probable absence of suitable reference lines in this region of the stellar comparison spectrum which was used in the determination of wave-lengths. The other principal discrepancy to which attention is drawn is between the doublet of the laboratory spectrum at λ 4887, 4916, and that of the comet 4846, 4879; the former was noted by me only as "an indication of a faint band," and it should not have been identified with a cometary band so far distant. Considerations as to the arrangement of the bands in series, however, make it probable that the comet doublet λ 4846, 4879 really belongs to the carbon monoxide spectrum, as will appear later. The want of agreement in the wave-lengths, as a whole, is not greater than might be expected from the nature of the photographs of the

cometary spectrum given by a prismatic camera, and from the use of the spectrum of *Vega* as the only term of comparison.

The arrangement of the bands in series by Pluvinel and Baldet seems to call for further consideration. I wish to suggest that the less refrangible bands which they have included in the brighter (A) series should not be regarded as part of this series at all, but as forming a distinct series in themselves, probably related to the other series in the manner indicated by the work of Deslandres in the case of nitrogen. Taking my own values for the more refrangible components of the doublets of the A series (i.e., Pluvinel and Baldet's A_2) the positions are given very closely by the equation

$$n = 65008 - 13 \cdot 5(m + 0 \cdot 444)^2,$$

where n is the oscillation-frequency in air, and m has successive integer values ranging from 51 to 56. The resulting calculated values are compared with the observed wave-lengths in Table I.

TABLE I

m	λ calc.	λ obs.	O−C
56	4545·4	4545·4	0.0
55	4253·9	4253·2	−0.7
54	4001.3	4001.3	0.0
53	3781.0	3781.0	0.0
52	3587.0	3587.0	0.0
51	3415·3	3415·0	−0.3

The satisfactory agreement in the case of these six well-established bands suggests that the formula may be used to predict the approximate positions of any less refrangible bands which may form part of the same series. These work out at 4887, 5292, 5779, and 6375; but with the possible exception of the first, it is to be expected that they would be too faint for observation, and none of them agrees with Pluvinel and Baldet's measurements of the cometary bands.

A very similar result is obtained even if the probably less accurate wave-lengths given by Pluvinel and Baldet are made the basis of calculations. The bands corresponding to the first, third, and fifth, of Table I, give the equation

$$n = 73392 - 11 \cdot 25(m + 0 \cdot 6)^2.$$

The positions calculated from this equation are shown in the second column of Table II, which also indicates the observed *minus* computed wave-lengths, compared with those derived by Pluvinel and Baldet from their own formula.

TABLE II

m	λ calc.	λ obs. in Comet (P & B)	O−C	O−C (P & B)
67	4549.2	4549.2	0.0	+17.8
66	4256.8	4256.9	+ 0.1	+ 5.4
65	4003.4	4003.4	0.0	− 3.0
64	3781.6	3782.6	+ 1.0	− 7.4
63	3586.0	3586	0.0	−11.6
62	3412.3	3436	+24	+10.5
61	3257.0	3269	+12	− 1.5

The positions of probably fainter bands of the same series given by this formula are 4890, 5292, 5774, and 6362, which do not differ materially from those calculated from my own wave-lengths. The important point is, however, that the five best determined bands can be represented much more closely than by the use of Pluvinel and Baldet's formula, in which the constants are based in part on the positions of less refrangible bands which they have assumed to belong to the same series. These less refrangible bands, if they belong to carbon monoxide at all, would therefore seem to constitute a separate series.

The fainter (B) series of doublets is apparently related to the A series in the usual manner; namely, that on the frequency-scale, one of the series may be superposed on the other, at least approximately, by an appropriate displacement. In other words, the constant a in Deslandres' formula $n=a+bm^2$ is alone different for the two series. Thus, for the more refrangible components of the B series of doublets, using my own wave-lengths, the equation becomes

$$n=62822-13\cdot5(m+0\cdot444)^2.$$

The calculated and observed values are compared in Table III.

The errors are not greater than might be expected from the nature of the data on which the calculations are based. The formula predicts other possible doublets of the B series with their

more refrangible components about 5984 and 6614, but these would probably be relatively faint.

From analogy with other band spectra, it is possible to predict additional series of bands of the tail spectrum, but until more accurate measures become available for the known bands it would be unwise to attach undue importance to the results. The value of the constant a for the extra series which is of most immediate interest may be derived from the corresponding numbers for the A and B series thus: $a_1=65008$; $a_2=62822=65008-2186$; $a_3=62822-(2186-2\times 13.5)=60663$. The assumption that the second difference between successive values of this constant is identical

TABLE III

m	λ calc.	λ obs.	O−C	Remarks
57	5472.3	5473	+0.7	
56	5047.4	5049	+1.6	
55	4690.0	4688.5	−1.5	
54	4384.8	......	......	Confused with other bands in laboratory spectrum
53	4121.2	......	......	
52	3892.3	3891	−1.3	Doubtfully observed in laboratory spectrum
51	3690.9	3693	+2.1	

with that in the A and B series is not necessarily true, but they are usually not very different, and no great errors are likely to be introduced by taking them to be equal only when considering a series which is adjacent to the two which have been observed. Hence a third series may be approximately represented by the equation

$$n=60663-13\cdot 5(m+0\cdot 444)^2.$$

Doublets having their more refrangible components in the neighborhood of λλ 6872, 6205, 5665, 5218, and 4843 are therefore possible. It is probably this series which accounts for the less refrangible cometary bands included by Pluvinel and Baldet in their A series, their computed positions for the corresponding bands being 6853, 6205, 5674, 5230, and 4854. There are no very great differences in the positions of these bands calculated by the two processes, but my own values have been obtained without sacrificing the accuracy with which the well-established bands in the blue

and violet may be represented by Deslandres' formula. The well-marked cometary band λ 4846 is also more accurately represented by the λ 4843 of my computation than by the λ 4854 given by Pluvinel and Baldet's equation. The accuracy of the comet measures is probably not sufficient to distinguish between the respective values for the remaining bands. From these considerations, however, it is extremely probable that the less refrangible bands of the comet's spectrum had the same origin as those in the blue and violet.

In what has gone before, no account has been taken of the less refrangible components of the doublets.[1] From my own measurements, and from analogy with certain other band spectra, there is every reason to believe that the intervals between the two components are constant throughout, if measured on the frequency-scale. Hence, the less refrangible components of the doublets should be represented by the same equations as the more refrangible ones if the separation of the components, amounting to about 118 on the frequency-scale, be subtracted from them.

If estimated in Ångström units, the intervals between the components of the doublets would vary as the squares of the wave-lengths, and would range from 14 Å at λ 3415 to 56 Å at λ 6872. Since the dispersion in the prismatic spectrum is nearly in inverse ratio to the cube of the wave-length, the separation on the photographic plates should be nearly inversely proportional to the wave-length. The doublet intervals observed in the comet spectrum by Pluvinel and Baldet, or computed by their formulae,[2] are of corresponding orders of magnitude for the bands on the more refrangible side of $H\beta$, but are considerably greater for the less refrangible bands. The discrepancies are probably to be attributed to the difficulty of obtaining accurate measurements in the case of the comet, but if Pluvinel and Baldet's formulae were correct, there should have been no difficulty in resolving the bands near 5218, 5665, and 6205, which their plates failed to show as doublets. My

[1] Professor Newall has found that, in the laboratory spectrum, each component of the doublets is itself a close pair, but this feature may be disregarded in the present discussion.

[2] The formulae are given in *Comptes Rendus*, 148, 760–761, 1909.

own formulae, however, place the components closer together on the photographs than in the case of the bands in the blue, and therefore accord better with the comet observations; the formulae would not, however, admit of the comet bands 6848.4 and 7027.4 being regarded as the separate components of a doublet, as is assumed by Pluvinel and Baldet.

Although the identity of the cometary bands with those of low-pressure carbon monoxide may be considered as beyond question, it may be of interest to give the additional direct demonstration which is afforded by the photographs reproduced in Plate IV. For the photographs of the comet and its spectrum I am indebted to Dr. H. D. Curtis of the Lick Observatory, who has kindly placed enlargements on glass at my disposal. The photographs were taken at Santiago, Chile, on March 20, 1909, and the objective spectrograph employed had sufficient resolving power to show clearly the four principal bands of the comet as doublets.[1] The carbon monoxide spectrum is placed alongside that of the comet for easy comparison, and the identity of the two spectra will be evident at a glance. The strong band on the left in the terrestrial spectrum is the negative band of nitrogen 3914, due to an impurity in the carbon monoxide, and it will be seen that this is appropriately represented by a single band in the comet; other comets which have shown the carbon monoxide doublets, however, have not shown the nitrogen band.

It should be noted in conclusion that out of the five or six recent comets which have shown the low-pressure carbon monoxide spectrum, it was only in comet Morehouse that the bands were bright in the head as well as in the tail. The bands in question may therefore be regarded as especially characteristic of the tails of comets.

IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY
SOUTH KENSINGTON
December 6, 1911

[1] See Lick Observatory *Bulletin* No. 163.

"ON THE BEST VALUE OF THE SOLAR CONSTANT"

By C. G. ABBOT and F. E. FOWLE, Jr.[1]

It seems rather discouraging, after having spent ten of the best years of our lives on the determination of the solar constant of radiation, to be told[2] that we have relinquished reformed methods having a rational basis given them by Langley in favor of methods which do not differ essentially from those of Pouillet; that we have abandoned an essential principle recognized by many able investigators; that our value of the solar constant is to all intents and purposes the same as that of Pouillet, whose observations would yield our result if known corrections were applied for defects in the water pyrheliometer; that the admirable agreement of our separate measures and the small apparent probable error of our final result are but characteristics of the original Pouillet model, not to be regarded as evidences of the trustworthiness of our work; and finally that it is known that reliable measures of solar radiation may be made within the atmosphere which exceed the supposed value outside the atmosphere, as will be shown by Very in a subsequent paper.

We await with great interest the last item, namely: "reliable measures of solar radiation made within the atmosphere which exceed" 1.93 calories per square centimeter per minute. Our own pyrheliometers we thoroughly believe to be trustworthy instruments, for we have spent about seven years in establishing the standard scale of radiation. We have read our pyrheliometers in three successive years on Mount Whitney, altitude 4420 meters, the highest point in the United States, with exceptionally pure and dry air above us, and never yet have obtained readings exceeding 1.75 calories per square centimeter per minute. Had we used Ångström's pyrheliometer, adopted by the Solar Union as a standard instrument, our readings would have been 5 per cent lower.

[1] Published by permission of the Secretary of the Smithsonian Institution.

[2] F. W. Very, *Astrophysical Journal*, **34**, 371, 1911.

Ångström himself observed on Teneriffe,[1] at an altitude of 3683 meters, and never obtained values exceeding 1.63 calories.

To grasp the essential features of our determination of the solar constant of radiation the reader must consult: (1) the theory of our method given in *Annals of the Astrophysical Observatory*, Vol. II, pp. 13 to 17, 18 and 19; (2) the discussion of sources of error, pp. 58 to 82; (3) the procedure we pursue, pp. 50 to 57; (4) the measurements, pp. 83 to 98. Also, in recent years we have made very important improvements and measurements additional to those given in Vol. II of the *Annals*, as stated in this Journal, **29**, 281; **33**, 125; **33**, 191, and **34**, 197. Our work comprises spectrobolometric and pyrheliometric measurements made at high and low sun according to the method of Langley, as stated by him in his *Report of the Mount Whitney Expedition*, pp. 135 to 142, and Table 120, values 1 to 5. Our measurements have been made at Washington (sea-level), 1902 to 1907; Mount Wilson (1750 meters), 1905 to 1911; Mount Whitney (4420 meters), 1908, 1909, 1910; and at Bassour, Algeria (1160 meters), 1911. The total number of spectrobolometric solar-constant determinations now exceeds 600. All agree within narrow limits of variation to fix the value of the solar constant in terms of our standard water-flow pyrheliometric scale at about 1.93 calories per square centimeter per minute.

This value is supported to within the limits of his error of measurement by the determinations of Langley at Lone Pine and at Mount Whitney, where he obtained 2.06 and 2.22 calories respectively. We have proved (*Annals*, **2**, pp. 119 to 121) that the value 3 calories which Langley gave as the result of his Mount Whitney expedition is erroneous because of an error of logic which he committed in his discussion in the *Report of the Mount Whitney Expedition*, pp. 143 to 145.

This is a summary of the work against which Mr. Very has made the charges above mentioned. We should naturally expect that, having made such charges, he would have proceeded at once to attack one or more of the essentials of our work above enumerated, and have shown distinctly and numerically wherein, by errors of logic or of measurement, the work fails, and how large resulting errors have arisen.

[1] *Nova Acta R. Soc. Sci. Upsala*, 20, I, 1900.

But far from attacking our main work in this straightforward fashion he begins by misleading the reader to suppose that our value of the solar constant depends on raising to the 26th power some coefficients of water-vapor transmission obtained by Rubens and Aschkinass. If this matter formed any part of our definitive determination of the solar constant we should proceed to show how unfairly Mr. Very has used what we said about it. As it forms no part at all, we merely invite the reader to see what we did say from p. 168 clear through to p. 172 of Vol. II of the *Annals*, and then compare with what he says we said.

According to Mr. Very the reflection of *Mars* for total radiation is 0.27, that of *Venus* 0.92, and that of the earth "is more likely than not" to exceed the mean, or about 0.60. We have no confidence in any of these figures, and see no reason why a solar constant of 3 calories or upward must be conceded to allow them to stand. When he chose to blow hot instead of cold, Mr. Very attempted to show that *Neptune* could be maintained at 50° C. (!) by the solar radiation,[1] although the earth at less than 1/30 of *Neptune's* distance from the sun is only at 15° C. If planetary atmospheres can make such differences as this, we hardly think our careful direct measurements of the solar constant can be overthrown by computations made from the terrestrial temperature by the aid of guesswork about the reflecting and emitting power of the earth.

Mr. Very goes on to say that the temperature of the moon, as determined by him, requires a higher solar constant than 2 calories. We remember a very good story by Newcomb of a person who believed gravitation extended no farther than the atmosphere, and fell far short of the moon. Newcomb ascertained that his caller had never been to the moon to see, and told him that as he had never been there either, he doubted if they could agree! We feel some skepticism as to the temperature of the moon, and incline to praise the moderation of Langley, who, in summing up his classical investigation of it (in which Mr. Very assisted), says:[2]

[1] *Phil. Mag.* (6), 16, 478, 1908.

[2] *National Academy of Sciences, Third Memoir*, Vol. IV, Part 2, p. 193; read 1887, published 1889.

The conclusion of the whole matter is, that we have been dealing with a subject almost on the limit of our power of investigation with the present means of science, and have reached no conclusion which we are absolutely sure of. . . . . If beyond this we can be said to be sure of anything it is that the actual temperature of the lunar soil is far lower than it is believed to be; but the evidence does not warrant us in fixing its maximum temperature more nearly than to say it is little above 0° Centigrade.

But Mr. Very admits no such uncertainty. For him the effective temperature of the moon's equatorial sunlit surface is 454° absolute, and nothing less than 3 calories will do for the solar constant to correspond. He maintains this, notwithstanding that Coblentz has shown that the moon is probably a very bad radiator![1]

Mars, according to Mr. Very, has a temperature too high to admit of a solar constant of 2 calories. We suppose he has no direct information as to the radiating power of that planet or its absorption of solar radiation. If we had not already had some experience of the unpleasantness of discussing *Mars*,[2] we should go on to say what we think about its temperature. We confess, however, that we know very little about *Mars*. Still, if *Neptune* can be maintained at 50° C. by solar radiation of about 1/1000 the intensity that reaches the earth,[3] we should think *Mars* might have some chance without a solar constant of 3 calories.

Mr. Very gives a meteorological argument which according to him shows that we have underestimated the loss of solar radiation in our atmosphere, and thus have derived too small a value of the solar constant. He gives an expression for the insolation of the earth, into which expression the average transmission of the atmosphere for solar radiation enters. He says the distribution of temperatures on the earth June 21 agrees with the requirements of his formula better when one takes the transmission at less than 0.25 than for higher values. He considers the best value 0.18 as the average transmission over the whole sunlit surface of the earth. For this he requires a solar constant exceeding 3 calories.

Mr. Very does not give sufficient details of this comparison to enable us to understand clearly what he has done. But at all

[1] *Physical Review*, **23**, 247, 1906.

[2] *Science*, **31**, 987, 1910.

[3] F. W. Very, *Phil. Mag.* (6), **16**, 478, 1908.

events we are of the opinion that the earth's surface temperature is a complicated function of many variables besides the insolation. Among these are cloudiness, distribution of land and water, mountains, ocean currents, and winds. Their effect is shown by the well-known differences of temperature at equal latitudes, as for instance between Europe and America. To choose that value of the atmospheric transmission which makes an insolation-curve match best with a temperature-curve, without any regard to these other factors, seems to us as indefensible as it would be to determine the reading of a Pouillet pyrheliometer merely by the rise of temperature on exposure to the sun, neglecting altogether the cooling due to the surroundings. We therefore can attach no weight at all to this method of fixing the solar constant.

In the remainder of this article we propose to take up the criticisms which Mr. Very makes of our work. He claims that we have abandoned Langley's methods and employ methods which do not differ essentially from those of Pouillet. *Pouillet observed only with the pyrheliometer.* We employ the pyrheliometer, as did Langley, only to determine the scale of energy of our spectro-bolometric determinations. Like Langley we determine the form of the solar energy-curve at different solar altitudes, correct it for instrumental absorption, determine numerous coefficients of atmospheric transmission from high- and low-sun observations on nearly homogeneous rays, determine thereby the form of the energy-curve outside the atmosphere, and assume, like Langley, that there is no water or oxygen absorption in the sun. We reduce the results to calories per square centimeter per minute as did he by the aid of the simultaneous readings of the pyrheliometer. Great improvements have been made in 30 years. We have the advantage of a better pyrheliometer than Langley, an automatic recording bolometer free from drift, and we can determine in 15 minutes all the data that it took Langley several successive days to obtain, and far more besides. We use many more atmospheric transmission coefficients than Langley did, and can determine them more accurately. We have made the measurements at sea-level, 1750 meters, and 4420 meters. In one thing only do we fail to follow Langley's example—we do not build a 2-calory solar constant up to 3 calories. We cite pp. 14 to

16 and 119 to 121 of Vol. II of the *Annals*, as against pp. 143 to 148 of Langley's *Report of the Mount Whitney Expedition*, as our defense for this course.

Mr. Very says Pouillet's result, if corrected for known errors in his pyrheliometry, will not differ essentially from ours. It will differ by about 10 per cent from ours, and for the reason that Pouillet made no spectrum observations.[1]

It will be a doctrine new to physicists that close agreement of independent determinations made under circumstances so diverse as those in our measurements at Washington, Mount Wilson, and Mount Whitney is no recommendation of the probable accuracy of the result; yet Mr. Very implies as much. As to his reflections on the effective radiating temperature of the earth and the transmission of water-vapor for long-wave rays, we think there is too much guesswork and too few facts, both on his side and ours, to make an argument about it worth while. After about five years more of experiments, we hope we may be in a condition to talk with him about these complicated questions without having to piece out our data with assumptions. We are surprised, however, by the roughshod manner in which Mr. Very overrides our value of the earth's reflection of total radiation. He merely remarks that he has "no hesitation in saying this is too small," without taking up the data by which we determined it. He suggests a mean between 0.27 and 0.92. We took more pains to obtain our value.

Mr. Very differs with us for our saying "we know that in general the lower layers of air have smaller transmission coefficients than the upper ones, owing to the generally low level of the larger quantities of dust and humidity." He says this "is true only for unsifted radiation and does not apply to the actual residual radiation which has already experienced its greatest absorption by aqueous vapor in the upper air." We reiterate our statement, both for Mr. Very's excepted kind of radiation and for homogeneous rays. If he still doubts it, let him look about in London, or look down on the dust layers above Pasadena from Mount Wilson. We should have thought his long residence near Pittsburgh would have convinced him that we are right. Certainly we still believe that light is more

[1] See Abbot's *The Sun*, Appleton & Co., 1911, pp. 293 to 296.

hindered near sea-level than in the higher atmospheric layers. If further verification is needed, see Table 118 of Langley's Mount Whitney Report.

Mr. Very objects to our process of evaluating the energy which would be found in the solar spectrum outside the atmosphere, beyond the wave-length where our observations stop in the infra-red. Perhaps he can suggest a better. At all events we hardly think he will claim that he can build up the solar constant to 3 calories out of energy beyond the wave-length 2.5 μ.

Mr. Very says we ought not to lay much weight on our observations made when the sun is near the zenith, and that we ought to observe at lower sun. He says we abandon low-sun observations "because low-lying mists render such measures uncertain" and prefer "midday observations on the plea that the sky is then clearer." He is quite wrong in both respects. We observe in the morning until about 10 A.M as a rule, and then stop because no decrease in air-mass worth waiting for occurs afterward, at the latitude of our stations in our observing season, May to November. We do not begin observing before the sun's altitude is 15° because, as we have shown (*Annals*, **2**, 63 to 64), the air-masses are uncertain at lower altitudes. We seldom observe when the sun is less than 40° or more than 75° from the zenith. We see no occasion to change our practice in these respects.

Mr. Very also regrets that we do not observe in winter. So do we, because we strongly suspect that the sun is a variable star. But if we did observe in winter we should require a *cloudless* region in or near the Southern Hemisphere, where the sun is *high* during our winter months. We see no advantage in extrapolating from air-mass 2 to air-mass zero when the extrapolation may begin at air-mass 1.2 just as well. However, there is no evidence from our winter observations at Washington that any systematic difference in the solar-constant values would arise on account of winter conditions.

Mr. Very intimates that even our narrow linear bolometer does not discern all the lines of atmospheric absorption, so that conditions may be conceived to exist under which our methods might give results appreciably too low. We have discussed this possi-

bility at considerable length elsewhere[1] and will not take space here to repeat that discussion in full. Such a condition would exist if what is sometimes called "the general absorption" of the atmosphere was in fact made up of nearly complete absorption and nearly complete transmission following one another in the spectrum in innumerable bands too narrow to be observed separately. We freely admit that the selective absorption of water-vapor in the great infra-red bands, and that of oxygen in its great bands, is of this character. Following the practice of Langley, we employ a special procedure for these bands, making the assumption, as he did, that none of them would exist in a spectrum taken at the outer limit of the atmosphere. We therefore draw a smooth curve for the energy spectrum outside the atmosphere where the terrestrial bands appear. This gives the highest possible solar-constant value. But as regards other regions of the spectrum, Rayleigh has shown satisfactorily that the so-called "general absorption" is really probably a scattering effect of small particles and molecules in the air, plus a diffuse reflection by larger particles. Schuster has shown that our atmospheric-transmission coefficients determined on Mount Wilson are almost exactly what could be predicted by Rayleigh's theory of the scattering of light by the molecules of air. Diffuse reflection and scattering is, according to Rayleigh, a continuous function of the wave-lengths. Hence our linear bolometer amply suffices to estimate the atmospheric transmission in regions where selective atmospheric absorption does not exist. But Mr. Very would have us admit an atmospheric band at $0.40\ \mu$ to $0.46\ \mu$. Our own work gives no certain intimation of this.[2] We cannot, however, see how an inspection of the extra-atmospheric energy-curve we have determined[3] could allow anybody to believe that any appreciable increase of the solar constant could come from smoothing the curve from $0.40\ \mu$ to $0.46\ \mu$, as if there were an atmospheric band there, in the manner we employ for the water-vapor bands of the infra-red.

[1] See *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **2**, 64–65.

[2] *Ibid.*, Plate XVII.

[3] *Astrophysical Journal*, **34**, 206, 1911.

CONCLUSION

Mr. Very evolves a value of the solar constant of radiation from such unknown or fragmentary data as the reflection and emission of the earth, moon, and *Mars*, the temperatures of the two latter, and the dependence of terrestrial temperature on insolation. To clear the way for this he accuses us of doing what we have not, misrepresents what we have done, and suggests certain sources of error in our methods which we had already quantitatively discussed. We base our value on ten years of painstaking work, both theoretical and experimental, in laboratory and afield, at sea-level and high altitudes, comprising over 600 independent determinations by the most approved method.

Astrophysical Observatory
Smithsonian Institution
Washington, D.C.
January 1912

THE STARK-DOPPLER EFFECT FOR HYDROGEN CANAL RAYS IN AIR

By GORDON S. FULCHER

It has been found that light is produced by the collision of canal rays with gas molecules.[1] An attempt was made to explain the details of the Stark-Doppler effect, assuming that the collisions of canal rays with gas molecules obey the laws of ordinary, perfectly elastic impact. This was found to involve two other rather improbable assumptions: namely, that neutral canal rays do not cause the emission of an appreciable amount of light; and that it is the hit molecules, not the hitting, which emit the light showing the Stark effect. Also the fact that α particles are but slightly scattered in ionizing over 10^5 air molecules each, suggested that the assumption of perfectly elastic collision was wrong. The following experiment was devised to test this.

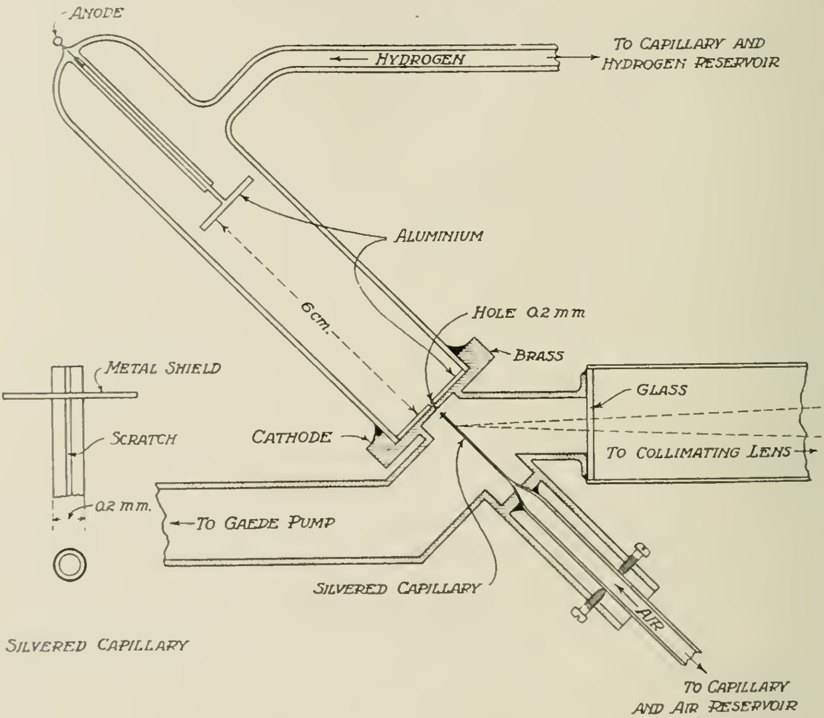
APPARATUS

The apparatus was arranged so that a fine beam of canal rays, after passing through a 0.2 mm hole in the cathode, was projected axially into a capillary about 0.3 mm in diameter. Hydrogen was supplied continuously to the discharge chamber through a second capillary, and air could be sent through the main capillary back of the cathode at a rate regulated by a third capillary and gas reservoir. The gases mixed in the chamber back of the cathode and were pumped away continuously by a Gaede pump. The stream of air was sufficient to prevent any appreciable quantity of hydrogen backing up into the capillary. Also very little air got into the discharge chamber, though that was much less important. In the main capillary, then, the only possible sources of the hydrogen lines were the hydrogen canal rays, luminescent as a result of their collisions with air molecules. It was necessary only to examine the hydrogen lines for the Stark effect to determine whether or not the hydrogen rays were stopped by their collisions

[1] G. Fulcher, *Astrophysical Journal*, **33**, 28, 1911.

with air molecules—whether they retained their energy, or handed it on to the hit molecules as previously assumed.

The main capillary was first silvered and the coating scratched off along a line parallel to its axis. Then it was set in place and adjusted to its proper position by means of three screws. The



APPARATUS FOR OBSERVING THE STARK EFFECT WHEN HYDROGEN CANAL RAYS BOMBARD AIR MOLECULES.

FIG. 1

scratch served as the slit of a spectrograph, which was placed so that the axis of the collimator made an angle of about 45° with that of the capillary. Two prisms were used; one, whose angle was 45°, was placed so that the angle of incidence was about 75°, considerably greater than for minimum deviation; the other, a 60° prism, was placed for minimum deviation. With this arrangement, the width of the lines was less than half the width of the

slit. A curious fact noticed is that if the order is reversed so that the 60° prism comes first, the dispersion is considerably less for the two prisms than for the 60° prism alone. Lenses of 45 cm and 55 cm focal length were used. The dispersion at λ 4300 was about 15 Å per mm. The depth of focus obtained is remarkable. Lines one cm long on the plates are of practically uniform width, though the slit, as is stated above, was inclined at an angle of 45° to the collimator. The light was so feeble that exposures of from 10 to 24 hours were necessary.

RESULTS

Parts of three of the first spectrograms obtained with the above apparatus are shown enlarged about five times in Fig. 2, Plate V. Some spectrograms obtained more recently are shown enlarged about twice in Fig. 3, Plate V. When hydrogen canal rays bombard hydrogen molecules, the series lines $H\beta$ and $H\gamma$ show the Stark effect very clearly. The "rest line" is seen to be distinctly separated from the broad "displaced line" by an "intensity minimum" (No. 160). When, however, the hydrogen canal rays bombard air molecules, the *displaced line alone is obtained*. No. 167 is interesting for another reason also: though it represents an exposure of 24 hours and the series lines of hydrogen are quite dense, it shows not a trace of the compound lines. If present, they must be relatively very much weaker than in the ordinary canal ray spectrum (No. 160). The cathode fall of potential was in all cases between 4000 and 5000 volts, and a continuous current was obtained by the use of a condenser and high resistance as in previous experiments.

These results mean that all the sources of the hydrogen lines under these circumstances have a considerable velocity, between 2×10^7 and 5×10^7 cm per second. What conclusions can we draw from this fact? Conceivably the emission of light by the canal rays may be due (1) to their collisions with air molecules, or (2) to their collisions with free electrons produced in connection with the ionization of the air by the canal rays. There are no other alternatives since the intensity of the light has been shown to depend directly on the pressure of the gas through which the rays are

passing (*loc. cit.*). Collisions with air molecules must occur of course, since the gas becomes ionized and the canal rays are stopped within a certain range. Let us first suppose that these collisions are perfectly elastic. The hit molecules will be given a considerable momentum, and all the light they emit as a result of the collisions will show a Doppler effect; the hitting rays, however, will emit most light when stopped or reflected back by the collision, and the light so emitted will show a broadened rest line, but no shifted line. But neither a Doppler effect for the nitrogen bands nor a rest line for the hydrogen lines was obtained in the above circumstances. Either, then, the collisions with air molecules do not result in an appreciable emission of light, or the collisions are not perfectly elastic.

Now consider the possibility of collisions with electrons. With ordinary discharge tube pressures, the ratio of the number of electrons present to the number of air molecules is less than one in a million. The chances of collisions with electrons are of course relatively much greater because of the electric field surrounding the charged rays. The deflection experiments with crossed magnetic fields made by J. J. Thomson,[1] and Koenigsberger[2] and his co-workers, have shown that at very low pressures such collisions predominate, perhaps, over molecular collisions. Such is certainly not true at the pressures used here (a few tenths of a millimeter). Also the impulse involved in such a collision would be over a thousand times less than in the case of a molecular collision. The effect would be the same as when hydrogen ions are neutralized by slow cathode rays whose energy is due to a potential difference of less than a volt. Such slow rays cannot produce ionization and in case of such recombinations would not cause sufficient disturbance to produce any appreciable luminescence.

It seems very probable, then, that canal rays emit light chiefly as a result of collision with gas molecules; and the fact that the light so emitted shows no rest line seems to prove that such collisions are not perfectly elastic, but that the rays retain a large proportion of their momentum after the collisions and do not impart much momentum to the hit molecules. The collisions then seem

[1] *Phil. Mag.*, **18**, 825, 1900.

[2] *Physikalische Zeitschrift*, **11**, 666–668, 1910; *Verhandlungen der deutschen physikalischen Gesellschaft*, **12**, 995–1017, 1910.

intermediate in type between the perfectly elastic collisions of ordinary gas molecules and the almost perfectly inelastic collisions of α rays with gas molecules.

Geiger[1] has determined the most probable angle through which an α particle is deflected by passing through certain thin metal foils, and also the way this angle varies both with the velocity of the α ray and the molecular weight of the metal causing the scattering. From these data he has computed the most probable angle of deflection resulting from collision with a single gold molecule. This involved certain assumptions, and the exact meaning of the word collision was not stated; but the results are very interesting, and accurate enough for our purposes. Extrapolating from his data down to lower velocities and smaller atomic weights, we find that when a helium canal ray with a velocity of 10^8 cm per second hits an air molecule, we may expect a most probable deflection of about 5°. For a velocity half as great the most probable angle should be about 40°. We may expect that hydrogen canal rays would be less deflected, in agreement with the conclusion stated above. It would be very interesting to determine the constants of scattering for canal rays of various kinds and velocities, and thus get deeper insight into the structure and internal electric fields of molecules. Only when this has been done will a quantitative explanation of the details of the Stark effect be possible.

The loss of energy resulting from the collisions may account for the fact that the maximum velocity of the sources of the shifted lines is always less than that computed from the cathode fall of potential.[2] The existence of the intensity minimum seems to show that the intensity of the light emitted as a result of any collision depends on the energy of impact, which will on the average vary directly with the energy of the bombarding rays,[3] falling to zero perhaps at a certain velocity depending on the molecules involved. It would be interesting to determine how the width of the intensity minimum depends on the kind of gas molecules bombarded. That it varies greatly is suggested by Strasser's results

[1] *Proc. Roy. Soc.*, (A) **83**, 492–504, 1910.

[2] F. Paschen, *Annalen der Physik*, **23**, 257, 1907; J. Stark, *Physikalische Zeitschrift*, **8**, 399, 1907.

[3] G. Fulcher, *Astrophysical Journal*, **33**, 40, 1911.

with mixed gases.[1] Here seems to be another example. Stark[2] reports that the Doppler effect in mercury is clearly obtained only with high potential differences, above 40,000 volts. Yet with mercury canal rays bombarding hydrogen molecules, I obtained a shift of 0.5 Å for $\lambda\lambda$ 4359 and 4047 with a cathode fall of less than 5000 volts (see below).

Reichenheim[3] found that in the spectrum of light from the path of anode rays, some strontium and calcium lines showed the Stark effect without any rest lines. The conditions were in fact quite similar to those obtained with the above apparatus; light was probably produced mainly as a result of the collision of metallic anode rays with molecules of iodine or some other electro-negative gas. His results tend to show that strontium, calcium, and probably other metallic anode rays with high velocities may retain a large part of their momentum after colliding with iodine or other gas molecules.

When nitrogen canal rays (5000 volts) bombard hydrogen molecules the negative bands alone are produced, and these show a Doppler effect (No. 162). The following are measurements made on two spectrograms, (*a*) *H* canal rays in *H* (trace of air) (No. 160), (*b*) *N* canal rays in *H* (No. 162).

Lines	(*a*) No. 160—in *H* mm	(*b*) No. 162—in *N* mm	(*a*)−(*b*) mm
H_{II} 4205.26	27.196	27.194	+ .002
H_{II} 4212.07	27.725	27.724	+ .001
Ca 4226.9	28.740	28.741	— .001
N−4256.2	30.775	*30.715	+ .060
N−4259.4	30.995	*30.938	+ .057
N−4262.3	31.184	*31.122	+ .062
N−4265.0	31.370	*31.314	+ .056
N−4278.0	32.243	*32.185	+ .058
H_γ shift line	*35.92 *36.12	———	———
H_γ 4340.6	36.220	36.212	+ .008
Hg 4358.6	*37.274	37.304	— .030
H_{II} 4412.42	40.452	40.457	— .005

* The starred lines are shifted.

[1] *Annalen der Physik*, **31**, 890–918, 1910.

[2] *Astrophysical Journal*, **25**, 180, 1907; *Annalen der Physik*, **21**, 439, 1906.

[3] *Annalen der Physik*, **33**, 747–761, 1910.

Each measurement given is the mean of five; the mean deviation from the mean is less than .002 mm, yet irregularities in the film cause a somewhat greater uncertainty. The width of the lines is .06 mm. The maximum shift for the $H\gamma$ line, λ 4340.6, is .30 mm (a); the mean shift for the N negative band, λ 4278, is .059 mm (b); while the mercury line λ 4358.6 is shifted .030 mm (a). The mercury line λ 4046.8 was also shifted on No. 160 a corresponding amount. These shifts were verified on other spectrograms. No. 160 was made with the discharge chamber in direct connection with the pump so that it contained mercury vapor from the pump, whereas in the case of No. 162 the only mercury vapor present was that which diffused over with the gases from the manometers connected to the gas reservoirs. Multiplying these shifts by the square roots of the atomic weights, $1, \sqrt{14}, \sqrt{200}$, we get 0.300, 0.223, and 0.423. Taking instead, $1, \sqrt{28}$, and $\sqrt{100}$, we get 0.300, 0.312, and 0.300. The shifts point to sources having velocities corresponding to singly charged nitrogen molecular rays and doubly charged mercury atomic rays. The cathode fall of potential was between 4000 and 5000 volts.

In the case of canal rays in air or pure nitrogen, both Stark[1] and Hermann[2] state that the nitrogen bands do not show a Doppler effect. There are two things to consider: First, the number of molecular canal rays compared to the number of atomic rays decreases as the voltage increases. I have never obtained any trace of any nitrogen spark lines with voltages less than 5000, whereas at higher discharge potentials the spark lines appear. The Doppler effect for the bands is then weaker for higher voltages and may be masked by the intense negative bands emitted as a result of the ionization of nitrogen molecules by secondary cathode rays or electrons produced by the canal rays.[3] Secondly, the scattering may be so much greater in the case of collision with nitrogen molecules than with hydrogen molecules that in the first case only a broadening of the lines results, whereas in the second a distinctly shifted line is obtained.

[1] *Göttingen Nachrichten*, p. 463, 1905; *Physikalische Zeitschrift*, **6**, 894, 1905.

[2] *Physikalische Zeitschrift*, **7**, 568, 1906.

[3] Cf. G. Fulcher, *Astrophysical Journal*, **34**, 388, 1911.

No trace of any positive bands could be found on any of the spectrograms (air into H), though long exposures were made with less dispersion. They are, then, relatively much weaker in comparison with the negative bands than in the ordinary canal ray spectrum. The negative bands of nitrogen, then, are emitted by the nitrogen molecules when ionized by cathode rays,[1] and also as a result of collision with hydrogen molecules.

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February 1, 1912

[1] Cf. G. Fulcher, *Astrophysical Journal*, **34**, 388, 1911.

SPECTROSCOPIC STUDIES ON HYDROGEN

By HARVEY BRACE LEMON

Introduction.—The question as to whether any analogy exists between the behavior of line spectra and of continuous spectra, and if so, what are its limitations, has long been a mooted one in spectroscopy. For continuous spectra, excited solely by temperature, we have had a firm theoretical basis developed by Boltzmann, Wien, Planck, and others, upon which to work. Experiment has in a remarkable manner confirmed the deductions of theory. In the case of line spectra of gases and vapors, however, the situation is far otherwise. Until we have a more definite picture of what the emitting mechanism is, theory has no foundation upon which to build. To picture the mechanism we must have first of all agreement as to the phenomena it produces. As yet we have not even succeeded in this very preliminary requisite. We are not agreed upon the data at hand, still less are we prepared for their interpretation. With the large number of variables associated in the production of spectra of this type, it is not surprising that our data are as yet fragmentary and deal with very special cases. The importance of correlating these, however, in the hope that they may fit together and give us some clue to the mechanism beneath can not be overestimated. It may be that spectroscopic data alone will be insufficient—that perhaps a study of the ionization processes taking place simultaneously with the production of light may be necessary. The general aim of the present work will be confined to an attempt to bring the spectroscopic data available into concordance.

Because of the apparent simplicity of at least one portion of its spectrum and on account of its predominant character in the spectra of many stars, hydrogen has been one of the most widely studied elements, especially as to the question of analogy between the behavior of its series spectrum and that of continuous spectra. Kayser[1] and Langenbach[2] in 1903 attempting this parallel between

[1] "Zur Temperaturbestimmung strahlender Gase," *Boltzmann Festschrift*, p. 38 (July 1903), Barth, Leipzig, 1904.

[2] "Ueber Intensitätsverteilung in Linienspectren," *Annalen der Physik*, **10**, 789, 1903.

continuous and discontinuous spectra believed they detected a shift in the energy maximum of emission toward the violet with increased electrical excitation. The order of magnitude of this effect was considerable. If the same effect had been observed in the temperature radiation of a black body it would have corresponded to a rise in temperature of the radiator from 2200 to 2760 degrees absolute. This does not say, however, that such was the temperature of the hydrogen. In fact, looked at from other angles there seems to be little ground for any analogy whatever, for the comparison is between a so-called "luminescent" radiation excited solely by an electrical discharge and one whose origin lies in temperature alone. Kayser's and Langenbach's results have meaning only as showing a relation between the radiation and the intensity of excitation. Moreover, Nutting and Tugman[1] studying the effect of electrical conditions, primarily current and pressure, upon the hydrogen radiation, reach conclusions not in accordance with those of Kayser and Langenbach. Jungjohann[2] on the other hand agrees qualitatively with Langenbach as to the effect of pressure on the energy-maximum shift but disagrees with Kayser as to the correspondence of this shift with that of a black body. All of these experiments have been carefully conducted, but the methods used have been widely divergent, especially as to the mode of excitation employed; and to this and to one other cause about to be mentioned must the disagreements be ascribed.

Continuous background.—In making photometric measures on the spectrum of hydrogen one very important factor seems to have been universally overlooked, and it may be the neglect of this factor, more than experimental differences in method, has been the greatest cause of the discrepancies among workers in this field. This factor is the continuous spectrum of hydrogen. There are present in all ordinary forms of vacuum tubes filled with pure hydrogen three spectra—the series line spectrum, the compound line spectrum, and the continuous spectrum. The last, very faint

[1] "The Intensities of Some Hydrogen, Argon, and Helium Lines in Relation to Current and Pressure," *Bulletin Bureau of Standards*, 7, 49, 1911.

[2] "Ueber Emission und Absorption leuchtender Gase bei hohen Stromdichten unter Verwendung von Gleichstrom," *Zeit. f. Wiss. Photog.*, 9, 84, 105, 141, 1910.

in the red, becomes, even at low pressures (1–3 mm), quite strong in the blue and very marked in the violet. Obviously when examined under the low dispersion of a spectrophotometer it doubtless consists of a mixture of true continuous background and unresolved compound line spectrum. In any case measures made on the series lines $H\beta$, $H\gamma$, and $H\delta$, etc., will be largely in error unless correction is made for this continuous background on which these lines appear. A glance at the following table will show clearly the magnitude of this correction.

TABLE I

MAGNITUDE OF THE BACKGROUND CORRECTION

	α	β	γ	a	b	α_g	β_g	γ_g
Uncorrected	.084	.130	.169	.033	.094	....	....	....
Corrected	.084	.081	.035	.033	.045	.000	.049	.134

The figures represent the photometric intensities of the series lines $H\alpha$, $H\beta$, $H\gamma$, and of a group of secondary lines, a, in the red ($\lambda = .600–.603\ \mu$), b, in the blue ($\lambda = .492–.495\ \mu$), relative to the intensity of the corresponding portions of the spectrum of a Nernst glower. The gas was pure, under a pressure of .318 cm and was carrying a current of 6 milliamperes; the form of tubes and the apparatus is described fully below. The first line in the table gives the intensities as measured without any consideration being taken of the background. The second line shows that whereas the red lines α and a are unchanged, owing to the absence of the background in the red, the blue lines have their intensity values changed by 40 to 50 per cent and the violet line by as much as 80 per cent. The intensity of the background immediately adjacent to the red side of the series lines, denoted in the table by α_g, β_g, γ_g, is seen to rise very rapidly from zero in the red to a magnitude almost equal to that of γ in the violet. At lower pressures the background is not so strong, at higher pressures it is much more marked, in fact, it becomes so strong that $H\gamma$ is seen with difficulty, if at all, on it. That so large a source of error in the intensities of the hydrogen lines should have been overlooked by previous workers seems very strange. However, none of them in any place makes mention of

observations on the continuous background or of any correction of the line intensities due to it. Plate VI, a reproduction of photographs of the spectra given by some of the hydrogen tubes used, shows clearly the magnitude of the continuous ground. *A* is taken with the large dispersion of a 2-meter focus, $3\frac{1}{4}$-inch concave grating ruled by Professor Michelson; *B* is taken with a small prism for the dispersion piece, and shows the effect of pressure in strengthening the background.

Outline of experiments.—The original aim of these experiments was an attempt to throw some light upon the open question as to the effect of temperature alone on the emission of hydrogen. The method was the very simple one of changing the temperature of the discharge tube through a large range and making photometric measures on the intensities of the various portions of the spectrum. Electrical conditions were of course to be held as nearly constant throughout as was possible. In the course of these first experiments there came to light the above-mentioned possible cause of the disagreement of previous workers on the question of the effect of electrical conditions, and the original aim of the undertaking has been broadened to a somewhat more extensive one. This will include the study of the effect of electrical conditions on the radiation of hydrogen, helium, and perhaps other elementary gases, as well as the effect of temperature alone.

Part I*a*

THE EFFECT OF TEMPERATURE UPON THE HYDROGEN SPECTRUM AS PRODUCED BY ALTERNATING CURRENT

Apparatus.—The apparatus consisted of the hydrogen generator and purifier, the discharge tubes, the electrical exciting equipment, and the spectrophotometer. A general view is given in the photograph, Plate VII. Fig. 1 is a detail drawing of the hydrogen generating and purifying apparatus. It was made entirely of glass with a minimum number of stopcocks. The gas was generated by the electrolysis of a dilute solution of orthophosphoric acid in tube *A*. So prepared, it contains a trace of oxygen which is absorbed in tubes *B* and *C*, each containing an alkaline pyrogallol solution.

The formula due to Hempel was used, as this seems to be one of the few satisfactory solutions for completely taking up oxygen without giving off carbon monoxide. It consists of 15 gms pyrogallol in

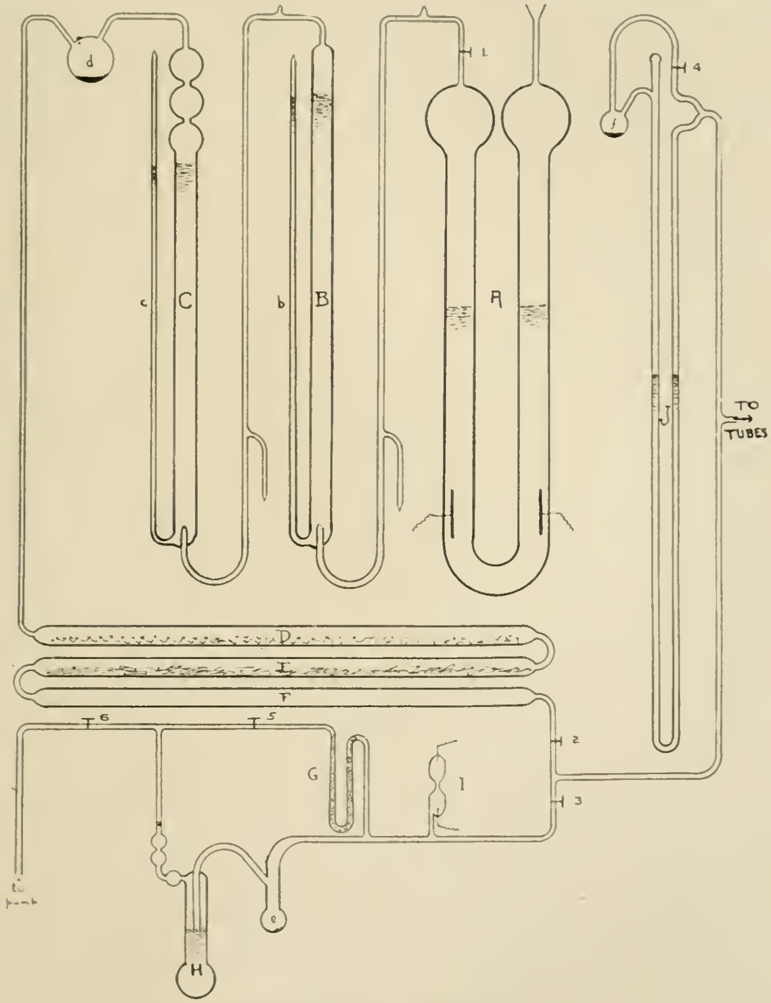

Fig. 1. —Hydrogen Generator

45 cc water; 360 gms potassium hydrate (prepared without use of alcohol) in 240 cc water. These two solutions are made separately, then thoroughly mixed, and quickly transferred to the tubes

prepared for them without any of the solution which entered the tubes having come in contact with the air. A very brief exposure to air will oxidize the solution and render it almost worthless. To avoid the presence of air in the apparatus at the start it was all completely exhausted before any of the liquids were admitted to it. The side tubes *b* and *c* make it possible to replenish the solutions when exhausted without contamination of the contained gas and without using stopcocks. The hydrogen, when thus freed from oxygen was collected in the tubes *D*, *E*, and *F*, which contained, respectively, calcium chloride, potassium hydrate sticks, and phosphorous pentoxide, and which serve as driers and as a storage chamber.

The stopcock 2 admits the gas to the experimental tubes which are in connection with a self-exhausting sulphuric acid manometer, *J*. When cock 4 in the manometer is open both sides can be completely exhausted, after which the cock is closed. On readmitting gas the difference in level assumed by the two arms indicates the pressure. The trap *f* serves to protect the cock from acid accidentally thrown over by the use of too high a pressure. Exhausting the apparatus takes place through cock 3 by means of a Gaede pump. To prevent diffusion of mercury vapor back from the pump into the discharge tubes two mercury traps, *G* and *H*, were provided. For all pressures down to .03–.04 cm cock 5 was left closed and the trap *H* used. This consisted simply of a tube just dipping beneath the surface of sulphuric acid. Exhaustion through this could take place down to a pressure approximately represented by the amount of submersion of the tube. For pressures below this amount cock 5 was opened and exhaustion took place through tube *G*, which was closely packed with gold leaf. At the low pressures for which *G* was needed the free path of the mercury molecules was large enough to prevent their diffusion through the spaces between the gold without hitting it. Thus all are caught and held by the latter. Obviously at higher pressures this might not be the case. These two traps were completely successful in preventing the diffusion of mercury back into the apparatus. During a year's intermittent use the test discharge tube at *I* never showed any trace of the mercury spectrum.

Fig. 2 shows a general plan of the rest of the apparatus. The light from the discharge tube, *D*, was compared with that from a Nernst glower, *N*, by a modified form of a Brace spectrophotometer, *B*. The selective absorption of the silver strip through the middle of the photometer prism was found to be negligible for such measures as these experiments involved. Constant slit-widths were used in both collimators, but the beam coming from the Nernst was plane polarized by means of a Nicol at *P*. After having traversed

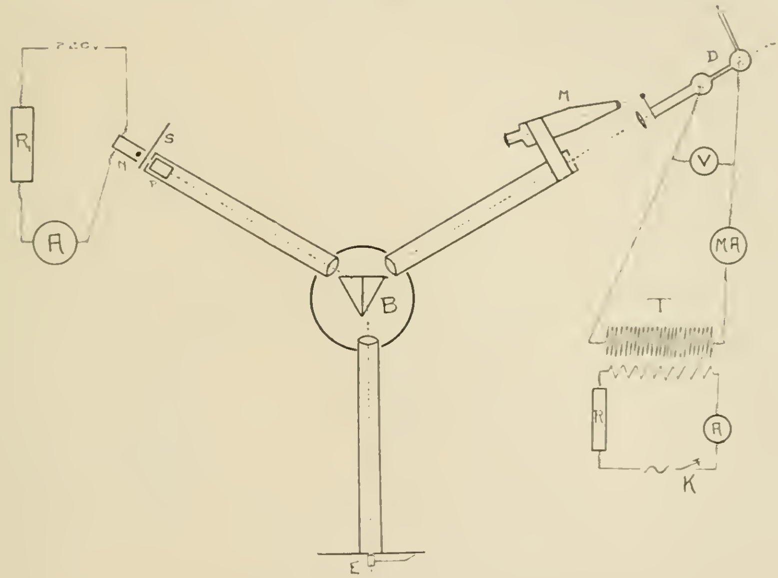

FIG. 2.—General Plan of Apparatus

identical paths through the prism (except for the reflection) the two beams unite in the telescope and the two spectra formed superimpose when viewed with an eyepiece at *E*. An eye-slit at this point cuts out all light except that of the spectral line or group being measured, and the background behind it. On removing the eyepiece and looking through the eye-slit with the eye approximately in the principal focus of the telescope lens the surface of the prism appears illuminated uniformly, except for the silver strip, which is of different intensity from the rest. Now by means of an analyzing Nicol in front of the eye the intensity over the rest of

the surface can be made to match exactly that of the silver strip, when, by careful adjustments, the latter disappears. The speed and accuracy of the settings obtainable with this form of spectrophotometer are too well known to require mention.

The Nernst comparison lamp, N, was a seasoned 90 v., 0.5 amp. glower held by means of a series resistance, R_1 at a constant current. A heavy metal shutter, S, screened the collimator slit from the intense radiation except while settings were actually being made. The discharge tubes had the form shown in section in Fig. 3. The

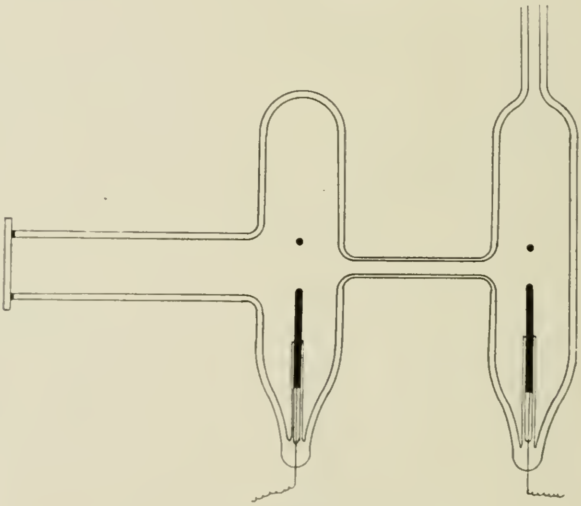

Fig. 3.—Discharge Tube

top part of the electrodes was bent into the form of a ring so that the light from the capillary could be viewed "end-on" through the side arm, which was closed with a quartz or glass window. The tubes were excited by means of the secondary circuit of a 5 kw. transformer (see Fig. 2). This circuit also contained as indicated a Weston milliammeter and a Brown electrostatic voltmeter. The primary of the transformer was fed through a series resistance, R, with alternating current from a 5 H.P. twin motor-generator set which had separately excited generator fields and could be held very constant. In fact, the current through the tubes never fluctuated by amounts readable on the milliammeter. An open

circuit program of observations was adopted, the switch *K* being open and no current flowing through the tubes except at the moments settings were being made. Thus there was no heating of the tubes due to the discharge.

Several of the tubes were held simultaneously in a double-walled container which permitted the part containing the discharge to be entirely surrounded with liquid air when desired. The side arm of the tube, projecting through the walls of the container, gave an uninterrupted view into the interior of the tube at all times. On the window was attached by means of hard wax a fine needle point which was observed in a micrometer microscope, *M*, clamped to one of the collimators. Any shift between collimator and tube was, therefore, easily detected and carefully corrected by means of adjusting screws. Moreover, the various tubes in the container could thus be successively brought into alignment, moved out, and brought back again accurately to their original positions. Reliable measures were found impossible without this arrangement, since the changes in temperature to which the container was subjected would warp it slightly, thus bringing into view light from a slightly different portion of the capillary and causing thereby variations of selective intensity considerably larger than those due to the causes under consideration.

Method of observation.—The plan of observation finally adopted as the most reliable was to make at a given pressure two successive comparisons of the radiation of the discharge tube at room temperature with that of the Nernst glower. Then after cooling the tube two successive comparisons were again made with the glower, the gas density in the tube being kept the same and the electrical conditions remaining constant throughout. Fresh, spectroscopically pure gas was readmitted for every set of observations. The gas density was controlled in two ways: (1) Gas was admitted to the tube at the desired pressure and at room temperature, and the tube then sealed off from the rest of the apparatus. Observations were then made first at room temperature then at that of liquid air. Owing to the possibility that occlusion at the low temperature might change the gas density, only few observations were made using this method. (2) In most of the observations gas was

admitted to the tubes to the desired pressure and observations with the tube at room temperature taken. The tube was then cooled while remaining in connection with pump and gas supply and the pressure now so adjusted that the gas density was just the same as before, i.e., the pressure was made proportional to the absolute temperature. Observations at the low temperature were then made. No difference was observed in measurements resulting from these two different methods of control, except at very low pressures.

The current through the tubes being kept constant and the mass of gas between the electrodes being always the same, the potential difference between the electrodes would not be expected to change. Earhart[1] has shown, however, that the behavior of hydrogen is irregular in this respect, a fact which has been also observed in these experiments. For pressures below 6–7 mm the potential difference across the electrodes rises somewhat when the tube is cooled. At pressures above this value it falls somewhat on cooling. Test experiments have shown, however, that these variations are wholly incapable of producing in the radiation from the tubes such large changes as have been observed.

To sum up then: Under electrical conditions which have been controlled as closely as the nature of the case permits, indeed to within a few per cent, measurement has been made of the effect of a decrease in the temperature of the discharge tube from about 300° absolute to about 100° absolute, upon the photometric intensities of those parts of the hydrogen spectrum above mentioned, i.e., the series lines, α, β, γ ($\lambda = .656, .486, .434\mu$), the secondary groups, a, and b ($\lambda = .601, .493\mu$ approx.), and the continuous ground immediately adjacent to α, β, and γ, which is very near also to a, and b. The readings actually taken are successively on the intensities of (1) $\alpha+\alpha_g$; (2) α_g; (3) $a+\alpha_g$; (4) $b+\beta_g$; (5) β_g; (6) $\beta+\beta_g$; (7) γ_g; (8) $\gamma+\gamma_g$. The subtraction of (2) from (1) and (3), of (5) from (4) and (6), and of (7) from (8) give the true values of α, β, γ, a, and b corrected for background. (2) has always been observed equal to zero and therefore has been omitted from the tables.

[1] "The Effect of Temperature on Electrical Discharge in Gases," *Physical Review*, 31, 652, 1910.

Results.—A summary of the results obtained is given below. Three different tubes were used which differed in general dimensions and especially in the bore of the capillary. Under a total current of 6 m.a. the different sized capillaries resulted in the following current-densities for the different tubes:

Tube 1.	2.3 amp/cm²
Tube 0.	0.2 "
Tube 2.	0.04 "

Five different pressures were used, viz., 0.08, .13, .31, .70, 1.37 cm (*Hg*) respectively. The figures tabulated are the ratio

Intensity at T=100°: Intensity at T=300°.

Two independent measures of the numerator and two of the denominator were made. The four possible combinations of these which give the ratio are shown, as illustrating the experimental error, and the average taken. The total number of settings involved in the data of each table is given with it.

TABLE II

A

Pressure, 0.08 cm — Tube 2

Current-Density, 0.04 amp/cm² — 140 Settings

α	β	γ	a	b	β_g	γ_g
.51	.55	.40	1.55	1.64	1.17	1.35
.49	.56	.38	1.60	1.47	1.22	1.24
.55	.50	.52	1.41	1.70	1.33	1.27
.53	.51	.49	1.45	1.52	1.38	1.17
Av. .52	.53	.45	1.50	1.58	1.27	1.26

B

Pressure, 0.08 cm — Tube 1

Current-Density, 2.3 amp/cm² — 105 Settings

α	β	γ	a	b	β_g	γ_g
.58	.31	.39	1.22	1.24	1.13	1.07
...	..	...	1.33	1.28	1.13	1.04
Av. .58	.31	.39	1.27	1.26	1.13	1.05

TABLE III

A

PRESSURE, 0.18 CM — TUBE 2

Current-Density, 0.04 amp/cm² — 210 Settings

a	β	γ	a	b	β_g	γ_g
.64	.52	.64	1.36	1.21	1.30	1.28
.61	.62	.57	1.41	1.31	1.28	1.28
.53	.54	.53	1.36	1.08	1.19	1.23
.51	.63	.57	1.41	1.17	1.17	1.23
(.48	.40	.87	1.40	1.02	1.28	1.19)
Av. .56	.56	.58	1.38	1.18	1.24	1.24

B

PRESSURE, 0.18 CM — TUBE 1

Current-Density, 2.3 amp/cm² — 140 Settings

a	β	γ	a	b	β_g	γ_g
.28	.28	.33	1.18	1.54	1.57	1.38
.28	.33	.34	1.30	1.47	1.57	1.77
.32	.30	.40	1.09	1.63	....	....
.32	.26	.41	1.20	1.56	....	....
Av. .30	.32	.37	1.19	1.55	1.57	1.57

TABLE IV

A

PRESSURE, 0.32 CM — TUBE 2

Current-Density, 0.04 amp/cm² — 280 Settings

a	β	γ	a	b	β_g	γ_g
51	56		2.00	1.85	1.59	1.40
52	56		2.00	2.16	1.40	1.47
42	.50		1.57	1.78	1.68	1.40
43	50		1.58	2.08	1.48	1.47
Av. 47	.53		1.78	1.97	1.54	1.44

B

PRESSURE, 0.32 CM — TUBE 1

Current-Density, 2.3 amp/cm² — 140 Settings

a	β	γ	a	b	β_g	γ_g
37	52	45	1.12	1.30	1.00	0.92
36	49	37	1.12	1.30	1.00	0.92
38	53	.44	1.00	1.38	0.95	1.03
37	50	35	1.00	1.38	0.95	1.03
Av. 37	51	40	1.06	1.34	0.97	0.97

C

PRESSURE, 0.32 CM — TUBE 0

Current-Density, 0.2 amp/cm² — 140 Settings

α	β	γ	a	b	β_g	γ_g
.64	.74	...	1.51	1.55	1.49	1.11
.64	.79	...	1.41	1.52	1.29	1.21
.67	.59	...	1.45	1.52	1.43	1.07
.67	.63	...	1.33	1.30	1.27	1.17
Av. .66	.69	...	1.42	1.42	1.36	1.14

TABLE V

A

PRESSURE, 0.7 CM — TUBE 2

Current-Density, 0.04 amp/cm² — 210 Settings

α	β	γ	a	b	β_g	γ_g
.62	.47	...	2.47	1.78	1.33	1.42
.62	...	...	2.56	....	1.25	1.37
.67	.83	...	2.44	1.99	1.34	1.42
.66	...	...	2.52	....	1.26	1.37
.60	.87	...	2.37	1.73	1.32	1.42
.59	...	...	2.46	....	1.24	1.37
Av. .63	.71	...	2.47	1.83	1.29	1.40

B

PRESSURE, 0.7 CM — TUBE 1

Current-Density, 2.3 amp/cm² — 70 Settings

α	β	γ	a	b	β_g	γ_g
.45	.38	.60	1.93	1.84	1.94	2.10
.31	.43	.55	1.86	1.95	1.56	1.96
Av. .38	.40	.58	1.90	1.90	1.75	2.03

TABLE VI

A

PRESSURE, 1.37 CM — TUBE 2

Current-Density, 0.04 amp/cm² — 140 Settings

α	β	γ	a	b	β_g	γ_g
.47	.23	...	2.38	2.44	1.25	1.02
.50	.34	...	2.24	....	1.00	0.98
.31	.45	...	1.94	1.69	1.20	0.97
.33	.66	...	1.82	....	0.96	0.94
Av. .40	.40	...	2.10	2.06	1.10	0.97

TABLE VII

SUMMARY

Current-Density, 0.04 amp/cm² Tube 2

Press.	α	β	γ	a	b	β_g	γ_g
.08 cm	.6	.5	.4	1.5	1.6	1.3	1.3
.18 "	.6	.6	.6	1.4	1.2	1.2	1.2
.31 "	.5	.5	..	1.8	2.0	1.5	1.4
.70 "	.6	.7	..	2.5	1.8	1.3	1.4
1.37 "	.4	.4	..	2.1	2.1	1.1	1.0

Current-Density, 2.3 amp/cm² Tube 1

Press.	α	β	γ	a	b	β_g	γ_g
.08 cm	.6	.3	.4	1.3	1.3	1.1	1.1
.18 "	.3	.3	.4	1.2	1.5	1.6	1.6
.31 "	.4	.5	.4	1.1	1.3	1.0	1.0
.70 "	.4	.4	.5	1.9	1.9	1.8	2.0

It is seen from these tables, summed up in Table VII, that the results, for all pressures and the two widely different values of current-density used, are in quite good agreement. In all cases at the lower temperature, the series spectrum weakens to about 0.5 its normal value, and the compound line spectrum strengthens to 1.5 or 2.0 times its normal value, while the continuous background increases somewhat, but less than the compound spectrum. Inasmuch as the background consists of true continuous spectrum plus some faint compound spectrum, it is quite possible that the true background changes little if at all, the faint compound spectrum mixed with it being responsible for the slight rise shown.

The radiant energy, then, as the discharge tube is cooled, tends to leave the series lines and to go into the compound line spectrum. This is in accord with the general behavior of series and many-lined spectra. The former are characteristic of substances which radiate at temperatures considerably above their melting or boiling points or under the most powerful electrical conditions; the latter are characteristic of substances such as iron, titanium, etc., whose condition when radiating approaches the solid or liquid state. Continuous spectra are characteristic of glowing solids and liquids. Hydrogen exhibits simultaneously these three kinds of spectra.

When its temperature falls and approaches those at which liquefaction can occur its predominating spectra become those characteristic of substances near the liquid or solid conditions, i.e., the compound line spectrum and the continuous spectrum, to a less degree. The photograph, Plate VIII, of the spectra of tube 2, shows clearly this transfer of energy from the series spectrum to the many-lined spectrum with a decrease in the temperature. Five pressures, approximating those at which the visual observations were taken are shown. The top spectrum, A, in each case is taken with the discharge tube at room temperature, the bottom one, B, with the tube immersed in liquid air. The positions of the series lines, α, β, γ, are marked as well as those of the groups a and b of the compound spectrum. The relative weakening of the series lines and strengthening of the many-lined spectrum at the low temperature, B, is very clear. Also as the pressure rises the large increase in the amount of continuous spectrum present is illustrated.

As to the behavior of the series lines with reference to each other, it is seen to be the same for all, to within the errors of measurement. No selective changes in their intensities are observed. In tube 2, which is the more reliable, the current-density was not high enough to bring γ into visibility for measurement at pressures higher than .3 cm. In tube 1, however, γ is observed throughout, and in all cases where there is enough data to rely upon the intensities of α, β, γ, respectively, all changed in the same way and to the same degree. Preliminary experiments which have not been made use of in this work pointed to the same result.

It is concluded, therefore, that as regards the effect of temperature on the spectrum of hydrogen in a vacuum tube, excited by alternating current, (1) a variation in the temperature of the discharge tube of from 300° to 100° Abs. largely affects the relative amounts of energy in the series and in the compound line spectrum, the latter being much enhanced at the lower temperature, (2) this variation of temperature does *not* affect the relative intensities of the series lines at least to within the limits of error of the experiment, which have been guarded by a painstaking control of electrical conditions and careful photometric measurements.

The author wishes to acknowledge his indebtedness to the

members of the staff of the Ryerson Physical Laboratory, and especially to Professor A. A. Michelson, for the interest taken in this investigation and for the ample facilities that have been placed at his disposal. Part I*a* of the work, herein described, was originally suggested by Dr. Henry Gale. To the firm of William Gaertner & Co. are due thanks for the loan of the excellent Brace spectro-photometer which was the basis for the modified form of instrument used.

Ryerson Physical Laboratory
August 1911

SPECTRA AND COLORS OF RED STARS

By J. A. PARKHURST

The writer has been engaged since 1906 in determining the relation between the spectra and color-indices of stars, and has found a good agreement between the two, at least as far as Type III or Harvard Class M; so good that the Harvard spectral classes and color-indices, when platted, fall nearly in a straight line. The slope of this line is in fair agreement with that found by E. S. King[1] from brighter stars.

Exceptions to this rule are therefore of special interest and have been investigated by the writer at various times, usually with negative results. For example, some of the most striking exceptions in the list published by Franks[2] were found to be quite normal. A list of "Fourth-Type Stars not Red" given in a letter from Professor E. C. Pickering, dated June 5, 1911, therefore demanded attention. In order to get a better idea of the spectra of a variety of stars of Type IV, the list was extended from *Harvard Circular*, No. 145, "Stars having Spectra of Type VI, Class R," also from a list of red stars whose spectra had been photographed with the 40-inch Yerkes refractor, and the results published by Hale, Ellerman, and the writer in 1903.[3] This extension of the list was made possible by the efficient assistance rendered by Professor A. H. Joy of the Syrian Protestant College of Beirut, Volunteer Research Assistant for the summer of 1911; and of Mr. C. H. Gingrich, Fellow in Astronomy at this observatory.

Ellerman had photographed the spectra of the brighter stars of Type IV in two parts; the blue region on Cramer Crown and Seed 27 plates, and the yellow region on Cramer Isochromatic plates at a different setting of the Brashear spectrograph, using a camera of 271 mm focal length, and three 60° prisms. For reproduction these spectra were enlarged, and at the same time broad-

[1] *Harvard Annals*, 59, 180. [2] *Monthly Notices*, 70, 191, 1909.

[3] "Spectra of Stars of Secchi's Fourth Type," *Publications of the Yerkes Observatory*, 2.

ened by a pendulum motion of the plate, so that they showed a great number of fine lines, very much like spectra of Type II. These spectra are therefore not as well adapted to account for the color-indices of the stars as if the blue and yellow regions had been taken on the same plate. Nevertheless, being the only large-scale spectra of extremely red stars so far published, they are very useful in themselves, and a comparison with the objective-prism spectra is given later in Plate IX, Fig. 1, to aid in co-ordinating the two sorts of spectra. Some specimens of the spectra taken with the 40-inch telescope therefore follow.

Fig. 1 includes the blue region of Types II, III, and IV, the stars, reading from the top downward, being *280 Schjellerup*, Type IV; Sun, Type II; *μ Geminorum*, Type III, and *74 Schjellerup*, an advanced specimen of Type IV. A close correspondence between the finer metallic lines in all four spectra is evident, but the prominent characteristic of the spectra of Type IV is the broad dark band extending from about λ 4650 to λ 4750, with a bright band of nearly equal width on the less refrangible side and a less intense but broader bright region on the more refrangible side.

The yellow region of the same spectra is shown in Fig. 2. Here the correspondence in the metallic lines is somewhat masked by the carbon flutings in the spectra of Type IV. The most prominent characteristic of the red stars in this region is the bright band having its more refrangible edge sharply defined at about λ 5640.

An inspection of these two figures will show that the color-index of stars of Type IV will depend mainly on the strength of the bright band in the yellow as compared with the bright regions bordering the dark band in the blue.

In Fig. 3 we have a comparison of Types II and IV extending farther into the violet, to about λ 4200, and in Fig. 4 a still greater extension to a little beyond λ 3900. These show a falling off in intensity of the Type IV spectra at λ 4300 (the solar G group) and another at λ 4227. Fig. 4 is remarkable in that it shows the extension of the spectrum of the red star *19 Piscium* beyond the Fraunhofer H and K lines. This plate was taken by Ellerman with the Brashear spectrograph on the 24-inch reflector, requiring an exposure of 24 hours and 40 minutes on four nights.

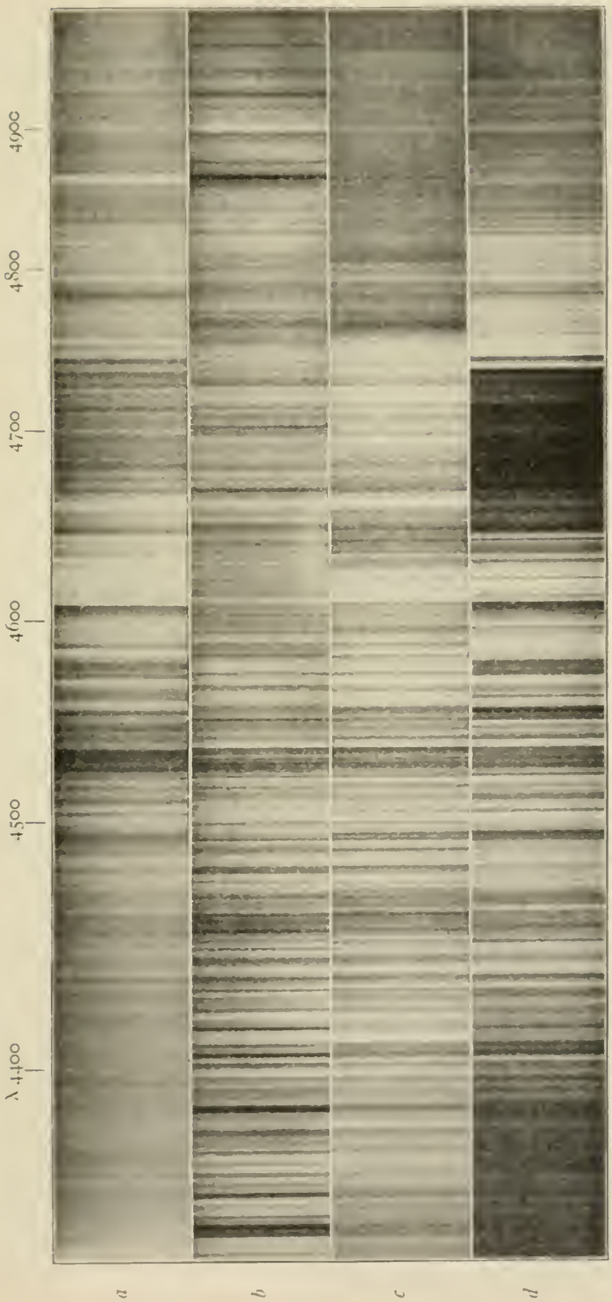

FIG. 1.—Blue Region of Stars of Types II, III, and IV

(*a*) 280 *Schj.* (*b*) Sun. (*c*) *μ Geminorum.* (*d*) 74 *Schj.*

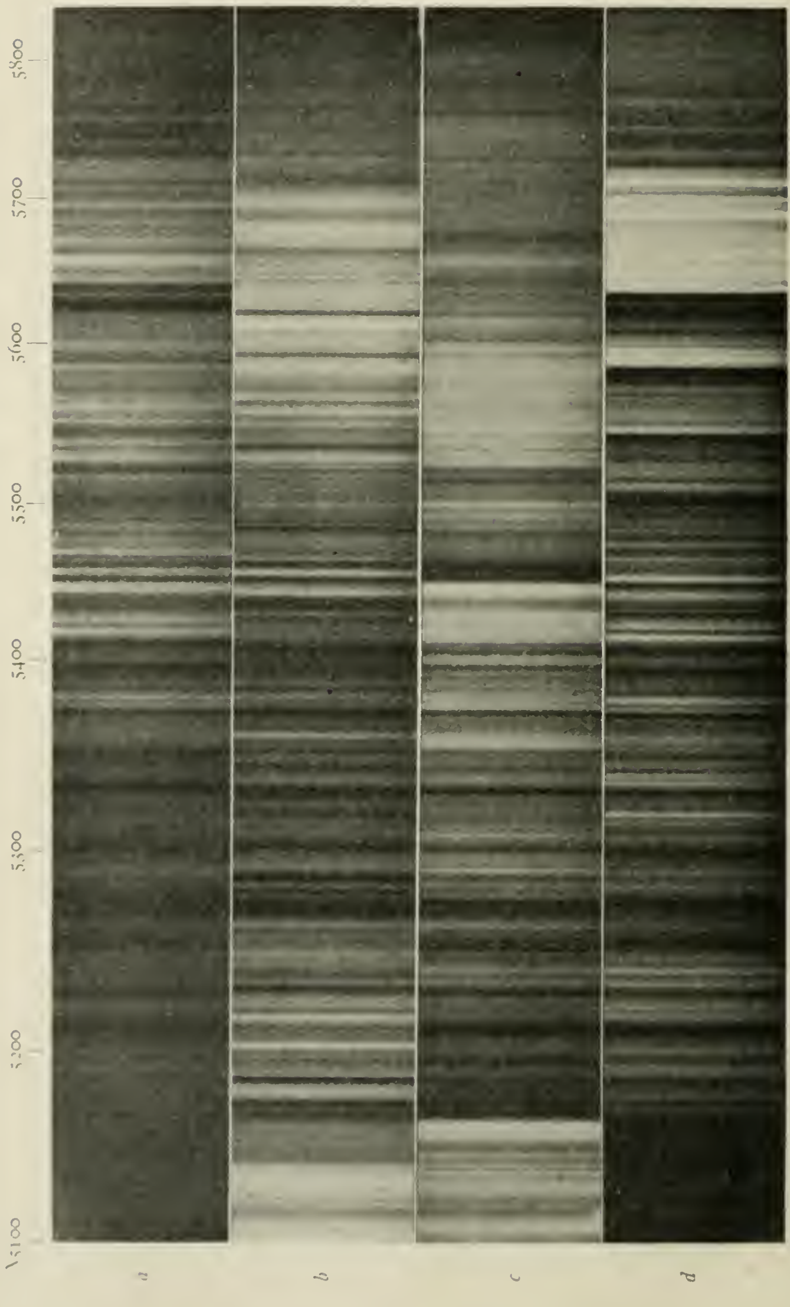

FIG. 2.—Yellow and Green Regions of Stars of Types II, III, and IV
(a) 280 *Schj.* (b) Sun. (c) *μ Geminorum.* (d) 74 *Schj.*

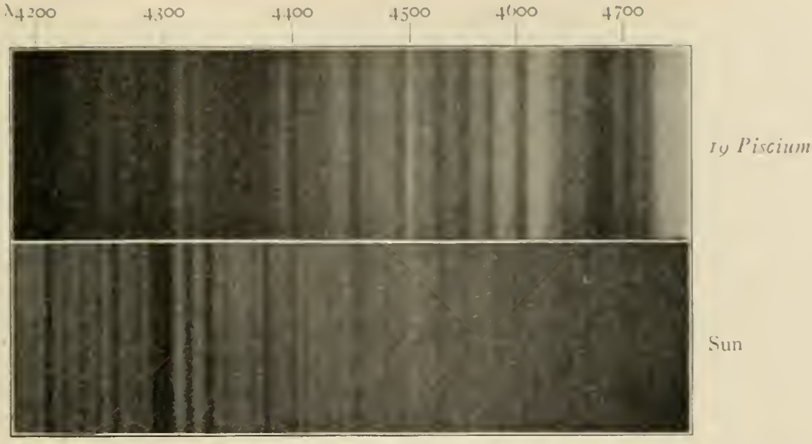

FIG. 3.—Blue Region of Types II and IV

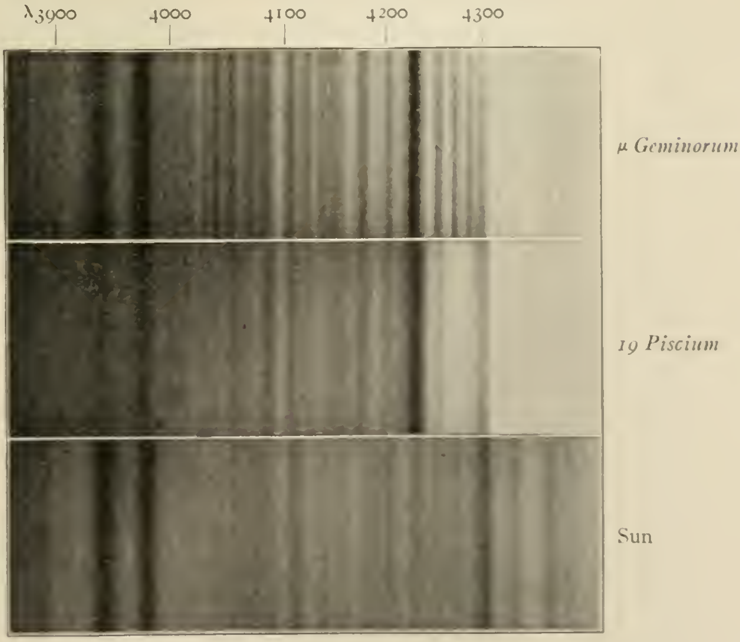

FIG. 4.—Violet Region of Types II, III, and IV

Fig. 5 shows both the blue and yellow regions of a series of stars of Type IV. In the blue region the regular progression in intensity of the dark λ 4700 band suggests an evolutionary series in the stars. This progression is not as evident in the yellow region, though it probably exists there also. This figure is a composite of Plates VIII and IX in the paper above quoted in Vol. II of the Yerkes Observatory Publications. The separation between the blue and yellow regions in the figure corresponds quite closely with the green region in the spectrum to which the plates used were not sensitive, and which therefore does not appear on the photographs. The star

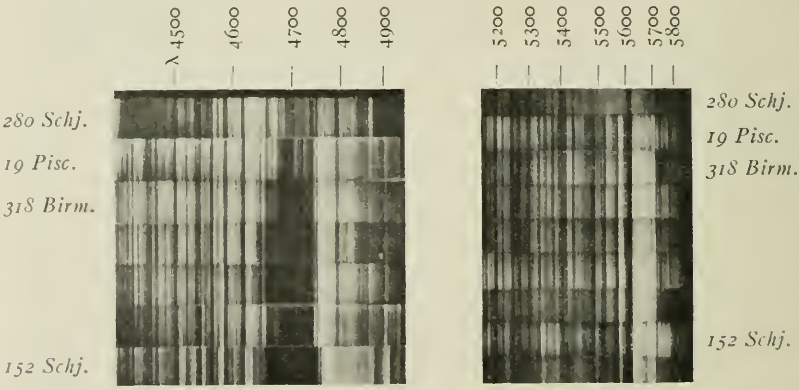

Fig. 5.—Spectra of Fourth-Type Stars Taken with Slit Spectrograph

names shown on the margins of the figure are those for which values of the color-index are given in the table following.

The spectra of most of these red stars were photographed with the Zeiss camera of 145 mm aperture and 814 mm focal length, provided with a doublet lens and 15° objective-prism of ultra-violet glass. These spectra were broadened by the diurnal motion to a width of about 0.6 mm, sufficient to show the lines. To give an idea of the relative appearance of the 40-inch and the objective-prism plates, the blue region of the spectrum of *19 Piscium* has been enlarged to the same scale from plates of each kind, and the two shown in Plate IX, Fig. 1. The objective-prism plates have a greater extension in each direction, but show only the features which are most prominent on the 40-inch plates. The broad

absorption band at λ 4700 is well shown on both, and the objective-prism plate shows the fall in intensity at λ 4300 and the extension of the spectrum to λ 4227. The similarity between the spectra of Types II and IV is thus evident in these bolder features as well as in the agreement of the fine metallic lines.

Fig. 2 of Plate IX shows a series of objective-prism spectra of red stars of Type IV, beginning with the short spectrum of *152 Schjellerup* at the bottom, and ending with the long spectrum of *B.D.* -10°5057 at the top. The two principal points to be noticed in this series are: first, the difference in the relative intensity of the light-action on each side of the dark λ 4700 band; and second, the difference in the length of the spectra. As the conditions of plate, exposure, and development were similar, these represent real differences in the stars. Going up from *152 Schjellerup*, the light on the red side of λ 4700 steadily diminishes in intensity as compared with the light on the violet side of the band. The upper spectrum, *B.D.* -10°5057, shows the Fraunhofer H and K lines and extends some distance beyond; but as Fig. 4 showed the spectrum of *19 Piscium* to H and K, it is probably a mere question of exposure to extend the other spectra to this region, except perhaps that of *152 Schjellerup*.

The upper spectrum, that of *B.D.* -10°5057, requires further mention, since the negative, even more than the engraving, suggests a composite spectrum. Mrs. Fleming seemed to have the same idea, for she wrote:[1] "The lines in the spectrum are not those due to hydrogen. In some photographs they are broad bands, while in others the lines appear to be double." Professor Barnard kindly examined the star with the 40-inch telescope, but could detect no duplicity. The spectrum resembles the solar type, with the addition of the dark λ 4700 band. The color-index, given in the following table, is 1.09, but little greater than that of an ordinary solar type star.

In the accompanying table of spectra and color-indices, the stars are arranged in order of the values of the color-index. The first column gives the name of the star, the second the *B.D.* designation, the third and fourth the place for 1900, the fifth the *B.D.* magnitude,

[1] *Astronomische Nachrichten*, **128**, 122, 1891.

the sixth the color and magnitude from the Potsdam Durchmusterung. The spectral classification given in the seventh column is usually from the Harvard lists, given as N, Na, or R. For the star *3 Schjellerup* Duner's classification, IV is used. The color in the eighth column, when expressed by one figure means Wendell's estimate made visually with the 15-inch equatorial, and communicated in Professor Pickering's letter. When a decimal place is given, the estimate is Duner's, taken from Krüger's *Catalog der farbigen Sterne*. The ninth column gives the color-index in magnitudes, as obtained by methods which will be presently explained. The tenth column gives the number of plates, R standing for the 24-inch reflector and C for the Zeiss camera.

SPECTRA AND COLOR-INDICES OF RED STARS

1	2	3	4	5	6	7	8	9	10	
		1900								
Star	*B.D.*	R.A.	Dec.	*B.D.* Mag.	P.D. Col. Mag.	Spectrum	Color	Color-Index	Plates R.	C.
		h m	° ′					M		
	−10° 5057	19 13	−10 53	7.0	....	R pec	...	1.09	4	3
	+85 332	19 44	+85 9	9.2	....	......	...	1.56	4	
	+53 66	0 19	+53 46	9.3	....	R	...	1.66	4	
	+20 5071	22 0	+20 34	8.7	....	R	...	1.82	4	
	+38 2389	12 55	+38 22	8.6	....	N	1	1.94	6	
	−16 5272	19 13	−16 5	6.8	....	Na	...	2.18	4	
	+ 6 3898	18 37	+ 6 43	9.0	....	......	...	2.37	4	
152 Schj. . .	+46 1817	12 40	+45 59	5.5	GR-5.24	Na	8.0	2.50	8	4
74 Schj. . .	+14 1283	6 20	+14 47	6.5	GR 6.26	N	...	2.54	2	3
	+76 734	19 25	+76 22	6.5	GR 6.24	......	...	2.63		5
249a Schj. . .	+34 4500	21 38	+35 3	6.2	GR 6.13	Na	4	2.74	6	4
19 Piscium	+ 2 4709	23 41	+ 2 56	6.2	GR 5.12	Na	...	2.80	4	3
280 Schj. . . .	+59 2810	23 56	+59 48	7.8	....	Na	6	2.88	5	
318 Birm. . .	+68 617	10 38	+67 56	6.1	RG 6.07	N	8.3	3.18	4	4
	+36 3243	18 39	+36 52	7.6	RG-7.62	Na	5	3.26	5	
3 Schj. . .	+43 53	0 15	+44 9	8.2	....	IV	9.3	4.56	4	
U Cygni . .	+47 3077	20 16	+47 35	var	....	pec	...	5.60	4	

The "color-index," the difference between the photographic and visual magnitude, was determined as follows: The photographic magnitudes were found from Seed 27 and 30 plates, those with the reflector being taken in focus, those with the camera being extra-focal. The "visual" magnitudes were found with Cramer Trichromatic plates taken in focus behind a Wallace "visual luminosity" filter, both with the reflector and the camera. The

extra-focal images were measured with the Hartmann "Mikrophotometer" and reduced with the writer's absolute scale. The diameters of the focal-images were measured under the microscope and reduced by the formula

$$\text{Mag.} = a - b\sqrt{D}.$$

The following conclusions can be drawn from the Table:

1. The three stars classed as having spectrum R in the Harvard lists have the smallest values of the color-index.

2. The nine stars classed N and Na have colors ranging from 1.94 to 3.26. Among these, Wendell's estimates of color range from 1 to 6, nearly in the order of the color-index.

3. Duner's color estimates, 8.0, 8.3, and 9.3, correspond to color-indices 2.50, 3.18, and 4.56; a decided difference in the range.

4. The star $B.D. - 10°5057$ stands in a class by itself.

5. There seems to be no sharp dividing line between Harvard Classes R and N.

6. Except the first two, all the stars in the list are as red or redder than *α Tauri* (for which E. S. King gives the color-index 1.64), so that the expression "Fourth-Type Stars not Red" seems inappropriate.

Yerkes Observatory
February 1912

THE PARALLAX OF *NOVA LACERTAE* 1910

By FREDERICK SLOCUM

Nova Lacertae was discovered by Espin December 30, 1910. The telegram announcing the discovery was received at Williams Bay on the afternoon of December 31, and the first plate for the investigation of its parallax was obtained with the 40-inch telescope soon after 6 o'clock that same evening.

The longitude of the star was 91° greater than that of the sun on that date, giving practically the maximum negative value for its parallax factor, −0.98.

Ten plates were secured as shown by Table I. The first two were taken when the star was east of the sun, the next four when it

TABLE I

Plates of *Nova Lacertae*

Taken with the 40-Inch Refractor of the Yerkes Observatory

No.	Date	Hour Angle	Observers	Quality of Images	Remarks
359.........	1910 Dec. 31	$+2^h8$	Sl, Sl	Fair	Telescope east
360.........	1911 Jan. 7	+2.8	Sl, Sl, Sl	Good	Telescope east
432.........	June 17	−2.0	Su, Sl	Good	
438.........	July 1	− .6	Su	Fair	
446.........	July 8	− .5	Su, Sl	Good	
498.........	Sept. 16	0	Sl, Su	Good	
531.........	Oct. 14	− .2	Sl, Su	Good	
536.........	Oct. 28	− .5	Su, Sl	Good	
548.........	Nov. 21	− .1	Sl, Sl	Good	
563.........	Dec. 1	− .2	Sl, Sl	Good	

was west of the sun, and the last four again to the east. The third column gives the hour angle of the star at the time the plate was exposed, plus indicating west of the meridian, and minus, east. So far as possible all observations for parallax are made near the meridian with the telescope west of the pier. In the case of the first two plates of this series, taken just after the *Nova* was discovered, the star had already passed the meridian by the time the sky was dark enough to allow a plate to be exposed, and it was

necessary to reverse the telescope and make the exposure with a relatively large hour angle, 2.8 hours.

In general two exposures were made on each plate, the guiding being done by the observers indicated in column 4: Sl=Slocum, and Su=Sullivan. On plate No. 360 there were three exposures and on No. 438 only one.

Six comparison stars were selected, as symmetrically situated as possible, and as near as possible to the mean brightness of the *Nova*. Table II gives additional data pertaining to the comparison

TABLE II

COMPARISON STARS

No.	Diameter	Approx. Magnitude	X (Longitude)	Y (Latitude)	Dependence
1...........	0.32 mm	8.8	−261.6	− 77.2	0.13
2...........	.22	9.4	− 76.5	+179.8	.21
3...........	.21	10.1	− 65.8	− 28.2	.15
4...........	.30	8.8	− 20.8	+ 42.8	.18
5...........	.24	9.4	+ 55.3	−247.5	.11
6...........	.27	9.4	+369.4	+130.3	.22
Nova........	.37 to .12	8 to 12	+ 23.46	+ 32.17	

stars. The second column gives the mean diameter of the star images in fractions of a millimeter. The image of the *Nova* diminished from 0.37 mm to 0.12 mm in the eleven months of observation, although the exposure time was increased from 5 to 15 minutes as the *Nova* decreased from somewhat brighter than the 8th down to the 12th magnitude.

Columns 4 and 5 show the co-ordinates of the comparison stars, in units of the scale of the measuring machine, referred to the mean of all. Thus in longitude four are below and two above the mean, while in latitude three are above and three below. The last column gives what Professor Schlesinger has well called the "Dependence" of each comparison star. The figures indicate the relative influence of each star upon the parallax to be found.

In the case of this star it was decided to measure the parallactic displacement parallel to the ecliptic rather than that parallel to the equator.

The usual method of reduction was followed. One plate was selected as a standard, in this case No. 446, and all of the others referred to this standard by a linear relation of the form

$$ax+by+c=x-x_0.$$

After a least square reduction for the a, b, and c for each plate, the residuals for the parallax star were found. These residuals involve the displacements due to parallax and proper motion plus a constant. Each plate then gives an equation of the form

$$P\pi+t\mu+c=m$$

from which the parallax and proper motion may be obtained.

TABLE III

REDUCTIONS FOR *Nova Lacertae*

Plate	Solution (m)	Weight (p)	Longitude Parallax Factor (P)	R.A. Parallax Factor	Time in Days (t)	Residual (v)	Residual in Arc
359........	0.000	0.8	−0.983	−0.765	−189	0.000	0″.000
360........	+ .002	1.3	− .978	− .697	−182	− .001	− .003
432........	+ .024	1.0	+ .984	+ .889	− 21	− .001	− .003
438........	+ .047	.5	+1.016	+ .792	− 7	− .022	− .059
446........	.000	1.0	+1.012	+ .727	0	+ .025	+ .066
498........	+ .036	1.0	+ .299	− .268	+ 70	− .009	− .024
531........	+ .046	1.0	− .176	− .648	+ 98	− .019	− .051
536........	+ .036	1.0	− .405	− .785	+112	− .009	− .024
548........	+ .001	1.0	− .736	− .911	+136	+ .026	+ .069
563........	+ .029	1.0	− .838	− .916	+146	− .001	− .003

NORMAL EQUATIONS

$$6.052\pi+\ .9160\mu-1.4093c=+.0081$$
$$13.8942\mu-1.4990c=+.1436$$
$$+9.600\ c=+.1975$$

$c=+.0200$

$\mu=+.0079=+0''.021$

$\pi=+.0048\ \pm\ .0051=+0''.013\pm0''.014$

Probable error corresponding to unit weight, $\pm.0123=\pm0''.033$.

In Table III, column 2 gives the residual for the *Nova* of each plate.

Column 3 gives the weight depending upon the number of exposures and the quality of the images.

Column 4 the parallax factors in longitude: two minus, four plus, and four minus.

Just for comparison the parallax factors in right ascension are given. For all except a few plates these run smaller than the corresponding factors in longitude.

The sixth column shows the time in days, reckoned backward and forward from the standard plate No. 446.

From these values ten equations are formed which yield the normal equations shown in Table III. The value of c is of no especial interest; μ is the proper motion in longitude for 100 days.

The relative parallax comes out $+0''.013 \pm 0''.014$ with a probable error from one plate of unit weight of $\pm 0''.033$. According to Kapteyn's table of average parallaxes, that of the comparison stars may be taken as $0''.005$, which would make the absolute parallax of the *Nova* $+0''.018$.

The above parallax is less accurate than might be expected from a normal star for two reasons: first, because it was necessary to take the first two plates with the telescope east; second, because the *Nova* diminished four magnitudes during the eleven months, thus rendering the possibility of influence from guiding error considerably greater. If the *Nova* is still bright enough, additional plates will be taken next summer, and a new reduction made, omitting the first two plates and using much fainter comparison stars.

So far as I have been able to learn parallax determinations of but two other novae have been made:

For *Nova Andromedae* (1885) Franz found, from heliometer observations, a parallax of $-0''.32 \pm 0''.12$.

Several determinations of the parallax of *Nova Persei* (1901) were made:

Hartwig found	$+0''.16 \pm 0''.06$
Chase	$-0''.012 \pm 0''.035$
Greenwich observers	$< 0''.1$
Bergstrand	$+0''.029 \pm 0''.008$

The first two determinations were made from heliometer observations, the last two by photography.

Without claiming too much accuracy for my result, we are safe in saying that the parallax of *Nova Lacertae* is exceedingly small.

If the value given above were correct, it would mean that the outburst observed in 1910 really occurred 180 years ago.

Yerkes Observatory
January 10, 1912

ON THE ORIGIN OF SELECTIVE CONTINUOUS ABSORPTION OF BAND AND SERIES SPECTRA

By J. KOENIGSBERGER

I wish here to enumerate briefly a few experiments which Mr. Küpfer and I have previously described at greater length. I am indebted to the Board of Trustees of the Elizabeth Thompson Science Fund of Boston for their assistance in these experiments. A great number of elements and compounds were heated in very highly evacuated flasks of soft and hard glass or of quartz. The absorption of the vapor was then examined. A good Rowland grating in the third order dispersed the transmitted light, making it possible to see what substances gave a band spectrum resolvable into lines, and what ones a selective continuous absorption. For the latter, $p\frac{e}{m}$ could be reckoned by the formula given elsewhere, and in the same way p represents the number of the vibrating parts of the molecule, and $\frac{e}{m}$ the ratio of electricity to mass.

The following fact was then observed: all colored vapors which possess no reversible chemical dissociation, i.e., a steady association

Substance	Absorption	Chemical Dissociation	Substance	Absorption	Chemical Dissociation
Indigo red........	S	—	Iodine trichloride..	B	+
Indigo blue.......	S	—	Selenium chloride..	B	+
Alizarine.........	S	—	Arsenic tri-iodide..	B	?
Anthrachinon derivatives.....	S	—	Tellurium dichloride......	B	+
Iodeosin.........	S	—	Tellurium dibromide......	B	+
Iron chloride......	S	—	Selenic acid.......	B	?
Chromium chloride	S	—	Manganese superchloride........	B	+
Tin iodide........	S	—	Chromium oxychloride.....	B	?
Nickel chloride....	S	—	Chlorine dioxide...	B	+
Iodine...........	B	+	Chlorine monoxide.	B	+
Bromine..........	B	+	Nitrogen dioxide...	B	+
Chlorine.........	B	+			
Sulphur..........	B	+			
Selenium.........	B	+			

S = Selective continuous absorption. B = band absorption. + = reversible chemical dissociation either at the temperature of the experiment, or at a somewhat higher temperature. ? = doubtful, as no observations on dissociation were made.

and dissociation, give a continuous selective absorption. All substances which, on the other hand, have band spectrum resolvable into lines, dissociate chemically either at a temperature at which they give the band spectrum, or at a higher temperature.

The metal vapors of zinc, cadmium, tin, arsenic, lead, mercury, and thallium show no absorption at 550°, neither continuous nor discontinuous, in the visible spectrum. The metals were placed in highly evacuated glass flasks entirely free from gas, and the flasks then closed. In this way all chemical reaction was out of the question. The metalloids, which, like sulphur, selenium, tellurium, are all polyatomic, show at a lower temperature a continuous, at a higher a band, absorption. Chlorine, bromine, iodine give a band spectrum in conjunction with a continuous selective absorption.

This behavior of compounds and the metalloids seems to prove the dependence of the band absorption upon processes which effect chemical dissociation, either with dissociation of the molecule or with its reunion. The experiments of E. J. Evans[1] on bromine and of J. I. Graham[2] on sulphur show that bromine Br_2, as well as S_8 and S_2 in these cases must be present if there is band absorption.

Therefore, the process of the dissociation of Br_2 into $2Br$, of S_8 into $4S_2$, and of S_2 into $2S$, or the union of $2Br$ into Br_2, as of $4S_2$ into S_8 points to the probability of vibrations in the bands. Which of these is the real cause we cannot at present say. Below we note that for line emission of the series-spectrum a similar hypothesis is possible. If substances show a band spectrum then polyatomic molecules must be present, which also holds for metalloids. We rather think that sodium vapor, which, as R. W. Wood has shown, has band absorption, is not strongly monatomic but possesses a complex molecule at a temperature at which it is ionized. Experiments on the mobility of the ions prove also the existence of such complex molecules, which, therefore, depends on the formation of the ions. On the other hand, for the same reason, it should not be possible to observe a band spectrum for the inert gases.

It is often not possible to prove chemical dissociation at temperatures at which band spectrum occurs, as in the case of bromine

[1] *Astrophysical Journal*, **32**, 291, 1910.

[2] *Proc. Roy. Soc.*, A, **84**, 311, 1910.

and iodine; but the band spectrum is weaker, the higher the temperature of the chemical dissociation. Then the process of dissociation and reunion takes place probably in immediate succession without the atoms *Br* or *I* existing as such free for any appreciable length of time. On the other hand, if the gas has become monatomic, a band spectrum is either not visible or only very seldom. The band spectrum which benzol, ammonia vapor, etc., shows in the ultra-violet, would have its cause in the rupture and the reunion of one or more chemical bonds. What bonds these may be, is a chemical question, and one which the author cannot answer.

The band absorption occurs on the boundaries of the continuous selective absorption and indeed chiefly on the side of the greater wave-lengths. The greater the density so much the more is the intensity of the band absorption attenuated and shifted, because the continuous absorption is increased. The cause of this is not due to the darkness caused by the absorption, which would prevent the band absorption from being seen, for we can prove that there is no band absorption if we diminish the thickness of the vapor layer and increase the density. On the other hand, by diminution of the density the continuous absorption becomes weakened and compressed. Therefore, if we wish to have a sufficiently intense band absorption, we must increase the density of the vapor layer; in this case the band absorption takes up the greater part of the spectrum. Band spectrum occurs by the rupture of those chemical bonds which give also continuous selective absorption. Whether the influence of the density on the band spectrum depends on the exterior collisions[1] or on a change of the dissociation process, it is not now possible to say. It appears as if the conditions in which the band spectrum is absorbed and emitted remain relatively longer in the case of smaller density.

Gases which dissociate with difficulty, as nitrogen and oxygen, show a band spectrum at a correspondingly higher temperature.

The continuous selective absorption in the visible part of the

[1] The strong damping of vibration which produces the continuous selective absorption is not caused by collision with other gaseous molecules. We could prove this by observations on the vapors of undissociated colored substances. In these substances the damping of vibrations is quite independent of the density, for it remains the same if the density is 1/10000 of the original.

spectrum results only through vibrating electrons so far as it has been possible to calculate. *Continuous selective absorption is due to the normal state of gas molecules; the band spectrum absorption and emission only to an intermediary non-stationary condition.*

The series spectra appear on the other hand to have an entirely different cause, viz., electric dissociation or ionization. We conclude that this is so from the following observations on canal rays when we take the hydrogen atom as the simplest example.

The canal rays emit light, as we have further found, only when they come into a space filled with gas; in high vacuum ($1 . 10^{-5}$ mm) no light can be seen, and, indeed, is so much the weaker, the higher the vacuum. Hence the light-emission is brought about by a process between the moving canal-ray particles and the gaseous molecules which are at rest. As W. Wien and J. J. Thomson have found, the canal rays change their charge and become absorbed. We have found that the change of charge gives no appreciable loss of velocity. The absorption is brought about through a diminution of the number of the canal-ray particles by diffusion and total or partial stopping. Emitted light which shows the Doppler effect, as discovered by J. Stark, cannot result from particles at rest, or partially stopped, but only from those moving in the direction of the canal rays with the original velocity. Hence the emission of light can occur not by absorption but only by the change of charge. This change of charge is of two kinds: (1) neutralization of the positive part by attraction of an electron, (2) dissociation of the neutral part into positive, by giving up of an electron.

It is evident that by magnetic deviation only the neutral ray emits light. The positive will not emit light so long as it becomes neutral again, through a partial change of charge. The experiment succeeds best at a pressure of about $5 . 10^{-3}$ to $4 . 10^{-2}$ mm. At a greater pressure the change of charge takes place too quickly, and the positive ray becomes neutral too quickly; on the other hand, at a smaller pressure the change of charge occurs too seldom and the light emission is too weak. Hence the dissociation of neutral into positive causes the light emission. The positive ion *in statu nascendi* emits light in the visible part of the series spectrum. This opinion agrees to a certain extent with that of J. Stark.

The distance, l, on which the newly formed ion emits light and therefore the length of time of the light emission is very short; l, according to our observations, is smaller than 0.5 cm, therefore $t=\frac{l}{v}2.5\cdot 10^{-10}$ sec. But this time corresponds to a large number of wave-lengths. We cannot say whether the emission is stopped after this time or whether the condition ceases in which the series spectrum is emitted and absorbed. The number of canal-ray particles is so small that an absorption of the light through them cannot be proved, as W. Wien has shown. The change of charge from positive to neutral brings about no light emission, as the experiments show.

FREIBURG IN BADEN
January 16, 1912

Minor Contributions and Notes

THE ASTRONOMICAL AND ASTROPHYSICAL SOCIETY

The thirteenth meeting of the Astronomical and Astrophysical Society of America was held in the auditorium of the Carnegie Institution of Washington on December 27, 28, and 29, 1911, in connection with the meeting of the American Association for the Advancement of Science. There were four sessions of the Society, presided over by Professor E. C. Pickering, president. Sixty-four members of the Society were in attendance. The program included the following papers:

E. W. Brown: "A Device for Facilitating Various Forms of Computation."
H. S. Davis: "The *Astronomischer Jahresbericht;* an Announcement."
H. S. Davis: "The Lesson of Joseph Piazzi's Life."
Joel Stebbins: "The Variability of *Polaris.*"
J. A. Parkhurst: "Magnitudes, Colors and Spectra of Standard Stars within Seventeen Degrees of the North Pole" (Lantern).
J. G. Porter: "A Comparison of Doctor Peters' Celestial Charts with the Photographic Charts of the Sky."
G. H. Peters: "The New Twin Photographic Telescope of the United States Naval Observatory" (Lantern).
Miss S. F. Whiting: "The Use of Special Topics in Teaching Astronomy."
J. C. Duncan: "The Orbit of the Spectroscopic Binary *β Scorpii.*"
Frederick Slocum: "The Dissolution of Solar Prominences" (Lantern).
Frederick Slocum: "The Parallax of *Nova Lacertae* (1910)."
W. J. Humphreys: "A Simple Pyrheliometer."
F. W. Very: "The Violle Actinometer as an Instrument of Precision."
Miss A. J. Cannon: "The Revised Draper Catalogue."
H. N. Russell: "Notes on the Calculation of the Elements of *Algol* Variables."
H. N. Russell: "The Eclipsing Variables, W *Centauri* and W *Ursae Majoris.*"
J. S. Plaskett: "The Solar Rotation" (Lantern).
F. E. Ross: "The Moon's Mean Parallax."
Eric Doolittle: "The Secular Variations of the Elements of the Orbits of the Four Inner Planets."
C. F. Talman (introduced by W. J. Humphreys): "The Language of Meteorology."

F. H. Loud: "May Astronomy Derive Any Benefit from the Dissemination of Esperanto?"

W. S. Eichelberger and H. R. Morgan: "On the Flexure of a Meridian Circle."

E. S. King: "Tests with Standard Electric Lamps."

J. A. Brashear: "Recent Interviews with Optical Glass Manufacturers of France and Germany."

E. E. Barnard: "Some Observations with the 60-Inch Reflecting Telescope of the Mt. Wilson Solar Observatory" (Lantern).

E. E. Barnard: "Photographic Observations of Brooks' Comet of 1911" (Lantern).

F. B. Littell: "Personal Equation Apparatus of the 9-inch Transit Circle of the Naval Observatory."

Asaph Hall: "Observations of the Satellites of *Uranus* and *Neptune* Made at the Naval Observatory, 1908–1910."

W. S. Eichelberger: "Paris Conference of October, 1911."

Zaccheus Daniel and F. Schlesinger: "The Spectrum and Orbit of β *Scorpii*."

Reports were presented by the Committees on Comets and on Photographic Astrometry, and from the Committee on Co-operation in the Teaching of Astronomy.

A new Committee on Asteroids was formed, consisting of E. W. Brown, J. H. Metcalf, G. H. Peters, and A. O. Leuschner.

There were also two sessions of Section A of the American Association, presided over by Edwin B. Frost, vice-president of the Section. At these sessions were presented the address of the retiring vice-president of the section, Professor E. H. Moore, "On the Foundations of the Theory of Linear Integral Equations"; and papers by Lewis Boss on "Recent Researches as to the Systematic Motions of the Stars," and by J. H. Metcalf on "The Asteroid Problem."

The next meeting of the Astronomical and Astrophysical Society will be held at the Allegheny Observatory in the coming summer.

F.

Reviews

The Sun. By Charles G. Abbot. New York and London: D. Appleton & Co., 1911. 8vo, pp. xxv+448, with 26 plates, 72 illustrations in the text, and 34 tables. $2.50.

This is the fourth book on the sun that has appeared within the past fifteen months, the other three being *Les théories modernes du soleil*, by J. Bosler, *Vorlesungen über die Physik der Sonne*, by E. Pringsheim, and Stratonoff's *The Sun*, printed in Russian.

This new book by Abbot is the first on this subject to be printed in English since the last edition of *The Sun* by Young in 1895. For fifteen years Young's book was the authority upon all matters pertaining to the sun. That interval was marked by great progress in solar investigation and revised editions of the book appeared at short intervals. Since 1895 there has been an even greater advance. As Abbot says in his preface:

> Within the last fifteen years we have seen the publication of Rowland's great table of solar spectrum wave-lengths, the establishment of the Yerkes, Kodaikanal, Mount Wilson, and other observatories largely devoted to solar researches, the photography of the spectrum of the corona and of the chromosphere at total solar eclipses, Hale's brilliant discovery of magnetic fields in sun-spots, the determination of the rotation periods of the sun at different levels, as well as at all solar latitudes, Langley's bolometric investigations of the sun's infra-red spectrum, and the recent Smithsonian determinations of the absolute intensity of the solar radiation outside our atmosphere. The great interest in such researches has been marked by the establishment of the International Solar Union, and its enthusiastic gatherings of the foremost investigators from all lands.
>
> The time seems ripe for collecting the splendid array of new solar knowledge which such unprecedented activity has produced, and for discussing the probable nature of the sun in the light gained.

Young's book is now out of print and it is the intention of the author that this book shall take its place. In fact, some of the illustrations and text of Young's book have been incorporated into the present volume, issued by the same publishers.

An idea of the general scope of the work may be obtained from the chapter headings: i, "The Solar System"; ii, "Instruments and Methods

Used in Solar Investigation"; iii, "The Photosphere"; iv, "Eclipses and the Outer Solar Envelopes"; v, "Sun-Spots, Faculae, and Granulation"; vi, "What Is the Sun?" vii, "The Sun as the Earth's Source of Heat"; viii, "The Sun's Influence on Plant Life"; ix, "Utilizing Solar Energy"; x, "The Sun among the Stars."

As will be seen, the sun is considered in three aspects: first, as the controlling member of the solar system; second, as a star, interesting in itself, and typical of a large class of stars; third, as the source of light and heat, and through them of life on the earth.

Thus a wide field is covered. In addition to the matter pertaining directly to the sun and to astrophysics, many facts and theories are given which will prove of interest to the meteorologist, geologist, botanist, and engineer. And even the student of household economics will find here something of interest in the section on solar cooking appliances.

To attempt to cover such a wide field in a single volume of this size is rather a bold undertaking, but Mr. Abbot has, in general, succeeded in treating each subject simply, clearly, and concisely. A few exceptions may be noted. Fig. 58 on p. 260 and the accompanying explanation are, to me, obscure. Again, on p. 53, the reason for inserting a prism into a plane grating spectroscope is not made clear. On p. 121, referring to solar prominences, we read: "And now the spectroheliograph has enabled us to recognize them frequently as dark hydrogen flocculi on the disk itself. A view of the sun through the $H\alpha$ (C) line is best adapted for this purpose. . . ." The fact that the prominences can, with sufficiently high dispersion, be equally well shown as dark *calcium* flocculi seems here to be ignored.

In the chapter entitled "What Is the Sun?" are given outlines of the views of Young, Halm, Schmidt, Julius, and Abbot. These views contain the most recent theories of the causes of solar phenomena, so far as such theories can be stated in simple, non-mathematical language.

The strongest parts of the book are naturally those dealing with bolometry, pyrheliometry, and the theories of radiation—subjects to which Mr. Abbot has for years devoted special study. But in trying to do full justice to these subjects he has been forced to treat others somewhat *stiefmütterlich*, as a German might say. For example, we find numerous figures of pyrheliometers but no illustration of a spectroheliograph. And the same set of bolographs is given on p. 83 and on p. 292.

The illustrations are, with one or two exceptions, very good. I am glad to see Langley's drawing of the sun-spot of 1873 used as a frontispiece. During the past thirty years this picture has been reproduced in

scores of astronomical books, and I trust that it will continue to be reproduced, at least until someone makes a better one. Photography has done wonders for the advancement of solar research, as is well shown by the many reproductions of photographs in this book, but it has not as yet superseded direct visual observations for the delineation of the details of sun-spots. Careful and persistent visual study of sun-spots is as important today as it ever was. This drawing should be a perpetual source of inspiration to amateurs and visual observers of the sun.

The book as a whole will be a welcome addition to the literature of the subject, and it is to be hoped that the author will follow Young's example and try to keep it up to date by new editions whenever necessary. The rapidity of the progress in this field is shown by the fact that some of the material in the book has already been superseded by results based upon later and better observations. This is true in regard to Table IX, p. 126, and Table XXXIV, p. 430.

The addition of many more references to original sources would, in my opinion, greatly increase the value of the book.

FREDERICK SLOCUM

Cours d'astronomie, Première partie: Astronomie théorique. Deuxième édition entièrement refondue. Par H. ANDOYER. Paris: A. Hermann, 1911. Pp. 375, with 83 figures. Fr. 12.

The revised edition of the first volume of M. Andoyer's work follows the plan of the first edition, which was reviewed four years ago in this Journal (**25**, 288, 1907). Even to a larger extent than in the first edition, all the theoretical discussions are based upon a small number of fundamental theories, which are completely developed. Thus we find in chap. i an excellent development of spherical trigonometry and in chap. iii a very full discussion of change of co-ordinates. The work now possesses, both in its composition and also in its printing, the scholarly finish and excellence which are characteristic of French scientific books.

F. R. MOULTON

THE
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THE SPECTROSCOPIC DETERMINATION OF AQUEOUS VAPOR[1]

By F. E. FOWLE

The purpose of this research is to make possible the determination of the amount of aqueous vapor present in the atmosphere by the observation of the absorption at certain wave-lengths produced by this vapor in the spectra of bodies observed through it. This requires the noting in the laboratory of the absorption resulting at these wave-lengths from known amounts of water vapor at as nearly as possible the same conditions as to pressure, density, and temperature that exist in the atmosphere.

ABSORPTION BANDS

The water-vapor bands in the infra-red spectrum known as Φ ($\lambda = 1.13\,\mu$) and Ψ' ($\lambda = 1.47\,\mu$) have been chosen for this purpose. Fig. 1*a* shows the bottom of Φ, Fig. 2*a*, Ψ with Ψ' as these bands appear in the solar energy-spectrum. Fig. 3 shows these same bands as well as Ω and the band at about $2.25\,\mu$, also due to water vapor, as they appear in the energy-spectrum of a Nernst glower when observed through vapor corresponding to 0.101 cm precipitable water.[2] The form of these bands depends not only on the

[1] Published by permission of the Secretary of the Smithsonian Institution.

[2] In this discussion the amount of absorbing vapor will be stated, for brevity, as so much precipitable water, meaning the depth of water which, if evaporated into a column of the same section, would produce the absorbing layer of vapor. This should not be construed as meaning that the liquid water produces the same amount of absorption as the corresponding vapor.

amount of aqueous vapor absorption but also on the purity of the spectrum as influenced by the widths of the slit and of the bolometer and the character of the spectroscope. A line has been drawn

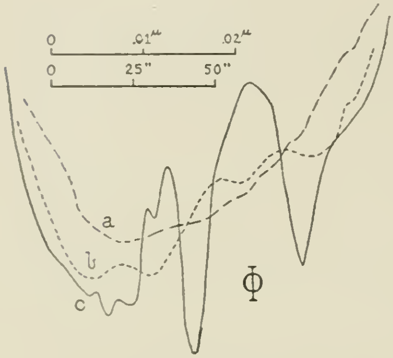

FIG. 1.—Energy-curves of bottom of water-vapor band Φ.

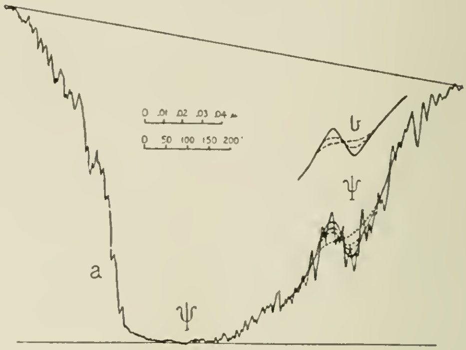

FIG. 2.—Energy-curves of Ψ with Ψ′

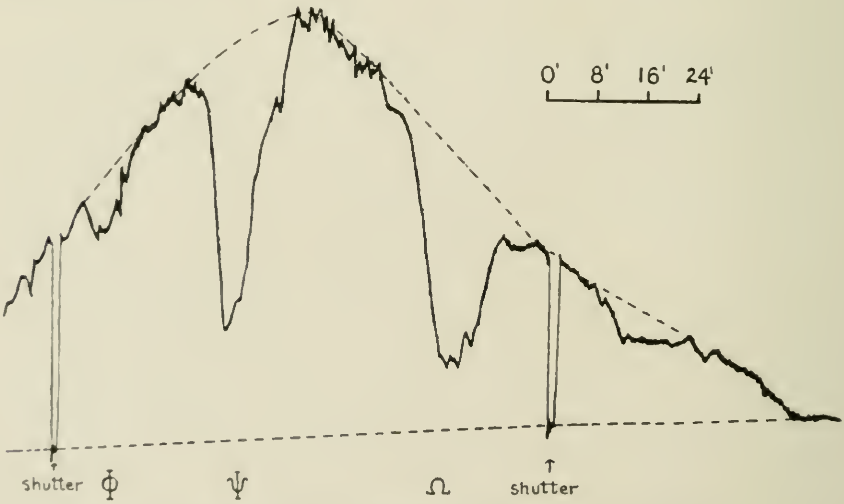

FIG. 3.—Energy-spectrum through water vapor of Nernst glower

over the tops of the large bands as indicated in Figs. 2 and 3, and the transmissibility of radiation taken as the ratio of the ordinates of the energy-curve at the chosen places to the ordinates of this line directly above. Since slight absorption due to aqueous vapor

may exist in the spectrum on either side of the bands Φ and Ψ, this ratio may not give exactly the true transmissibility. But the error, if any, should affect the laboratory work and its applications in the same sense and, therefore, should be almost wholly eliminated in the applications of this method.

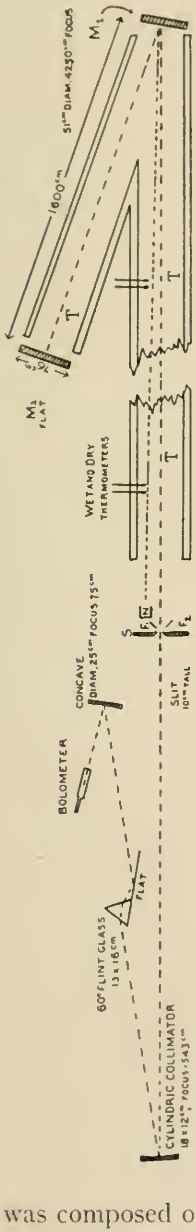

Fig. 4.—Plan of apparatus

APPARATUS AND METHOD OF OBSERVING

Light from the source N, composed of Nernst lamps, passed through the tube T, containing water vapor, to the concave mirror M_1, 51 cm in diameter and 42.7 meters focus, thence, collimated, to the flat mirror M_2, 76 cm in diameter, back to M_1, and then to focus on the slit of the spectroscope at S. Sometimes, before entering the spectroscope, the beam was returned over the course through the water vapor by the two flats F_1 F_2 beside the slit. The first arrangement gave a path through the water vapor in the tube of about 117 meters, to which must be added the path through the spectroscope, giving a total path of 128.5 meters; the corresponding value for the second case was 245.5 meters (806 feet). This long path greatly reduced the intensity of the lamp's image at the slit of the spectroscope. There also resulted a serious vibration of this image due to the consequent magnification of the small tremors of the mirrors. These two causes went far to limit the accuracy attainable in the observations.

The lamp used as a source of radiation was composed of two layers of Nernst alternating-current, 110-volt

lamps, four glowers wide, five long. The rear layer was so adjusted as to shine through the interstices of the front layer, presenting a glowing surface about 90 by 5 mm. These glowers were placed in the rear of a rectangular cavity, 100 by 10 mm, and 40 mm deep, cut into a block of soapstone.

The spectroscope was supplied with a 60° prism of flint glass described in the first volume of the *Annals* of this Observatory, p. 45. The bolometer was 0.1 mm wide, 12 mm tall, and subtended 27″ (about 0.006 μ in the region of the spectrum under study). The slit was from 0.5 to 2 mm in width by 100 mm in height, subtending from 20″ to 78″. The other details of the spectroscope may be learned from Fig. 4. The galvanometer had a time of single swing of about 3 seconds. Its indications were recorded on a moving photographic plate as described in the above *Annals*, p. 58.

The large galvanized-iron tube for the water vapor was open at its ends, double-walled, and covered with a canvas tent to protect it from rapid temperature changes. A rotary blower served to stir the vapor within the tube and free it from stratification just before an observation. It was not feasible to run the blower during an observation because of the tremors communicated to the mirrors and to the galvanometer. Steam could be forced in by the blower to increase the amount of aqueous vapor, but most of the measures were taken with the amount of vapor normally present in the tube. All the measures were at atmospheric pressure.

MEASUREMENT OF THE QUANTITY OF AQUEOUS VAPOR

The amount of aqueous vapor was determined by wet and dry thermometers at the spectroscope, at the mirror shelters and at several places in the tube. These were read while the air was stirred by the fan. Check determinations were made several times by Mr. L. B. Aldrich, who absorbed in tubes of calcium chloride and phosphorus pentoxide the water vapor from known volumes of air taken from the tube. The following table gives the water per cubic meter as measured by the two methods:

By Wet and Dry Thermometers	Absorbed by $CaCl_2$ and P_2O_5
3.25 grams per cu. m	3.29 grams per cu. m
3.82 " " "	3.85 " " "
7.96 " " "	8.76 " " "

The following shows a determination of the amount of water vapor in the large tube just preceding and just after an observation on the transmissibility of radiation through the water vapor:

August 11, 1911: Barometer 762 mm

Dry: Wet:	34°6 26.1	37°4 27.5	36°45 26.8	35°65 C. 26.6	$11^h\ 45^m$
Grams per cu. m before observation..............	19.5	20.7	19.8	19.8	Mean 20.0
Dry: Wet:	35°8 26.55	38°85 27.75	38°1 27.45	37°15 C. 26.85	$12^h\ 30^m$
Grams per cu. m after observation..............	19.7	20.3	20.6	19.5	Mean 20.0

CORRECTIONS FOR THE WIDTHS OF THE SLIT AND BOLOMETER

Perhaps the greatest obstacle to the use of the spectroscope for the quantitative determination of vapor producing an absorption line or band lies in the allowance for the purity of the spectrum. The greater the purity, the greater generally is the observed absorption at the bottom of a band. With the spectroscope used here the resolving power, as dependent upon the optical system, is such that in the region of the spectrum studied, two lines separated by about $3''$ ($0.0007\,\mu$, $60°$ flint-glass prism) may be resolved. The bolometer subtends an angle of $27''$ ($0.0046\,\mu$ at Φ, $0.0062\,\mu$ at Ψ); at the same places in the spectrum, the slit, from $20''$ to $78''$ ($0.0034\,\mu$ to $0.0134\,\mu$ at Φ, 0.0046 to 0.0179 at Ψ). The limiting angle of resolution of the prism, $3''$, is therefore negligible in the following discussion compared with the sum of the angles subtended in the spectrum by the slit and bolometer, $47''$ to $105''$. In the spectroscope used for solar-constant observations at Mount Wilson the bolometer-plus-the-slit subtended about $92''$ ($0.022\,\mu$ at both Φ and Ψ, $60°$ ultra-violet glass prism).

In spectro-bolometric researches energy-curves are obtained with a bolometer and slit of finite widths and an attempt is generally made to correct the form of the observed curves to represent the distribution of energy which would have been obtained had the bolometer and slit been indefinitely narrow. Professor Runge has published a method for obtaining such corrections.[1] Although

[1] *Zeitschrift für Mathematik und Physik*, **42**, 205, 1897.

some improvement in the observations is thus made, evidently no mathematical process can replace high purity for revealing spectrum details. Fortunately in the present research a more satisfactory expedient is available. It is desired to reduce the observations made with the water vapor in the tube, not to a grade of purity corresponding to zero slit and bolometer, but to a grade of purity corresponding to that of some other spectroscope. This Observatory has published[1] the form of the bands Φ and Ψ (Figs. 1 and 2 of this article) as determined with apparatus far better as regards purity than the apparatus in use for the present research. By measuring areas included under those old curves between ordinates separated by any chosen spectrum intervals, the ordinates which would have been observed with a spectroscope in which the slit and bolometer combined would cover the corresponding spectrum interval could be determined. In this way the old curves with high purity have been transformed to the conditions of the apparatus prevailing during the tube work at Washington or during the solar-constant work at Mount Wilson. From the results of such transformations it is possible to reduce the measures of the present research and of the Mount Wilson work to any desired condition of purity short of those which prevailed in the old work published in Vol. I of the *Annals*.

Curves computed for various degrees of purity as just described are indicated by the dotted lines in Figs. 1 and 2. For instance, in curve *b*, Fig. 1, the slit plus the bolometer covered a region in the spectrum of $0.013\,\mu$, curve *c*, $0.025\,\mu$. It is interesting to note the peculiar positions of the minima in curve *b* as compared with those of curve *a*.

These corrections for the finite widths of the slit and bolometer are a function of the amount of absorption as well of the purity of the spectrum. To determine this second effect, a new curve (*b*, Fig. 2) was determined from curve *a* of the same figure using the formula[2] $d_w = d_0 a^w$, where d_w is the deflection observed through an amount of water vapor w; d_0 that with no vapor, and a the transmissibility with unit thickness of vapor. For the computed curve w was taken as equal to $\frac{1}{3}\,w$ of the observed curve. By such means

[1] *Annals*, Vol. I, Plates XX, XXI A and B.

[2] *Smithsonian Miscellaneous Collections*, 47, 1, 1904.

a series of corrections was obtained as known functions of the purity and of the observed coefficients of transmission.

OBSERVATIONS

The accompanying table contains the observations on the transmissibility through water vapor of the radiations of the wave-lengths 1.13 (Φ) and 1.47 μ (Ψ′).

Date	Barometer	Temperature of Vapor	Slit-Width	Log. of Transmission Observed*		Log. of Transmission Corrected†		Water‡ Vapor	Observations
	mm	C	mm	1.13 μ	1.47 μ	1.13 μ	1.47 μ	cm	
Sept. 2, '09..	764	24°	(1)	0.977	0.984	0.979	0.985	0.013	......
Aug. 24, '09..	766	29	.....	......	.983	......	.983	.015	......
Aug. 21, '11..	767	29	0.5	......	.966?	......	.970?	.016	......
Aug. 25, '09..	...	31	1.0	.974	.970	.977	.972	.020	......
Sept. 2, '09..	764	28	(1)	.905	.910	.914	.917	.101	2, 2
May 13, '11..	765	32	2.0	.897	......	.903	......	.128	......
Aug. 21, '11..	767	32	0.5	.876	.923?	.891	.931?	.132	1, 2
Aug. 20, '09..	757	35	.....	......	.913?	......	.913?	.165	......
Aug. 24, '09..	766	33	.....	......	.895	......	.895	.171	......
Aug. 25, '09..	...	35	1.0	.855	.875	.869	.884	.192	2, 2
Aug. 9, '11..	762	36	2.0	......	.901	......	.899	.209	......
Aug. 10, '11..	764	36	1.0	.855	.853	.866§	.873§	.210	3, 3
	...	..	0.5	.828	.870	......	......	......	......
	...	..	0.5	.854	.857	......	......	......	......
Aug. 24, '11..	764	35	1.0	.806	.862	.852	.871	.233	2, 2
	...	..	0.5	.869	......	......	......	......	......
May 13, '11..	765	38	2.0	.848	.896	.857	.894	.244	......
Aug. 19, '11..	761	29	1.0	.808	.838	.825	.849	.249	......
Aug. 21, '11..	767	30	1.0	.799	.820	.837§	.851§	.259	4, 4
	...	29	1.0	.834	.885	......	......	......	......
	...	31	1.0	.807	.826	......	......	......	......
	...	31	0.5	.824	.834	......	......	......	......
Aug. 11, '11..	762	38	1.0	.797	.852	.824§	.849§	.262	3, 3
	...	38	0.5	.804	.805	......	......	......	......
	...	40	0.5	.807	.866	......	......	......	......
May 12, '11..	761	35	2.0	.840	.866	.849	.863	.275	......
July 29, '11..	765	32	2.0	.826	.825	.836	.822	.369	2, 2
Aug. 19, '11..	761	27	1.0	.781	.838	.800	.848	.370	3, 3
Aug. 10, '11..	764	38	1.0	.699	.774	.810§	.807	.392	3, 3
	...	38	1.5	.840	.795	......	......	......	......
	...	38	2.0	800	.840	......	......	......	......
	...	34	2.0	.784	.793	.795	.789	.398	......
Aug. 24, '11..	766	32	1.0	.779	.811	.798	.824	.422	3, 3
May 19, '11..	760	33	2.0	.778	.839	.789	.836	.438	......
Aug. 11, '11..	762	35	1.5	.755	.774	.772	.791	.492	2, 2
	...	36	1.0	.740	.787	......	......	......	......
	...	39	1.5	.730	.784	.748	.791	.540	......

* Logarithm of the percentage radiation transmitted, as observed with the slit indicated in the table.

† Same corrected to the spectroscope in use at Mount Wilson for solar-constant determinations; slit + bolometer covers 0.022 μ in the spectrum at 1.13 and 1.47 μ.

‡ Depth of water layer which, if evaporated into column of the same section, would produce the amount of absorbing vapor.

§ Weighted mean.

Figs. 5 and 6 give graphical representations of these results for Φ and Ψ' reduced to spectroscopic conditions where the slit plus the bolometer covers a region 0.022 μ in the spectrum. These are approximately the conditions fulfilled by the spectroscope in use at Mount Wilson during 1910 for solar-constant determinations.

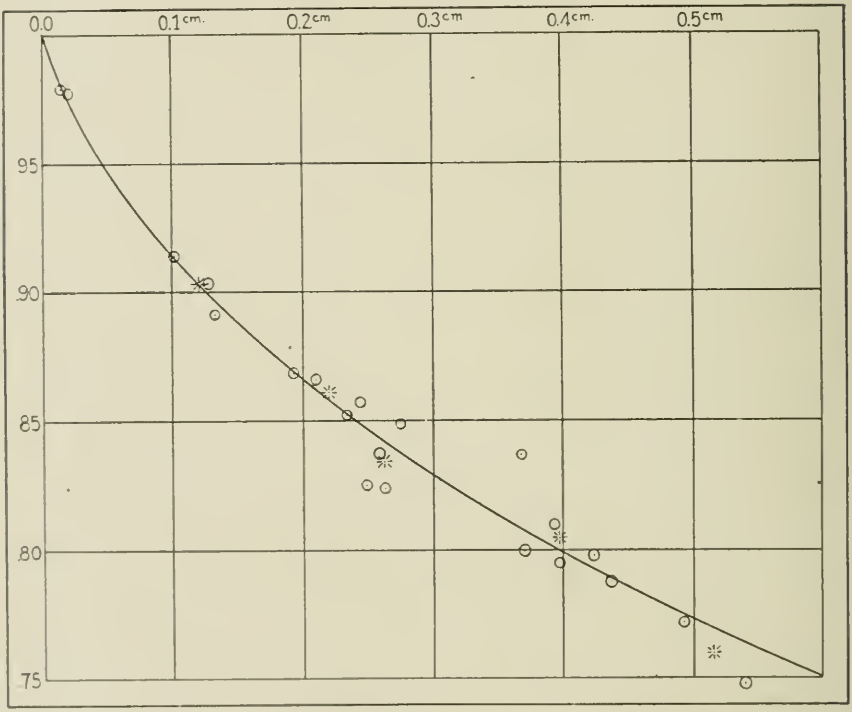

FIG. 5.—Abscissae, precipitable water; ordinates, logarithm of transmissibility of radiation at Φ (1.13 μ).

The ordinates are the logarithms of the percentage transmissibility of radiation at 1.13 μ and 1.47 μ, respectively, the abscissae, the corresponding quantities in centimeters of precipitable water. Besides the separate observations there are indicated on the plot by stars the mean results of all the determinations reduced by groups.

EXTENSIONS OF THE RESULTS FOR GREATER AMOUNTS OF WATER VAPOR

The tube experiments just described extend over a range of precipitable water up to 0.5 cm. According to Humphreys[1] the precipitable water in the path of a beam from the zenith to sea-

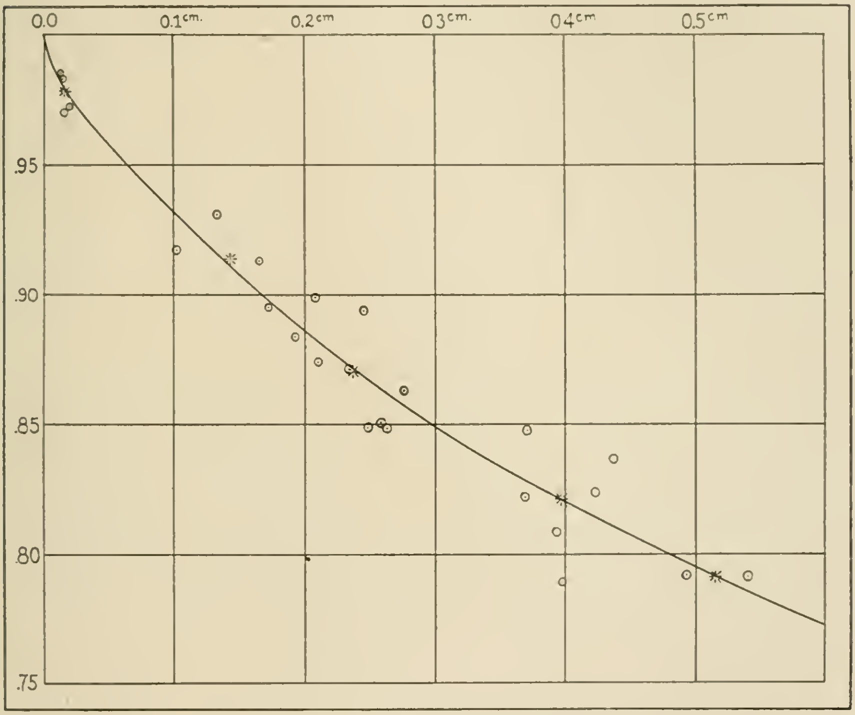

Fig. 6.—Abscissae, precipitable water; ordinates, logarithm of transmissibility of radiation at Ψ^1 (1.47 μ).

level may often reach 1.5 cm. It was not feasible to extend the tube experiments directly to such quantities; but from the Mount Wilson bolographs taken at high and low sun the curves of Figs. 5 and 6 may be extended, provided days may be found when the aqueous vapor in the air remained nearly constant for two or three

[1] *Bulletin of the Mount Weather Observatory*, 4, 121, 1911.

hours. For the relative lengths of the paths passed through by the radiation from the sun may be computed, being proportional to the secant of the zenith distance of the sun; then if the absolute amounts of the water vapor traversed during the high-sun observations are found by means of Fig. 5 or 6, the corresponding amounts of vapor in the path of the lower-sun observations may be determined.[1] If the earlier portion of the line determined by means of these solar observations coincides with the line from the tube work we may feel considerable certainty that the portion beyond furnishes a reasonable extrapolation. Having extended the curve one step, the new portion may be used likewise for still further extension. It is worth noting that on the days chosen by the criterion that the earlier portion of the plotted observations should agree in curvature with the tube work, the readings of the wet and dry thermometers at the top of the mountain indicated a greater constancy of vapor-pressure than usual.

As a result of such extensions Fig. 7 was obtained. The full curve represents the data for Ψ', the dotted line, for Φ. The ordinates are the logarithms of the percentage transmission of radiation at Φ and Ψ', respectively, the abscissae, the corresponding precipitable water. There are also given on this diagram the corrections necessary to reduce the curve for Ψ to various other spectroscopic conditions, namely: where the slit plus the bolometer covers 0.025, 0.020, 0.015, and 0.010 μ in the spectrum. On this subsidiary plot the abscissae correspond to the ordinates of the

[1] For example, the following data on the transmission at Ψ were obtained from observations at Mount Wilson:

Secant zenith distance	1.00	1.50	2.00	3.00	3.50
Logarithm transmission	0.824	0.783	0.740	0.670	0.636

The first two of these transmissions lie within the range of Fig. 6 and give for the corresponding amounts of vapor: 0.386 cm precipitable water and 0.547 cm. The latter figure is for a path 1.50 times as long as for the first and reduced to the same path, gives 0.365; the mean of 0.386 and 0.365 is 0.375; multiplying this mean by the relative paths as given in line one of this note we have for extending the curve:

Amount precipitable vapor	0.375	0.562	0.750	1.125	1.312 cm
Logarithm transmission	0.824	0.783	0.740	0.670	0.636

larger plot (log. transmission), the ordinates, the amount by which the curve must be lowered (raised for 0.025 μ). In other words, the ordinates are the logarithmic percentage corrections plotted on the same scale as used for the logarithms of principal line.

The laboratory observations on the transmissibility at Φ and Ψ', as given in columns 5 and 6 of the table, probably have nearly equal weight. When they are reduced to the conditions of the Mount Wilson spectroscope the figures for the latter have considerably the greater weight. At Ψ' the spectroscopic conditions in the laboratory and at Mount Wilson were nearly identical as to purity, and the correction was therefore small. For Φ, however, since the dispersion in this region is considerably smaller in the Mount Wilson spectroscope, the corresponding correction is much larger and, because of the shape of the band, is more doubtful.

VARIATION OF THE ABSORPTION WITH PRESSURE

It may be objected that the laboratory work has all been done at atmospheric pressure while some of the vapor of the atmosphere is under somewhat reduced pressures. Unfortunately the variation of the transmission with the pressure has not been determined for the bands employed here. But Miss Eva von Bahr[1] gives for the water-vapor band at 2.7 μ the following values for the absorption of a constant amount of aqueous vapor under varying pressures:

105 mm	4.6 per cent	405 mm	8.5 per cent
235	7.2	570	10.6
370	8.6	755	12.0

The increase in pressure was produced by dry air which exercises practically no absorption at this place. Miss von Bahr found that the absorption due to a vapor depended to a great degree upon the total pressure exerted upon it, not upon its own partial pressure. She also states that the "absorption as dependent upon the total pressure is in general, for the same gas, the same in the different bands."

[1] *Ueber die Einwirkung des Druckes auf die Absorption ultraroter Strahlung durch Gase*, p. 68, Upsala, 1908.

It may therefore give a fair estimate of the magnitude of this pressure effect in the region of Φ and Ψ′ to use these observations made at 2.7 μ. Using the distribution of aqueous vapor at different altitudes as given by Humphreys (*op. cit.*), a vertical column of air which would give a transmission of 88 per cent, with the pressure uniform throughout at 760 mm, would give, with a distribution of pressures such as actually exists in the atmosphere, according to the measures of Miss von Bahr, 90 per cent in summer, 89 in winter. With the distribution of vapor above Mount Wilson, the transmission comes out 90 per cent for both summer and winter. This computation indicates that it would take a slightly greater amount of vapor to produce an absorption noted in the spectrum of a celestial body than the curves of Fig. 7 would show. If the observations are made at the surface of the earth, the difference would be 1 or 2 per cent and about 3 per cent if made at Mount Wilson.

Before concluding the writer wishes to express his gratitude to Mr. Abbot for his criticisms and suggestions while preparing this matter for publication.

SUMMARY

By laboratory experiments on the transmissibility of radiation through long columns of air containing known amounts of water vapor the dependence of transmission on the water-vapor content has been determined for the infra-red bands Φ and Ψ′. The direct determinations cover quantities of water vapor up to a depth of 0.5 centimeters of precipitable water. Beyond this the determinations have been extended by aid of solar observations made on Mount Wilson. This extension does not require assumptions as to the actual quantities of water vapor in the solar beam, but only as to the relative quantities as fixed by the length of path of the beam. As the purity of the spectrum enters into the results it has been necessary to determine the dependence of transmission on water vapor for different values of combined slit- and bolometer-width. While the experiments have been made only at atmospheric pressure, a computation is given which shows that the results are probably applicable with slight correction to the actual

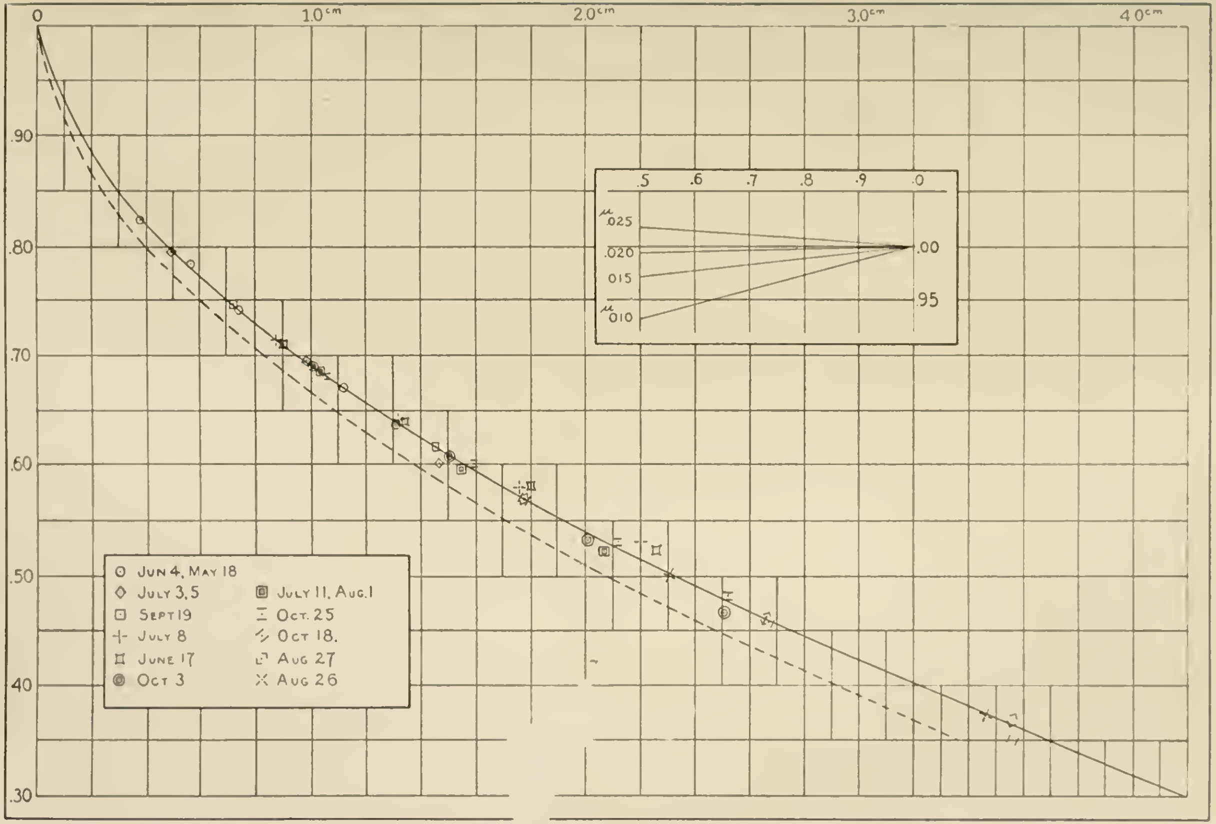

Fig. 7.—Abscissae: precipitable water. Ordinates: log. percentage transmissibility for radiation. Continuous curve for Ψ'; dotted curve for Φ.

Subsidiary plot: abscissae, log. percentage transmission at Ψ'; ordinates, log. percentage correction to Ψ' curve for slit + bolometer = 0.010 μ, 0.015 μ, 0.020 μ, and 0.025 μ.

pressures at which water vapor occurs in the atmosphere. Accordingly, a method has been established by means of which the total quantity of water vapor between the observer and the sun may be easily determined by spectro-bolometric observations. It is proposed in subsequent papers to give applications of the method.

Astrophysical Observatory
Smithsonian Institution
February 1912

THE THREE-PRISM STELLAR SPECTROGRAPH OF THE MOUNT WILSON SOLAR OBSERVATORY[1]

By WALTER S. ADAMS

The original design of the 60-inch reflector provided for the use of three stellar spectrographs in connection with the three principal mirror combinations. The first of these to be completed was the powerful spectrograph of 5.5 m focal length used with the coudé combination of telescope mirrors at the equivalent focus of 45.7 m. This instrument was employed for the investigation of the spectra of some of the brighter stars under high dispersion.[2] In the following year a small low-dispersion spectrograph was constructed for use at the primary focus of the large mirror. On account of the presence of the Newtonian plane mirror it was necessary to mount this spectrograph on the side of the tube of the telescope and to introduce an auxiliary reflection between the slit and the collimating lens. The instrument proved extremely efficient for qualitative work upon the spectra of faint stars, and the radial velocity results obtained with it were so promising as to warrant the construction of an instrument of similar type mounted directly in the axis of the telescope. In this way the loss of light at two reflecting surfaces is avoided and much greater mechanical stability is insured. This spectrograph is now nearing completion in the Observatory instrument shops.

Intermediate between these two spectrographs, one of very high and the other of low dispersion, is the three-prism spectrograph mounted at the lower end of the telescope tube and employed with the Cassegrain combination of mirrors. At this point the equivalent focal length of the telescope is 24.4 m, or the ratio of aperture to focal length is 1 to 16. This is much the same ratio as that of most of the large refracting telescopes used for spectrographic work, and accordingly the dimensions of the optical system in the

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 59.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 50; *Astrophysical Journal*, **33**, 64–71, 1911.

spectrograph are similar to those of some of the larger stellar spectrographs employed in radial velocity determinations. The instrument was built by William Gaertner & Co. of Chicago in accordance with designs provided by the observatory, and has been in regular use on Mount Wilson during the past year.

The very massive character of the telescope mounting and the proximity of the spectrograph to the center of rotation of the telescope made the consideration of its weight of less importance than is usually the case. Accordingly the main frame of the instrument consists of a single heavily ribbed iron casting. The base of the casting is rectangular in shape and about 84×61 cm in size. It is accurately surfaced and is attached directly to a planed flange upon the frame of the telescope by means of a number of strong studs. The slit of the spectrograph is behind the face of the casting and so is well protected from possible injury during the process of changing of instruments. The opening in the telescope frame through which the light passes from the diagonal plane mirror is about 33 cm in diameter, being made sufficiently large to provide for direct photography at this point. The corresponding aperture in the spectrograph casting opposite the slit is about 15 cm square.

At right angles to this base and forming a part of the same casting is the large plate which constitutes the frame of the spectrograph. It is about three-quarters of an inch in thickness and is planed over a large portion of its surface. Fastened directly to this plate are the prisms and the tubes carrying the collimating and camera lenses. The plate is about 140 cm long and tapers slightly from its base toward the prism box.

The focal length of the collimating lens was determined by two considerations: first, the size of the prisms available for use; and second, in a less degree, by the fact that a very long instrument would prevent observations of the region of the sky near the pole on account of striking the floor of the dome. The difficulty of securing optical glass of a quality suitable for prisms of considerable size is very great at the present time, as is fully recognized by most spectroscopists. Fortunately, in the case of this spectrograph, prisms of good quality were known to be available. At the time

at which the five-foot spectroheliograph was designed four prisms 210 mm high and with faces 125 mm wide were ordered from Jena. Three of these proved to be of excellent quality. They are of glass No. O. 102, with an angle of 63° 29′, which provides for a deviation of 180° at $H\gamma$ when three prisms are employed. Since only two prisms are required for the work of the spectroheliograph the third prism became available for the stellar spectrograph. Accordingly it was cut in our optical shop into three prisms, each 67 mm high and with faces the same, of course, as those of the original prism; that is, 125 mm in width. A prism with faces of this size would utilize a beam 59 mm in width at minimum deviation for $H\gamma$. In view of these considerations a diameter of 64 mm (2.5 inches) was fixed upon for the aperture of the collimating lens, which with a ratio of 1 to 16 gives a focal length of 102 cm (40 inches). When more than one prism is employed there is an appreciable loss of light for wave-lengths other than that at minimum deviation on account of the spread of the beam, but as the spectrograph was designed largely for work with one prism it seemed desirable to retain the large aperture. The collimating lens is a cemented triplet corrected for the $H\gamma$ region, made by the J. A. Brashear Co., and has proved very satisfactory in use.

Two camera lenses have been employed with the spectrograph. The longer one of these is an uncemented triplet by Brashear of 88 mm aperture and 102 cm focal length. The other is a Cooke lens of the "Astro-photographic" type with an aperture of 102 mm and a focal length of 46 cm. Both lenses give excellent definition, and the latter, on account of the transparency and the thinness of its components, has proved exceptionally efficient photographically. For all work with a single prism the longer camera has been used, while with two prisms the shorter camera has usually been found sufficient.

An important advantage possessed by the form of construction adopted in the spectrograph is that of adaptability for different regions of the spectrum. The prisms are mounted in cast-iron cells which rest upon three legs. These pass through slotted openings in the main plate of the spectrograph and are clamped rigidly with nuts upon the other side of the plate. The slotted

openings are provided with scales and the prism mounting has upon it an index by which they may be read. The scale-readings for each prism were determined when the three prisms were originally adjusted for minimum deviation at a definite wave-length, and in case it is desired to set the prisms for any other wave-length, changes are made in the scale-readings corresponding to the difference of deviation. Similarly the camera is mounted on a heavy iron plate which swings through an angle corresponding to that of the third prism, and is clamped in position by powerful bolts. This simple arrangement has proved most satisfactory in use.

The great length of the camera regularly employed with the single-prism arrangement and the difficulty of supporting it with sufficient rigidity led to the interposition of a mirror between the prism and the camera lens. In this way the camera may be left in the same position as that used when three prisms are employed. The objections to this proceeding are: first, the loss of light by reflection at the mirror; and, second, the difficulty of supporting the mirror with sufficient stability. The first objection is not very serious, since the mirror is entirely inclosed, and the silver coat deteriorates very slowly and may be kept in excellent condition. Under these conditions the reflecting power for the region of the spectrum usually photographed is not far from 90 per cent. The second objection is met by making the mirror exceptionally thick and holding it in a strong cell in much the same way as a diffraction grating is supported. The photographs obtained with the spectrograph have shown no impairment as regards definition or accuracy of results since the mirror was employed. When two prisms are used the mirror cell is moved toward the camera lens along a slide provided with a graduated scale and is clamped in position.

For comparison spectrum purposes the iron arc is used. The arc lamp is fastened to the outside of the spectrograph case. The light passes through a mica window and falls upon a lens which renders it roughly parallel. This throws it upon a piece of opal glass which thoroughly diffuses it, and an image of this glass is thrown upon the slit by a second lens. The glass, accordingly, serves as the effective source of illumination for the spectrograph. A totally reflecting prism, which is moved in front of the slit by a

handle on the outside of the spectrograph, serves to reflect the light into the instrument. An occulting screen with small openings through which the light from the star or from the arc may fall upon the slit also is controlled by a rod projecting from the side of the spectrograph.

The entire spectrograph is inclosed in a wooden case, the walls around the prism-box being of double construction. For purposes of automatic temperature control the convenient device first employed by Professor Campbell has been adopted, a pair of Draper thermostat strips with platinum contacts acting through a relay to throw the heating current on and off. The heating coils are placed outside of the first wall of the prism-box and are distributed as symmetrically as possible about it. A small fan placed inside of the outer cover serves to distribute the air around the outside of the prism-box and prevents stratification. When the instrument is under temperature control, readings of a thermometer placed inside the prism-box indicate ranges of temperature rarely greater than $0^\circ.1$ to $0^\circ.2$ C. throughout an entire night.

For guiding purposes we have used the customary device of a reflecting slit, the jaws being inclined at an angle of about $2^\circ.5$ to the normal. The reflected light is first collimated by a small lens and then reflected by diagonal prisms through a long tube to a point beneath the spectrograph where it is observed through a telescope. The arrangement is very similar to that employed by Professor Frost on the Bruce spectrograph. A finder of 4 m focal length attached to the tube of the large reflector is used to bring the star within the field of view. The observer keeps the star upon the slit by means of slow-motion motors controlled by switch buttons arranged upon a fiber bar held in his hand. Similar buttons control a motor at the upper end of the telescope tube which moves the convex mirror inward or outward and thus enables the observer to correct for changes of focus during the night. If the large mirror has been protected throughout the day by the canopy, as is regularly the case, these changes are rarely of large amount unless there is a marked change of temperature during the night.

This brief description of the general features of the spectrograph

is perhaps sufficient to give a satisfactory conception of the instrument as a whole. Plate X shows the spectrograph attached to the 60-inch reflector. A part of the outer cover has been removed as well as both covers to the prism-box, so that the arrangement of the prisms and of the camera and collimating tubes is well shown. The tube through which the light reflected from the slit is observed is seen beneath the spectrograph in the lower right-hand corner of the photograph.

PROGRAM OF WORK AND EXPOSURE TIMES

A large part of the time of the spectrograph during the year it has been in operation has been devoted to a determination of the radial velocities of some selected lists of stars, mainly of types A and B, whose proper motions have been measured by Boss and whose velocities are of especial importance in studies of star streams. The lists have been prepared by Professor Kapteyn and consist of stars which for the most part lie between magnitudes 5.5 and 6.5,[1] although a few are brighter. The experience of numerous observers of stellar spectra has shown that by far the greater number of stars with spectra of types A or B can be studied to better advantage with moderate dispersion and linear scale than with high dispersion, on account of the diffuse and broad character of their lines. Some experiments with our spectrograph showed that an optical system consisting of two prisms and the 46 cm camera, or a single prism and the 102 cm camera, gave the most satisfactory results for the majority of the stars. Both combinations have been used and as between the two it is largely a question of the type of spectrum and conditions of seeing. For stars with numerous lines in their spectra the greater resolving power of the two-prism arrangement is preferable. For stars with few lines, however, the single prism is adequate, and the greater width of spectrum obtained with the long camera is an important advantage. In fact, under good conditions of seeing the time occupied in running the star's image along the slit sufficiently to obtain a measurable width of spectrum

[1] The magnitudes used in this article are those given in the "Preliminary General Catalogue of 6188 Stars for the Epoch 1900," by Lewis Boss, *Carnegie Institution of Washington Publication* No. 115. A comparison of these magnitudes with those of the Potsdam and the Harvard Photometry is given in the "Catalogue."

is a considerable drawback to the use of the short camera. The linear scale at *Hγ* of the photographs obtained with the two arrangements is as follows:

Two prisms and short camera, 1 mm = 18.0 Ångströms
One prism and long camera, 1 mm = 15.7 Ångströms

The three prisms and long camera as used for the spectra of the brighter stars give a linear scale at *Hγ* of 1 mm = 5.2 Ångströms.

The exposure times with the spectrograph vary widely, of course, with the conditions of seeing. Under good conditions, when the silver surfaces of the telescope are bright, a fully timed negative of a star of type A or B of magnitude 6.0 on Boss's system may be obtained in one hour, when one prism and the long camera are employed. The exposure times with two prisms and the shorter camera are slightly less. Under average conditions the exposure times are somewhat longer, and under the poorest conditions of the winter season may be several times as long. Usually an exposure about one-fourth longer is given to stars of types F, G, K, and M than to stars of types A and B. The difference would be greater but for the fact that the density of negative required for satisfactory measurement is less in the case of spectra containing numerous lines than for those of types A and B. Under very good conditions of seeing, a fully timed negative of *Groombridge 1830* (Mag. 6.5, Spectrum G) has been obtained in 75 minutes with a slit-width of 0.050 mm. A narrower slit has been employed upon some nights of exceptionally fine seeing, but this width has been used for a majority of the photographs.

METHOD OF REDUCTION

The range of spectrum in good focus upon the negatives and upon which measures may be made extends from about λ 4250 to λ 4900. As a rule in the case of stars of types A and B the measures are limited to the portion between *Hγ* and *Hβ*. Within this region fall several of the most important helium lines, the magnesium line λ 4481, and a large number of enhanced metallic lines whose appearance is so characteristic of a portion of the A-type stars. The fact that *Hγ* and *Hβ* are almost without exception measurable lines in the spectra of A- and B-type stars has led us to the use of auxiliary

tables which have enabled us to save much time in the reduction of the photographs.

The method employed is that used by Professor Frost and several other observers, in accordance with which each negative is reduced independently from measures of three standard lines in the comparison spectrum. It has the marked advantage of requiring no adjustment of the measures on account of changes of focus of the camera or collimator or variation in the scale of the spectrum due to the different temperatures of the prism-train. The chief objection to it is the amount of time required to compute the constants of the Cornu-Hartmann formula for each plate. For the reduction of our spectrograms we have constructed tables giving the values of these constants for the entire range of variation of scale which may occur. This is accomplished in the following way. Two suitable comparison lines are selected at the extremities of the region measured and another line intermediate between them, and these lines are measured upon all of the photographs. The lines selected for our purposes are λ 4337.216 near $H\gamma$, λ 4859.928 near $H\beta$, and the intermediate line λ 4531.327. Let us indicate these lines by λ'', λ, and λ', and the corresponding readings of the comparator by S'', $S+\delta$, and S', as follows:

4859.928 ...... λ	$S+\delta$
4531.327 ...... λ'	S'
4337.216 ...... λ''	S''

The solution of the Hartmann formula gives for the values of the constants:

$$\lambda_0=\lambda-\frac{a(\lambda-\lambda'')-b(\lambda-\lambda')}{a-b+\delta(\lambda'-\lambda'')}\ ,$$

$$S_0=\frac{aS'-bS''-\delta S'(\lambda-\lambda')+\delta S''(\lambda-\lambda'')}{a-b+\delta(\lambda'-\lambda'')}\ ,$$

$$C=(\lambda''-\lambda_0)(S''-S_0),$$

where $a=(\lambda-\lambda')\ (S''-S)$ and $b=(\lambda-\lambda'')\ (S'-S)$.

If we develop the values λ_0 and S_0 into series we obtain:

$$\lambda_0=\lambda-\alpha(1-\beta\delta+\beta^2\delta^2-\ .\ .\ .\ .)$$

$$S_0=c-d\delta+d^2\delta^2-\ .\ .\ .\ .$$

in which α, β, c, and d are constants. Both series are rapidly convergent for small values of δ. It is, of course, a simple matter to

adjust all of the spectrograms under the comparator in such a way that the reading upon λ 4337 shall always be the same, or S'' constant. Then for any value of $S'-S''$ we may obtain the values of λ_0 and S_0 corresponding to a set of readings $S+\delta$ from the series given above. Since δ never exceeds 0.025 mm for our photographs the term δ^3 is negligible, and if extreme values of λ_0 and S_0 are known, all intermediate values may be obtained by simple interpolation in which the second differences are constant. To illustrate the construction of a page of the tables we may consider a specific case. Let the readings be:

4859.928	$56.500+\delta$
4531.327	41.317
4337.216	30.000

A range in δ of 0.025 will readily take care of all the differences which may arise in the reading upon λ 4859 for a given reading upon λ 4531. Accordingly three Hartmann formulae are solved for readings of 56.500, 56.512, and 56.525. With these values of λ_0, S_0, and C we derive the values of the first and second differences for purposes of interpolation, and are enabled to compute rapidly the values of λ_0, S_0, and C corresponding to 56.501, 56.502, etc. This page of the table corresponds to the argument $S'-S''=$ 11.317. For the value $S'-S''=11.318$ a second page is computed, and the process is repeated throughout the range of scale which may occur. The measurement of a few photographs taken at different temperatures gives sufficient knowledge of the mean values about which the table should be constructed. In the case of our own photographs a table consisting of forty pages has been found sufficient to care for the entire range of variation of scale observed, and a table of this size was completed by a single computer with the aid of a calculating machine in about six days.

The constants of reduction for a given spectrogram being obtained by inspection from this table, the wave-lengths of the stellar and comparison lines are computed in the usual way from the constants, and the stellar wave-lengths are corrected according to the deviations of the comparison lines. The complete reduction of a spectrogram containing fifteen stellar and comparison lines occupies about twenty minutes.

TABLE I

STARS OF VARIABLE RADIAL VELOCITY

Boss Number Designation R.A. (1910) Dec. (1910)	Mag. and Type	No. of Plate	Observer	Date	G. M. T.	Radial Velocity	Measured by	Quality	Remarks
					h m	km			
Boss 67	5.6	29	A.	1910, Dec. 20	16 17	+ 5	L.	Fair	
Groom. 58	B 5	36	P.	1910, Dec. 21	15 32	− 35	L.A.		
$0^h 10^m 4$		161	A.P.	1911, Jan. 21	16 18	0	L.A.		
$+51°31'$		944	K.	1911, Dec. 11	15 19	+ 2	L.E.		
		988	A.	1912, Jan. 7	15 34	− 4	L.		
Boss 68	5.6	25	A.	1910, Dec. 18	16 14	− 18	L.	Poor	
12 Cassiop.	B 9	37	P.	1910, Dec. 21	16 42	− 49	L.A.		
$0^h 19^m 8$		160	A.	1911, Jan. 21	15 34	− 12	L.A.		
$+61°20'$		934	K.	1911, Dec. 9	17 10	+ 5	L.E.		
		1009	A.	1912, Jan. 11	15 24	− 19	L.		
Boss 91	5.4	624	P.	1911, Aug. 13	22 15	− 3	L.E.	Fair	
49 G Ceti	A 2	859	K.	1911, Oct. 13	19 42	+ 15	L.E.A.		
$0^h 25^m 9$		915	K.	1911, Nov. 5	16 48	+ 32	L.E.A.		
$-24°17'$									
Boss 159	5.5	30	A.	1910, Dec. 20	17 08	+ 18	L.	Fair	
23 Cassiop.	B 9	38	P.	1910, Dec. 21	17 40	− 30	L.A.		
$0^h 41^m 7$		135	A.	1911, Jan. 18	15 22	− 16	L.A.		
$+74°21'$		874	A.	1911, Oct. 30	20 05	+ 3	L.E.K.		
		935	K.	1911, Dec. 9	18 16	+ 5	L.E.		
		981	K.	1912, Jan. 5	15 55	+ 11	L.E.		
Boss 412	5.7	138	B.	1911, Jan. 18	17 37	+ 77	L.A.	Poor	
1 Persei	B 3	876	A.	1911, Oct. 30	22 05	− 12	L.A.		
$1^h 46^m 1$		945	K.	1911, Dec. 11	16 20	− 6	L.E.		
$+54°42'$		989	A.	1912, Jan. 7	17 40	− 23	L.E.		

Boss 425	5.2	20	A.	1910, Dec. 17	15 30	+ 12	L.A.	Good	
ω Cassiop.	B 9	139	B.	1911, Jan. 18	18 42	− 46	L.A.		
1^h40^m0		148	A.	1911, Jan. 19	18 25	− 44	L.A.		
$+68°15'$		995	A.	1912, Jan. 8	17 00	− 41	L.E.		
Boss 522	5.4	710	K.	1911, Sep. 9	21 37	− 40	L.E.	Fair	Probably composite spectrum
62 Andromedae	A	766	K.	1911, Sep. 17	21 18	− 21	L.E.		
2^h13^m5		801	A.	1911, Oct. 6	21 45	− 22	L.E..A.K.		
$+46°58'$									
Boss 641	5.5	70	P.	1910, Dec. 24	16 38	− 9	L.A.	Good	
π Arietis	B 5	146	A.	1911, Jan. 19	16 48	− 12	L.A.		
2^h44^m2		209	A.	1911, Feb. 10	15 51	+ 32	L.E.		
$+17°5'$		951	K.	1911, Dec. 12	15 50	+ 7	L.E.		
Boss 731	5.7	711	K.	1911, Sep. 9	22 39	− 4	L.E.	Good	Probably composite spectrum
Piazzi 9	A	767	K.	1911, Sep. 17	22 19	− 8	L.E.		
3^h9^m8		820	A.	1911, Oct. 8	21 46	− 17	L.E..A.		
$+30°13'$		860	K.	1911, Oct. 13	21 30	− 4	L.E..A.		
		959	K.	1911, Dec. 14	18 00	+ 2	L.E.		
Boss 740	5.5	12	A.	1910, Dec. 16	17 02	+ 23	L.A.	Poor	
30 Persei	B 5	32	A.	1910, Dec. 20	18 42	+ 12	L.A.		
3^h11^m7		40	P.	1910, Dec. 21	19 28	− 6	L.A.		
$+43°42'$		77	P.	1911, Jan. 7	15 15	− 14	L.A.		
		87	P.	1911, Jan. 8	14 36	+ 11	L.A.		
Boss 796	6.1	33	A.	1910, Dec. 20	19 40	+ 24	L.A.	Poor	
Brad. 480	B 8	41	P.	1910, Dec. 21	20 30	+ 2	L.A.		
3^h24^m2		97	P.	1911, Jan. 11	17 43	− 1	L.A.		
$+47°48'$		107	P.	1911, Jan. 12	18 01	+ 13	L.A.		
Boss 841	5.7	51	P.	1910, Dec. 22	18 12	− 59	L.A.	Poor	
13 Tauri	B 8	177	A.	1911, Feb. 7	16 12	+ 9	L.W.		
3^h37^m1		997	A.	1912, Jan. 8	19 05	+ 8	L.E.		
$+19°25'$									

TABLE I—*Continued*

Boss Number Designation R.A. (1910) Dec. (1910)	Mag. and Type	No. of Plate	Observer	Date	G. M. T.	Radial Velocity	Measured by	Quality	Remarks
					h m	km			
Boss 857	5.4	62	P.	1910, Dec. 23	17 28	+ 61	L.A.	Poor	
24 Eridani	B 8	110	P.	1911, Jan. 12	18 44	+ 75	L.A.		
$3^h 39^m.9$		127	B.	1911, Jan. 17	16 24	+ 41	L.A.		
$-1°29'$									
Boss 878	5.3	724	A.	1911, Sep. 11	22 40	− 19	L.E.	Good	
42 Persei	A 2	790	A.	1911, Oct. 5	23 14	+ 13	L.E.A.		
$3^h 43^m.8$		891	A.	1911, Nov. 1	21 25	− 7	L.		
$+32°49'$		967	K.	1911, Dec. 30	18 42	− 40	L.E.K.		
Boss 992	5.6	35	A.	1910, Dec. 20	21 59	− 35	L.A.	Poor	
Groom. 809	B 3	42	P.	1910, Dec. 21	21 46	+ 3	L.A.		
$4^h 13^m.4$		180	A.	1911, Feb. 7	18 41	− 42	L.W.A.		
$+50°42'$		271	A.	1911, Mar. 15	15 29	− 7	L.A.		
		1007	A.	1912, Jan. 9	19 35	− 40	L.E.		
Boss 1076	4.4	826	K.	1911, Oct. 9	21 48	− 43	L.E.A.	Good	
88 Tauri	A 2	904	K.	1911, Nov. 3	22 31	− 40	L.E.A.		
$4^h 30^m.7$		984	K.	1912, Jan. 5	19 54	+ 71	L.E.		
$+9°59'$									
Boss 1249	6.0	211	A.	1911, Feb. 11	18 06	+ 61	L.	Good	Two spectra present, B 5 and probably A 3. Measures on stronger spectrum B 5
Pulk. 801	B 5 p	272	A.	1911, Mar. 15	16 30	+ 64	L.A.W.		
$5^h 10^m.4$		281	A.	1911, Mar. 16	16 26	+ 53	L.A.		
$+34°13'$		309	P.	1911, Mar. 24	16 20	+ 63	L.W.		
		936	K.	1911, Dec. 9	20 38	+ 53	L.E.		
		998	A.	1912, Jan. 8	20 13	+ 64	L.E.		
Boss 1349	5.6	64	P.	1910, Dec. 23	19 18	+134	L.A.	Fair	
Green. 412 (1860)	B 2	112	P.	1911, Jan. 12	21 28	−138	L.A.		
$5^h 28^m.9$									
$-1°13'$									

Boss 1607	5.7	877	A.	1911, Oct. 30	23 02	− 55.8	L.E.A.	Good	
Groom. 1149	A 5	910	K.	1911, Nov. 4	22 08	+ 21.2	L.E.A.		
$6^h 18^m.8$		949	K.	1911, Dec. 11	23 28	− 75.8	L.E.		
$+56°20'$									
Boss 1906	5.9	68	P.	1910, Dec. 23	23 10	− 80	L.A.	Fair	Two spectra present
Brad. 1050	B 8	103	P.	1911, Jan. 11	23 40	+ 90	L.A.		
$7^h 15^m.5$		132	B.	1911, Jan. 17	21 58	− 6	L.A.		
$+55°29'$									
Boss 2400	5.3	930	K.	1911, Dec. 8	23 11	− 46	L.E.	Good	
62 Cancri	A 3	969	K.	1911, Dec. 30	23 08	− 35	L.E.K.		
$8^h 52^m.2$		1025	K.	1912, Jan. 24	21 58	− 13	L.E.A.		
$+15°40'$									
Boss 2407	4.4	923	K.	1911, Dec. 7	1 08	− 54	L.E.	Good	
a Cancri	A 3	931	K.	1911, Dec. 8	23 56	− 59	L.E.		
$8^h 53^m.6$		1026	K.	1912, Jan. 24	22 50	− 21	L.E.A.		
$+12°12'$									
Boss 2451	5.6	191	A.	1911, Feb. 8	19 24	+ 14	L.E.A.	Poor	
19 Hydrae	B 8	202	A.	1911, Feb. 9	19 38	+ 18	L.W.A.		
$9^h 4^m.3$		256	A.	1911, Mar. 11	18 16	+ 46	L.A.		
$-8°14'$		392	A.	1911, May 9	16 10	+ 4	L.A.		
Boss 2866	5.4	75	P.	1910, Dec. 25	0 03	− 2	L.A.	Poor	
42 Leo. Min.	B 9	104	P.	1911, Jan. 12	1 20	+ 40	L.A.		
$10^h 40^m.9$		157	A.	1911, Jan. 20	0 40	− 9	L.A.		
$+31°9'$									
Boss 3123	5.7	950	K.	1911, Dec. 21	0 32	− 90	L.E.	Fair	
95 Leonis	A 2	1027	K.	1912, Jan. 25	0 01	+ 7	L.E.A.		
$11^h 51^m.0$									
$+16°9'$									

TABLE I—*Continued*

Boss Number Designation R.A. (1910) Dec. (1910)	Mag. and Type	No. of Plate	Observer	Date	G. M. T.	Radial Velocity	Measured by	Quality	Remarks
					h m	km			
Boss 3138	5.4	183	A.	1911, Feb. 7	22 34	+ 16	L.W.	Good	
31 Crateris	B 3	258	A.	1911, Mar. 11	20 26	−116	L.A.		
$11^h56^m.2$		311	P.	1911, Mar. 24	20 15	+ 85	L.A.		
$-19°9'$		348	A.	1911, Apr. 12	18 56	− 21	L.E.		
Boss 3546	6.4	302	B.	1911, Mar. 19	22 22	+ 11	L.A.	Poor	
85 Virginis	B 9	312	P.	1911, Mar. 24	22 00	− 93	L.A.		
$13^h40^m.7$									
$-15°19'$									
Boss 3915	5.6	363	A.	1911, Apr. 14	21 30	+ 16	L.E.A.	Poor	
50 Boötis	B 9	509	A.	1911, July 6	18 38	− 3	L.E.A.		
$15^h18^m.2$		539	B.	1911, July 11	16 41	− 41	L.E.A.		
$+33°15'$									
Boss 3944	5.7	303	P.	1911, Mar. 19	23 55	− 23	L.E.A.	Poor	Possibly composite spectrum
35 Librae	B 3	314	P.	1911, Mar. 24	23 39	− 19	L.A.		
$15^h27^m.8$		489	A.	1911, June 17	15 57	+ 30	L.A.E.		
$-16°33'$									
Boss 3993	5.3	232	B.	1911, Feb. 17	0 30	+ 3	L.W.	Good	
χ Serpentis	A 3 p	333	P.	1911, Apr. 9	20 47	− 4	L.		
$15^h37^m.6$		478	A.	1911, June 15	16 10	+ 36	L.A.		
$+13°8'$									
Boss 4097	5.8	359	A.	1911, Apr. 13	22 22	− 13	L.	Poor	
11 Scorpii	B 9	496	A.	1911, July 4	17 45	− 30	L.E.A.		
$16^h2^m.6$		527	A.	1911, July 9	16 40	− 6	L.E.		
$-12°30'$		533	A.	1911, July 10	17 19	− 50	L.E.A.		

Boss 4353	6.2	442	B.	1911, June 10	21 26	− 14	L.A.	Fair	
Piazzi 303	A	464	P.	1911, June 11	19 48	− 21	L.E.A.		
17^h3^m6		497	A.	1911, July 4	19 07	− 39	L.A.		
$-0°58'$		540	B.	1911, July 11	17 56	− 8	L.E.A.		
Boss 4402	5.5	674	B.	1911, Sep. 6	60 00	− 18	L.E.	Fair	
70 Herculis	A	726	K.	1911, Sep. 12	15 22	− 26	L.E.A.		
17^h17^m2		812	A.	1911, Oct. 8	15 10	− 3	L.E.A.		
$+24°35'$									
Boss 4643	5.7	660	B.	1911, Sep. 4	16 07	− 94	L.E.A.	Good	
108 Herculis	A 3	714	K.	1911, Sep. 10	15 35	− 62	L.E.		
18^h17^m5		813	A.	1911, Oct. 8	16 09	− 28	L.E.A.		
$+29°49'$		822	K.	1911, Oct. 9	15 25	+ 38	L.E.A.		
Boss 4867	5.7	667	B.	1911, Sep. 5	16 59	− 42	L.E.A.	Poor	Probably composite spectrum
Piazzi 318	A 3	728	K.	1911, Sep. 12	16 36	− 6	L.E.A.		
19^h3^m0		906	K.	1911, Nov. 4	15 00	− 24	L.E.K.		
$+28°29'$									
Boss 4947	6.2	591	B.	1911, Aug. 9	18 21	+ 61	L.E.	Good	
2 Sagittae	A 3	879	A.	1911, Oct. 31	15 46	+ 18	L.E.A.		
19^h20^m3									
$+16°46'$									
Boss 5042	6.4	629	P.	1911, Aug. 14	19 18	− 27	L.E.A.	Good	
ψ Aquilae	A p	720	K.	1911, Sep. 11	17 29	− 2	L.E.A.		
19^h40^m4		763	K.	1911, Sep. 17	17 20	+ 16	L.E.		
$+13°5'$									
Boss 5113	5.6	518	A.	1911, July 7	21 44	− 65	L.E.A.	Poor	
Groom. 2984	B 3	886	A.	1911, Nov. 1	15 32	+ 23	L.E.A.		
19^h54^m1									
$+40°8'$									

TABLE I—*Continued*

Boss Number Designation R.A. (1910) Dec. (1910)	Mag. and Type	No. of Plate	Observer	Date	G. M. T.	Radial Velocity	Measured by	Quality	Remarks
					h m	km			
Boss 5178	6.0	519	A.	1911, July 7	22 44	− 16	L.E.A.	Poor	$H\beta$ doubly reversed
20 Vulpeculae	B 8 p	536	A.	1911, July 10	20 48	− 32	L.E.A.		
$20^h 8^m 2$		552	B.	1911, July 13	20 36	− 37	L.E.A.		
$+26°13'$		796	A.	1911, Oct. 6	17 50	− 15	L.A.K.		
Boss 5211	5.8	691	B.	1911, Sep. 7	18 58	+ 2	L.E.A.	Fair	
36 Cygni	A	738	K.	1911, Sep. 14	18 13	− 27	L.E.A.		
$20^h 15^m 1$		823	K.	1911, Oct. 9	17 03	− 10	L.E.A.		
$+30°43'$									
Boss 5292	5.5	706	K.	1911, Sep. 9	17 51	+ 45	L.E.A.	Good	
ι Delphini	A 2	770	K.	1911, Sep. 18	18 06	+ 18	L.E.		
$20^h 33^m 5$		894	A.	1911, Nov. 2	15 08	+ 24	L.E.A.		
$+11°4'$									
Boss 5322	5.8	692	B.	1911, Sep. 7	20 12	− 40	L.E.	Good	
Groom. 3258	B 9	755	K.	1911, Sep. 16	17 59	− 25	L.E.		
$20^h 38^m 7$		914	K.	1911, Nov. 5	15 34	− 16	L.E.A.		
$+41°24'$									
Boss 5573	6.2	593	B.	1911, Aug. 9	21 25	+ 33	L.E.	Fair	
76 Cygni	B 9	640	P.	1911, Aug. 15	21 48	− 7	L.E.		
$21^h 38^m 0$									
$+40°18'$									
Boss 5581	6.2	588	B.	1911, Aug. 8	20 40	+ 23	L.E.	Poor	
45 Capricorni	A 2	621	P.	1911, Aug. 13	19 40	+ 45	L.E.		
$21^h 39^m 1$		654	P.	1911, Aug. 17	19 34	− 3	L.E.A.		
$-15°10'$									

Boss 5692	5.9	852	K.	1911, Oct. 12	18 23	− 55	L.E.A.	Fair	*Hβ* bright
25 Pegasi	B 5 p	887	A.	1911, Nov. 1	16 48	− 59	L.E.A.		
$22^{h}3^{m}.6$		954	K.	1911, Dec. 13	15 32	− 36	L.E.		
$+21°16'$									
Boss 5738	6.4	601	B.	1911, Aug. 10	22 08	− 15	L.E.A.	Poor	
Br. 2938	B 8	858	K.	1911, Oct. 13	17 36	+ 28	L.E.A.		
$22^{h}11^{m}.0$									
$+62°43'$									
Boss 5833	5.9	608	B.	1911, Aug. 11	22 16	− 39	L.E.A.	Poor	*Hβ* unsymmetrically reversed
8 Lacertae	Oe p	632	P.	1911, Aug. 14	22 22	+ 7	L.E.		
$22^{h}31^{m}.9$		807	A.	1911, Oct. 7	19 44	+ 4	L.A.K.		
$+39°10'$									
Boss 5848	5.3	732	K.	1911, Sep. 12	21 42	+ 40	L.E.A.	Poor	
30 Cephei	B 9	784	A.	1911, Oct. 5	18 24	− 5	L.E.A.K.		
$22^{h}35^{m}.5$		866	K.	1911, Oct. 14	17 30	+ 16	L.E.A.		
$+63°7'$									
Boss 5856	5.4	648	P.	1911, Aug. 16	23 00	+ 3	L.E.	Good	
12 Lacertae	B 2	702	K.	1911, Sep. 8	21 49	+ 6	L.E.A.		
$22^{h}37^{m}.5$		756	K.	1911, Sep. 16	19 03	− 8	L.E.		
$+39°45'$		786	A.	1911, Oct. 11	19 34	− 43	L.E.		
Boss 6075	5.4	602	B.	1911, Aug. 10	23 28	+ 37	L.E.A.	Poor	
18 Andromedae	B 8	633	P.	1911, Aug. 14	23 30	− 3	L.E.		
$23^{h}34^{m}.8$		659	P.	1911, Aug. 18	0 07	− 1	L.E.A.		
$+49°58'$		799	A.	1911, Oct. 6	19 56	+ 1	L.E.A.		
Boss 6142	6.2	888	A.	1911, Nov. 1	18 12	− 53	L.E.A.	Fair	Spectrum probably composite
Br. 3184	Oe p	987	A.	1912, Jan. 6	15 54	− 11	L.E.		
$23^{h}51^{m}.0$		1003	A.	1912, Jan. 9	15 12	−133	L.		
$+56°55'$									

SOME RESULTS

A list of fifty spectroscopic binaries.—We have found the above 50 stars mainly of types A and B to have variable velocities in the line of sight. The initial given in the column headed "Observer" refers to Messrs. Adams, Babcock, Kohlschütter, and Pease, and in the column "Measured by" to Miss Lasby, Miss Ensign, Miss Waterman, and Messrs. Kohlschütter and Adams. The type of spectrum given is in most cases from our own observations. The column in the table preceding "Remarks" indicates roughly the general character of the spectrum for purposes of measurement. There is evidence of complexity of the hydrogen lines in the spectra of many of these stars, and no doubt more would be found were the density of the negatives made somewhat less. As a rule, however, considerable density of the continuous spectrum aids in the measurement of the broad hazy lines characteristic of the spectra of most of these stars.

In addition to the stars given in Table I we have secured observations which agree in confirming the variability of velocity of the following stars announced from other observatories:

Name	R. A. 1910	Dec. 1910	Mag.	Observatory
25 Serpentis	15^h41^m4	$-1°31'$	5.6	Yerkes
χ Ophiuchi	16 21.8	$-18°14'$	4.8	Lick
ζ' Lyrae	18 41.7	$+37°31'$	4.4	Lick
δ¹ Lyrae	18 50.6	$+36°52'$	5.7	Yerkes
θ Aquilae	20 6.7	$-1°5'$	3.2	Meudon
6 Lacertae	22 26.6	$+42°40'$	4.5	Yerkes

Stars with bright hydrogen lines.—The stars *20 Vulpeculae*, *25 Pegasi*, and *8 Lacertae* in Table I have one or more hydrogen lines bright. In *χ Ophiuchi*, as has been announced by Professor Campbell, both $H\gamma$ and $H\beta$ are bright. The following stars also have bright hydrogen lines:

Name	R. A. 1910	Dec. 1910	Mag.	Bright Lines
11 Camelopardalis	4^h58^m3	$+58°51'$	5.3	$H\gamma$ and $H\beta$
165 G Canis Majoris	7 20.6	$-16°1'$	5.3	$H\gamma$ and $H\beta$
25 Vulpeculae	20 18.2	$+24°10'$	5.7	$H\beta$

TABLE II

STARS WITH GREAT RADIAL VELOCITIES

Designation R.A. (1910) Dec. (1910)	Mag. Spectrum	No. of Plate	Observer	Date	Radial Velocity	Mean Radial Velocity	Velocity in Space	Angle to Line of Sight
					km	km	km	
Lal. 4855	7.2	β 74	A.	1910, Jan. 18	−120	−120	186	130°
$2^h 33^m.3$	G							
$+30° 28'$								
Lal. 5761	8.0	β 75	A.	1910, Jan. 18	−156			
$3^h 3^m.1$	F	β 91	A.	1910, Feb. 18	−153			
$+26° 0'$		β 104	A.	1910, Feb. 21	−150	−153	188	144°
Groom. 864	7.3	β 81	A.	1910, Jan. 21	+103			
$4^h 35^m.2$	G	β 86	A.	1910, Feb. 17	+102			
$+41° 58'$		β 97	A.	1910, Feb. 20	+101			
		β 105	A.	1910, Feb. 21	+ 97			
		γ 237	P.	1911, Feb. 18	+100.8			
		γ 245	P.	1911, Feb. 21	+103.7	+101	163	52°
Groom. 1830	6.5	γ 194	A.	1911, Feb. 8	− 99.7			
$11^h 47^m.7$	G	γ 247	P.	1911, Feb. 21	− 99.2			
$+38° 23'$		γ 362	A.	1911, Apr. 14	− 96.2			
		γ 399	A.	1911, May 11	− 95.8	− 97.7	343	107°
Lal. 28607	7.3	γ 323	P.	1911, Apr. 7	−166			
$15^h 38^m.2$	A	γ 368	A.	1911, Apr. 15	−168			
$-10° 39'$		γ 440	B.	1911, June 7	−172			
		γ 516	A.	1911, July 7	−175	−170	173	128°
31 b Aquilae	5.2	γ 335	P.	1911, Apr. 9	− 95.9			
$19^h 20^m.7$	G	γ 377	A.	1911, Apr. 16	− 97.0	− 96.4	119	144°
$+11° 45'$								
Lal. 37120–1	6.6	β 227	A.	1910, May 20	−162			
$19^h 30^m.1$	G	β 235	A.	1910, May 21	−163			
$33° 0'$		γ 364	A.	1911, Apr. 14	−161.9			
		γ 409	P.	1911, May 15	−161.8	−162	167	166°

Some stars with great radial velocities.—In the course of our observations of some stars of large proper motions with known parallaxes we have found a few stars with very great radial velocities. Most of these had previously been observed with the small focal plane spectrograph and approximate velocities determined. Accordingly in Table II the spectrograms obtained with the focal plane instrument are indicated by the series letter β and those with the large spectrograph by γ. The values obtained with the small spectrograph are of course subject to considerable uncertainty.

With the exception of *Groombridge 1830*, for which Professor Campbell has published a value of -95 km, no other observations are available for these stars. The star *Lalande 28607* is of especial interest because of its type of spectrum. No star of type A with a constant velocity approaching this in magnitude has been observed heretofore.

Since the parallaxes and the proper motions of these stars are known, a computation of their velocities and directions of motion in space becomes of interest. These are given in the last two columns of Table II, the requisite data being taken from the list of parallax determinations compiled by Kapteyn and Weersma.[1]

I am indebted for much assistance in connection with the results referred to in this communication. In particular I wish to express my appreciation to Mr. Pease for his great aid in the design of the spectrograph, many important features of which are due to his suggestions; to Mr. Babcock and Dr. Kohlschütter for observations with the instrument; and to Miss Lasby, Miss Ensign, and Miss Waterman for the difficult work involved in the measurement of the spectra.

Mount Wilson Solar Observatory
February 1912

[1] *Publications of the Astronomical Laboratory of Groningen*, No. 24.

THE EFFECT OF PRESSURE UPON ELECTRIC FURNACE SPECTRA

SECOND PAPER[1]

By ARTHUR S. KING

In a former paper[2] the writer reported the results of some preliminary experiments on the spectrum of the electric furnace when operated in an atmosphere of compressed gas. The displacements measured were chiefly for lines in two regions of the iron spectrum for a pressure of 9 atmospheres. The measurements, though not extensive enough to be given high weight, showed, when compared with such measurements of arc spectra under pressure as were available, that the lines in general were displaced much more in the furnace than in the arc at equal pressure.

It was obviously desirable to continue this investigation in such a way as to establish in a definitive manner the leading characteristics of the furnace spectrum when under pressure, and it is believed that sufficient material is now on hand for this purpose. About one hundred furnace photographs have been made since the preliminary set, the spectra studied being those of iron for the regions λ 4200 to λ 4500 and λ 5200 to λ 5500, titanium from λ 4250 to λ 4600, and vanadium from λ 4050 to λ 4600. The range of pressures has been up to 24 atmospheres for the two regions of the iron spectrum, up to 16 atmospheres for the spectra of titanium and vanadium. The leading features studied have been as follows:

1. The rate of increase of displacement with pressure and the mean shift per atmosphere for various groups of lines which permitted measurements of considerable accuracy.

2. The pressure effect in absorption as obtained by passing white light through the furnace tube when under pressure.

3. A search for possible variation of displacement with temperature.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 60.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 53; *Astrophysical Journal*, **34**, 37, 1911.

4. The observation of the effects due to large and small quantities of the radiating vapor, to the presence of foreign vapors, and to variation in the length of the column of vapor by the use of long and short tubes, the pressure being the same for these different conditions.

5. A comparison of the relative displacements of groups of lines with their response to temperature excitation and with their behavior in the magnetic field.

6. A consideration in the case of a few iron lines of the variation of displacement with wave-length.

7. Attention is directed to what is apparently a fundamental difference in the structure of arc and furnace lines which may explain the difference in absolute displacement given by the two sources.

EXPERIMENTAL METHOD

The work has been carried on in general according to the method described in the previous paper, with such improvements as experience showed to be feasible. The photographs were made in the second order of the vertical Littrow spectrograph in the Pasadena laboratory, the scale for the regions studied ranging from 0.93 to 0.96 Å per mm. The furnace was operated in vacuum to give the comparison spectrum, which was placed on each side of the pressure photograph by means of the occulting plate above the slit. This exposure with the furnace in vacuum was taken partly before and partly after the pressure photograph, the second exposure being made with increased length of slit on each side, so that the comparison photograph appears as the superposition of the two exposures, with weak extensions due to the second. This gave an exceedingly delicate test for instrumental displacement during the making of the complete photograph, better than is given by a double exposure with slit unchanged in length since the fainter extensions sometimes showed a very slight lack of continuity when the superposed portion of the line did not appear to suffer in sharpness.

The tubes used in the furnace, except for the special tests at the close of the investigation, were of Acheson graphite, 30.5 cm long, 12.5 mm inside diameter, and either 19 or 20.5 mm outside diam-

eter, 20.5 cm being heated between the graphite contact blocks which led in the current. The tube was protected by a split graphite tube and by carborundum powder outside of this.

For the compressed gas, both carbon dioxide and air were used, the latter for most of the work, as it gave better results than carbon dioxide, especially for pressures above 16 atmospheres. The writer was surprised to find that the oxygen furnished by the air seemed to be less than that given off by dissociation of the carbon dioxide when in contact with the hot tube and jacketing materials, as the tubes lasted better in air and less of the white smoke was generated which always proved very disturbing with carbon dioxide on account of weakening the light and clouding the window. A very efficient method of jacketing has been devised, so that there was little circulation of gas about the exterior of the tube and it wore thin slowly. As a rule the tube was renewed in preparation for each photograph which involved a run of the furnace under pressure and two comparisons in vacuum, but occasionally a tube could be used longer at moderate temperatures. In one case the same tube was used for three successive experiments, being heated for a total of $2^h\ 46^m$ in air at 16 atmospheres, besides comparisons in vacuum aggregating $1^h\ 6^m$. The tubes appear to be much more subject to oxidation outside than inside, the oxygen entering the tube apparently passing into combination before it reaches the highly heated portion.

The best results were obtained for temperatures which gave readings of 2200° to 2400° C. when a Wanner pyrometer was directed at the wall of the interior of the tube. The temperature of the radiating vapor is necessarily lower than this by an unknown amount, since the temperature of the graphite wall, obtained either in this way or by the melting-point method, must be higher than that of the inclosed vapor.

The pressure was measured by a new calibrated gauge reading to 500 pounds. Two other gauges, which were used in former work and by Mr. Gale with the pressure arc, showed a close agreement with this gauge except at the beginning of their scales.

When the carborundum jacket was used, there was always a certain amount of vapor, especially at the higher pressures, which

condensed as a white powder on the water-cooled metal parts. This was not tested chemically but was probably an oxide of some constituent of the carborundum. As has been noted, there was more of this vapor when compressed carbon dioxide was employed. Near the end of the investigation, it was desired to try the effect of foreign vapors purposely introduced and also of different lengths of the column of radiating vapor. In both cases this oxide from the jacketing would have complicated the conditions, so the experiments were made without any protection around the tube. Naturally, there was then a more rapid wearing-away of the tube and a very rapid loss of the heat to the water-cooled chamber and electrode pipes, requiring a large increase in the electric energy to keep up the temperature. It was possible, however, to make the desired series of experiments under these conditions for a pressure of 8 atmospheres.

Tubes for which the heated portion was only 51 or 64 mm long were used by setting the two contact blocks at the proper points along the copper-pipe electrodes. Other tubes with a total length of 35.5 cm, 25.5 cm being heated, were also employed.

A new method of studying the pressure effect, for which the furnace is especially adapted, was carried out by obtaining the lines under pressure as absorption lines. The comparison spectrum was obtained as usual, in emission, with the furnace in vacuum before and after the pressure exposure. For the pressure photograph a parallel beam of white light from a projection arc was directed by means of a short-focus lens into the window of the furnace opposite the spectrograph. This light passed through the vapor in the furnace tube, which was radiating under pressure, and thence to the spectrograph. Absorption lines are thus obtained which can be made very narrow if the continuous spectrum is strong. The temperature of the furnace tube may be low if only the stronger lines are desired, and the exposure time is much less than is required for the same lines in emission. The pressure displacements, as will be shown later, are in good agreement with those obtained when the tube is strongly excited so as to give self-reversed lines. The method promises to be very useful as a supplementary one, especially for spectra such as titanium, where a large number of lines are easily

produced by the furnace but do not readily show self-reversal. The edges of the absorption lines are liable to be somewhat ragged as the result of the large variation in temperature from the middle to the two ends of the furnace tube, with attendant differences in density of the vapor. A strong continuous spectrum will remedy this to a large extent. Care must also be taken when examining lines which are also given as sharp emission lines by impurities in the carbons of the arc, since if the pressure displacement be small, the violet side of the absorption line is affected by the presence of the bright line.

In measuring the displacements, a series of four or six settings was made on the comparison line and a like number on the line displaced by pressure. The plate was then reversed in the machine and a similar set taken in the opposite direction. All plates of sufficiently good quality were measured by the writer on a small Gaertner comparator and most of them also by Miss Sheldon.

RESULTS

For the purpose of establishing the main phenomena of the pressure effect for the furnace, a set of lines was selected in the iron, titanium, and vanadium spectra which can be measured with fair precision at various pressures. As a rule (the exceptions usually being noted in the tables) only lines for iron and vanadium are measured which are distinctly reversed. For titanium the lines were measured unreversed and in absorption. The list is thus limited to lines appearing at the lower furnace temperatures, the self-reversal being given by the absorption of the cooler vapor at the ends of the tube. The former paper showed that the number of iron lines which can be measured in the furnace spectrum, especially at pressures less than 10 atmospheres, is quite comparable with the number measurable in the arc, and this is true also for titanium and vanadium, but special conditions must be chosen for various sets of lines, since on any one plate only a relatively small number of lines will yield measurements of high weight.

As there is always a certain amount of widening to pressure lines, which in general increases with the pressure and often involves dissymmetry in the widening, a reversed line is preferable for measure-

ment, provided the reversal is narrow. If there is a tendency for the line to widen toward one side, the reversal may be expected to take part in this, but on account of its relatively small width, its position as measured cannot differ from the true position of the maximum by so great an amount as may easily happen when an unreversed and rather wide line is measured.

It is characteristic of reversed lines in furnace spectra that they do not have clean-cut edges to the reversals. There is a slow gradient in temperature and vapor-density from the center to each end of the tube which results in a gradual shading of the sides of the reversal. For this reason, personal judgment may enter to a considerable degree in making settings on the lines, and the opportunity for this increases if the reversals are not very narrow. Differences of considerable magnitude in the measurements for single lines have occasionally been observed which may fairly be ascribed to this cause. However, all of the conclusions to be drawn from the material in this paper are based on mean displacements for sets of lines which show shifts of the same order of magnitude, and such means do not appear to be greatly affected by personal differences. As a test, the mean displacements were compared for ten good plates measured by Miss Sheldon and myself. These plates embraced the spectra of all three elements and were for various pressures and different types of lines. The differences ranged from 9 per cent for unreversed titanium lines to exact agreement for a plate containing over 20 reversed vanadium lines. Most of the differences were well under 5 per cent and about equally divided as to which observer obtained the higher values. A large proportion of the whole number of plates was of about the quality of those compared, so that it seems highly improbable that the mean values presented are affected in any vital degree by peculiarities in personal judgment.

It was desirable to look farther into the large differences between furnace and arc displacements for the same pressure, which were indicated by the preliminary observations. Instrumental differences, such as must occur for work carried out in different laboratories, have been largely eliminated by comparing the later furnace results with the values for arc displacements obtained by Gale and

Adams[1] in an investigation carried out with pressures which were used also for the furnace and with the same spectrograph. At the close of the pressure-arc investigation, the spectrograph was used by the writer with the same adjustments as to focus, thus insuring that the photographs, as regards scale and definition, should be as closely comparable as possible. The change from the arc arrangements thus consisted in turning the vertical spectrograph around until its mirror faced the furnace. The image of the interior of the tube was focused on the slit, giving a cone of light slightly larger than the objective of the spectrograph. The focusing lens was never moved during the making of a pressure photograph with its two comparison exposures in vacuum.

The pressure values for all of the displacements to be given are total pressures, owing to the comparison spectrum being made with the furnace in vacuum. A careful test having shown (see Table II) that the furnace in vacuum and at atmospheric pressure shows a displacement of the lines proportional to that for higher pressures, the shifts may be compared with those for the same difference of pressure with the arc, which was usually operated at atmospheric pressure for the comparison spectrum.

IRON

In Table I the displacements are given in Ångström units for a number of iron lines in the blue region. These are all reversed by the furnace and were as a rule very favorable for close measurement. The furnace spectra at 8, 16, and 24 atmospheres are the regular reversed emission lines. At 12 atmospheres, one plate in absorption was measured and also one in which the iron lines were narrow and unreversed in a photograph of the titanium spectrum. The last column gives the displacements found by Gale and Adams for the arc with 8 atmospheres difference in pressure. The wavelengths are on the Rowland scale.

It is seen from Table I that the displacements of all the lines, with the exception of those marked *, are of the same order of magnitude for the furnace at any given pressure. The mean shift per atmosphere for these eight lines is also nearly the same except

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 58; *Astrophysical Journal*, **35**, 10, 1912.

for the two plates at 12 atmospheres, where the rather wide absorption lines of the one and the unreversed lines of the other made measurements difficult.

TABLE I

PRESSURE DISPLACEMENTS FOR IRON

λ 4250–λ 4462

λ	8 Atm. 4 Plates	12 Atm. (Absorption) 1 Plate	12 Atm. (Unreversed) 1 Plate	16 Atm. 2 Plates	24 Atm. 1 Plate	8 Atm. Arc
4250.945	0.037	0.062	.....	0.073	0.116	0.022
4271.934	0.039	0.054	0.053	0.074	0.102	0.022
4294.301	0.036	0.064	.....	0.076	0.113	0.030
4308.081	0.040	0.055	0.055	0.081	0.116	0.021
4325.939	0.038	0.065	0.053	0.080	0.121	0.020
*4376.107	0.020	0.028	0.029	0.039	0.052	0.018
4383.720	0.040	0.063	0.056	0.080	0.119	0.027
4404.927	0.041	0.059	0.060	0.078	0.121	0.021
4415.293	0.040	0.064	.....	0.072	0.115	0.018
*4427.482	0.019	0.029	0.028	0.038	0.044	0.017
*4461.818	0.018	0.026	0.026	0.034	0.051	0.015
SUMMARY, OMITTING * LINES						
Mean displacement	0.0389	0.0607	0.0554	0.0768	0.1154	0.0226
Displacement per atmosphere	0.00486	0.00506	0.00462	0.00480	0.00481	0.00282

Mean furnace displacement per atmosphere 0.00483
Ratio, Arc : Furnace = 282 : 483 = 0.584

The lines λλ 4376, 4427, and 4462 are obviously in a different class as regards displacement, their shifts being about half as large as for the other lines in the list. Other lines in this region show a shift about twice as large as the unstarred lines in Table I, λ 4260.640 being the strongest of this class; but they do not reverse in the furnace and are usually so hazy for pressures above 8 atmospheres that measurements on them are of low weight and they are therefore not included in the present list.

A comparison of the furnace and arc shifts may be made most directly by means of the first and last columns of displacements. It is seen that the furnace displacements are uniformly larger than those of the arc, but that the ratio for individual lines is by no means constant. The largest deviations are for λ 4294 and for the three lines above mentioned which have small furnace displace-

ments. The reality of this difference is not altogether certain. These four lines are given at low temperatures and so are relatively strong and well reversed in the furnace spectrum. In the arc they reverse with difficulty, if at all, so that the matter hinges on whether a definitive comparison can be made between a reversed and an unreversed line when the displacement is small and is attended by photographic differences owing to the spectra being photographed on different plates. Duffield,[1] who investigated the iron lines in this region for pressures from 5 to 100 atmospheres, obtained very irregular displacements for the three starred lines of Table I, but they have in general decidedly smaller shifts than the other lines in this table. λ 4294 was also found by Duffield to show variable shifts at different pressures, but usually gave values close to those found by him for the unstarred lines of Table I. The measurements of Humphreys[2] for 41 atmospheres, a pressure which in some respects should be more favorable for lines of this type, give a close relative agreement with the furnace displacements, λ 4294 having a shift of the same order as those measured by Humphreys for the unstarred lines of Table I, while $\lambda\lambda$ 4376, 4427, and 4462 have a mean shift per atmosphere of 0.00122, the mean value given by the furnace for 8, 16, and 24 atmospheres being 0.00225. Thus, if we allow for the character of the lines in the two sources, the lack of agreement in the measured displacements for furnace and arc may not be great enough to indicate a real relative difference.

Plate XI reproduces the iron spectrum from λ 4376 to λ 4462, showing two spectra at 8 atmospheres having different intensities for the continuous ground, an absorption spectrum at 12 atmospheres and emission spectra at 16 and 24 atmospheres. The relatively small displacements of the starred lines of Table I may readily be seen.

A large number of photographs have been taken for the region λ 5300 to λ 5500, as the strong lines in this region are especially favorable for examining the characteristics of furnace displacements. Eight lines occur in this region which are of about equal intensity and show pressure displacements of nearly the same magnitude.

[1] *Philosophical Transactions*, A **208**, 111, 1908.

[2] *Astrophysical Journal*, **26**, 18, 1907.

They reverse readily in the furnace at moderate temperatures, the width of reversal increasing progressively toward the violet. This group of lines has been studied at successive steps of 4 atmospheres up to 24 atmospheres, and also for the possible effect upon displacement of variations in temperature and other conditions of the furnace. The displacement values for various pressures are given in Table II.

Table II shows a maximum variation in the shift per atmosphere of less than 10 per cent for those conditions which may be regarded as standard, namely emission spectra in air at pressures of 4, 8, 16, and 24 atmospheres. The large values given by one plate for 12 atmospheres in absorption are of doubtful weight, owing to the width of the lines. Absorption spectra at 8 and 16 atmospheres with fairly narrow lines gave displacements close to the general mean. The large shifts measured for two plates in carbon dioxide at 8 atmospheres may be real and will be considered later in the discussion. The number and quality of the plates at disposal leave little question that there is a regular increase of displacement with pressure through this range of moderate pressures. The material thus supplements the data from arc investigations, in which as a rule many irregular values have appeared at pressures under 20 atmospheres, and in which the proportionality of displacement to pressure in this range was somewhat doubtful.

The measurements for one atmosphere require explanation, since the probable error is large in proportion to the displacement measured. The furnace was operated in a partial vacuum for the comparison and then at about atmospheric pressure, the interval being regulated by a mercury manometer for which the difference in level was kept equal to the barometric height. Three good plates having narrowly reversed lines were measured in each direction. Single determinations ranged from 0.002 to 0.008, the extreme limits being rare. As the eight lines show displacements of the same magnitude, each plate thus furnished 16 determinations of the interval in question, giving a total of 48 measurements to make up the final mean of 0.00525 for a pressure of one atmosphere. It is believed that fairly high weight can be given this value of the shift for this difference in pressure. It shows that the displace-

TABLE II

PRESSURE DISPLACEMENTS FOR IRON

λ 5328–λ 5456

λ	1 Atm. 3 Plates	4 Atm. 3 Plates	8 Atm. 3 Plates	8 Atm. (CO_2) 2 Plates	8 Atm. (Absorption) 1 Plate	12 Atm. 2 Plates	12 Atm. (Absorption) 1 Plate	16 Atm. 2 Plates	16 Atm. (Absorption) 1 Plate	20 Atm. 2 Plates	24 Atm. 2 Plates	8 Atm. Arc
5328.236..........	0.006	0.021	0.043	0.047	0.044	.....	.....	0.080	.....	0.097	0.124	0.029
5371.734..........	0.006	0.020	0.041	0.045	0.039	0.058	.....	0.079	0.091	0.109	0.121	0.029
5397.344..........	0.005	0.021	0.042	0.047	0.042	0.065	0.068	0.080	0.087	0.107	0.137	0.029
5405.989..........	0.006	0.022	0.041	0.043	0.035	0.059	0.069	0.075	0.080	0.094	0.102	0.027
5429.911..........	0.005	0.021	0.044	0.049	0.044	0.064	0.076	0.083	0.089	0.104	0.126	0.029
5434.740..........	0.004	0.019	0.036	0.042	0.036	0.053	0.070	0.074	0.073	0.093	0.115	0.027
5447.130..........	0.005	0.023	0.047	0.050	0.047	0.063	0.069	0.074	0.081	0.106	0.124	0.031
5455.834..........	0.005	0.020	0.040	0.042	0.039	0.058	0.066	0.071	0.079	0.099	0.106	0.028
Mean displacement	0.00525	0.0210	0.0418	0.0456	0.0407	0.0600	0.0697	0.0770	0.0829	0.101	0.122	0.0286
Displacement per atmosphere.......	0.00525	0.00525	0.00522	0.00570	0.00509	0.00500	0.00581	0.00481	0.00518	0.00505	0.00508	0.00357

Mean furnace displacement per atmosphere, 0.00522

Ratio, Arc: Furnace = 357:522 = 0.684

ments produced by any pressure as referred to the furnace in vacuum may be fairly compared with arc measurements, for which the pressure is usually taken as the excess above one atmosphere.

The arc displacements for 8 atmospheres as measured by Gale and Adams are given in the last column of Table II. There is a close agreement among the individual values as for the furnace. The difference between the mean shifts per atmosphere for the two sources is not so large as for the lines given in Table I, but the different appearance of the lines in arc and furnace for this region makes them more difficult to compare with accuracy. The lines of Table II are usually unreversed in the arc at moderate pressures, only λ 5328 and λ 5371 sometimes showing slight reversal. In the furnace the fact that they are low-temperature lines and are given by the vapor near the ends of the tube permits them to be clearly reversed. Another set of pressure-arc measurements for the lines of Table II is given by Humphreys.[1] His measurements for the eight lines under a pressure of 41 atmospheres gave a mean shift per atmosphere of 0.0023, or less than one-half of the mean furnace displacement. If we take the mean of Gale and Adams' and of Humphreys' values for the shift per atmosphere in the arc, it comes out 0.00293, giving a ratio of arc to furnace displacements of 0.561.

The strong lines $\lambda\lambda$ 5497.735, 5501.683, and 5507.000, are usually unreversed in the furnace, and were not so favorable for accurate measurement as the lines given in Table II. Such measures as have been made indicate that the displacements are of the same order of magnitude as those recorded for the neighboring lines.

Plate XII reproduces the lines of Table II, with the exception of λ 5328. Variations of the pressure effect are shown ranging from the slight lack of continuity at one atmosphere as compared to vacuum to the large displacements at 24 atmospheres. In all of them, the progressively increasing width of the reversal toward the violet is evident. For 8 atmospheres, both emission and absorption spectra are shown. For 16 atmospheres, two emission spectra with quite different degrees of reversal and an absorption spectrum are given. The spectrum reproduced for 24 atmospheres

[1] *Loc. cit.*

shows for most of the lines only the beginning of reversal. Another plate was obtained at this pressure with rather wide reversals.

TITANIUM

The titanium furnace spectrum was photographed for the region λ 4200 to λ 4600, in which two groups aggregating 20 lines were strong enough to measure up to 16 atmospheres. These lines are not reversed by the furnace at the temperatures employed. A good agreement appeared for sets of measurements by different observers, although the lines were not as satisfactory for measurement as if they had been reversed. An absorption spectrum at 8 atmospheres gave displacements agreeing closely with the values for emission spectra. The measurements are presented in Table III, the arc displacements found by Gale and Adams for 8 atmospheres being given in the last column. The mean furnace shift per atmosphere is weighted on account of the small number of values for 12 atmospheres.

Considering the character of the lines there is a good agreement between the displacements at 8 atmospheres for emission and absorption and also between the shifts per atmosphere at 8 and 16 atmospheres. The ratio of mean arc and furnace displacements is almost the same as for the iron lines of Table I. The arc displacements for the 11 lines beginning with 4512.906 are consistently somewhat higher than for the first 9 lines in the list, while the furnace displacements are of the same magnitude throughout.

As in the case of the iron spectrum, only the best lines are measured in this region. A much larger number can be obtained with varying degrees of accuracy, the precision in most cases becoming much less at higher pressures.

VANADIUM

Recent work by Rossi[1] has furnished measurements for the displacements of the stronger vanadium lines from λ 4000 to λ 4600 as given by the arc at pressures of 15, 30, 50 and 100 atmospheres. The writer has taken a series of ten furnace plates for the same region at pressures of 8 and 16 atmospheres. Compressed air was used, with metallic vanadium in the furnace tube. On the better

[1] *Astrophysical Journal*, **34**, 21, 1911.

photographs the lines were in almost all cases reversed, the comparison lines also being frequently reversed, so that close measurements were possible.

TABLE III

Pressure Displacements for Titanium

λ	8 Atm. 2 Plates	8 Atm. (Absorption) 1 Plate	12 Atm. 1 Plate	16 Atm. 2 Plates	8 Atm. Arc
4286.168....	0.045	0.049	.....	0.087	0.021
4287.566....	0.040	0.048	.....	0.099	0.024
4289.237....	0.050	0.049	.....	0.075	0.025
4291.114....	0.049	0.042	.....	0.091	0.022
4295.914....	0.046	0.054	.....	0.094	0.022
4298.828....	0.045	0.054	.....	0.095	0.025
4300.732....	0.046	0.046	0.081	0.099	0.021
4301.158....	0.048	0.050	0.076	0.088	0.024
4306.078....	0.044	0.048	0.073	0.091	0.024
4512.906....	0.047	0.043	.....	0.088	0.029
4518.198....	0.046	0.049	.....	0.099	0.029
4522.974....	0.047	0.045	.....	0.099	0.031
4527.490....	0.044	0.056	.....	0.100	0.029
4533.419....	0.043	0.047	0.077	0.097	0.031
4534.953....	0.047	0.052	0.074	0.094	0.034
4535.741....	0.043	0.046	0.082	0.102	0.029
4544.864....	0.048	0.050	.....	0.094	0.031
4548.938....	0.045	0.044	.....	0.099	0.031
4552.632....	0.045	0.046	.....	0.097	0.029
4555.662....	0.046	0.049	.....	0.090	0.029
Mean displacement.	0 0457	0.0483	0.0772	0.0939	0.0270
Displacement per atmosphere.....	0.00571	0.00604	0.00643	0.00587	0.00338

Mean furnace displacement per atmosphere, 0.00592

Ratio, Arc : Furnace = 338 : 592 = 0.571

Table IV gives the furnace displacements for vanadium, the arc displacements measured by Rossi for 15 atmospheres being given in the last column. The values in the third column, for unreversed lines at 8 atmospheres, were obtained from a plate made for the iron spectrum, but containing the stronger vanadium lines narrow and bright from impurities in the iron.

From Table IV it is seen that the displacements per atmosphere at 8 and 16 atmospheres agree within 10 per cent. As the reversals at 16 atmospheres are rather wide, a difference of this magnitude may fairly be ascribed to errors of measurement and the propor-

TABLE IV

PRESSURE DISPLACEMENTS FOR VANADIUM

λ	8 Atm.	8 Atm. (Unreversed)	16 Atm.	15 Atm. Arc
4090.728....	0.070	.....	....	0.071
4092.821....	0.040	.....	.....	.....
4095.633....	0.045	.....	.....	0.061
4099.941....	0.040	.....		0.049
4102.321....	0.060	.....	.....	0.047
4105.318....	0.047	.....	.....	.....
4109.905....	0.040	.....	.....	0.052
4111.940....	0.046			0.043
4115.330....	0.038	.....	.....	0.043
4116.634....	0.053	.....	.....	0.042
4123.539....	0.040	.....	.....	0.050
4128.251....	0.047	.....	.....	0.046
4132.100....	0.043	.....	.....	0.049
4134.589....	0.046	.....	.....	0.044
*4179.542....	0.016	.....	.....	.....
4182.755....	0.041	.....	.....	.....
*4330.189....	0.023	.....	0.039	0.042
*4332.988....	0.018	.....	0.034	0.041
*4341.167....	0.020	.....	0.045	0.052
*4353.040....	0.023	.....	0.039	0.040
4379.396....	0.042	0.049	0.086	0.046
4384.873....	0.042	0.040	0.087	0.046
4390.149....	0.048	0.044	0.085	0.043
4395.413....	0.040	0.046	0.084	0.047
4400.738....	0.035	0.044	0.091	0.047
4406.810....	0.037	0.042	0.081	.....
4407.810....	0.040	0.047	0.078	.....
4408.364....	0.040	0.040	.....	.....
4408.683....	0.057	0.047	.....	.....
4416.636....	0.035	0.042	0.084	0.043
4421.733....	0.044	0.037	0.096	0.046
4426.201....	0.044	.....	0.075	0.044
4428.711....	0.034	.....	0.065	0.046
4429.958....	0.044	.....	.....	0.047
4436.313....	0.034	.....	0.062	0.049
4438.006....	0.035	0.031	0.077	0.043
4441.881....	0.033	0.036	0.084	0.050
4444.566....	0.035	0.037	0.074	0.049
4457.600....	0.038	.....	0.054	.....
4459.922....	0.036	0.033	0.059	.....
4460.389....	0.041	0.035	0.080	.....
*4577.356....	0.019	.....	0.038	0.044
*4580.590....	0.018	.....	0.034	0.042
*4586.552....	0.020	.....	0.033	.....
*4594.297....	0.021	.....	0.036	0.041

SUMMARY, OMITTING * LINES

Mean displacement.....	0.0427	0.0406	0.0779	0.0478
Displacement per atmosphere......	0.00534	0.00507	0.00487	0.00319

TABLE IV—*Continued*

λ	8 Atm.	8 Atm. (Unreversed)	16 Atm.	15 Atm. Arc
Mean displacement of lines also in arc. .	0.0428	0.0406	0.0808	. . . . . .
Displacement per atmosphere of lines also in arc.	0.00535	0.00507	0.00505	. . . . . .

Mean furnace displacement per atmosphere, 0.00509
Mean furnace displacement per atmosphere for lines also in arc, 0.00516
Ratio, Arc : Furnace = 319 : 516 = 0.618

tional increase of the displacements in general is in harmony with the results for iron and titanium.

The ratio of the mean displacements per atmosphere for lines common to the furnace list and to Rossi's list for the arc is close to that found in Tables I and III. The last two columns of Table IV show the relative displacements in furnace and arc for nearly equal pressures. Nine furnace lines which have shifts much smaller than the average are starred and are not included in the averages at the end of the table. For the seven lines of this set which were measured by Rossi, there appears to be a distinct relative difference as compared to the unstarred lines. At 16 atmospheres the furnace displacements of the starred lines are close to those of the arc for 15 atmospheres while the displacements for the other lines approach a 2 : 1 ratio. These lines were measured on several good furnace plates and there can be no doubt of their large difference from the unstarred lines. So far as can be judged from the reproductions of Rossi's spectra, the starred lines in the arc spectra are comparable in quality with the others in his table. It is to be noted, however, that Rossi's measurements for 30 atmospheres, which gave the lines distinctly reversed in the arc, show consistently smaller values for the starred lines than for the others. Until a more direct comparison of furnace and arc photographs is possible, there is some question as to the certainty of a large relative difference for these lines.

Vanadium spectra at 8 and 16 atmospheres, accompanied by an

arc spectrum taken at atmospheric pressure, are reproduced in Plate XI. The four lines from λ 4577 to λ 4594 appear at the right, showing their displacements relative to those of the lines near λ 4400. The two parts of the spectrum at 8 atmospheres were enlarged from the same negative, so that photographic differences are eliminated.

EFFECT UPON DISPLACEMENT OF VARIATION IN FURNACE CONDITIONS

Since many modifications are possible in the arrangement and operation of the furnace, it seemed worth while to see what differences, if any, changes in certain variables might make in the pressure displacements. The region of spectrum selected for these experiments consists of the iron lines whose measurements for different pressures are given in Table II, with the addition of λ 5269.723 on some of the photographs. The lines of this group have the advantages of giving fairly large displacements, all of the same order of magnitude, so that the mean can be used to determine the effect of any special condition, and of appearing usually in reversal, which in the case of low temperature lines greatly increases the accuracy of measurement. The various modifications tried will be considered in turn.

1. *Temperature difference.*—A variation in pressure displacement inversely as the absolute temperature of the source would be of the proper order of magnitude to account for the difference observed in furnace and arc displacements. It has been possible to test this point with the furnace in such a way that a relation of this kind should have shown itself, but the results have failed to reveal a dependence of displacement upon temperature.

In these experiments a definitive test required a pressure high enough to give a large displacement and also a temperature difference as great as possible, both conditions acting against obtaining lines of the best quality for a close comparison. Pressures of 12, 16, and 20 atmospheres were used. At each pressure, a temperature was taken as low as would give clearly defined lines and then as high as could be employed. It was not possible to go to the upper limit of the furnace temperature, as wide reversals were given, and if the exposures were made long enough to narrow the reversals only

temperatures up to a certain limit could be used without generating large quantities of white vapor which cut off the light.

Little confidence could be placed in comparative measurements for the extreme low and high temperatures on account of the diffuseness of the lines at the pressures employed. To obtain good lines for measurement, the furnace was always operated at moderate temperatures (not above 2400° C.). The method adopted for the temperature comparison was to make exposures at the same pressure for low and high temperatures on the same plate, placing one outside of the other by means of the occulting device above the slit, and making vacuum exposures before and after to test for instrumental disturbance during the experiment.

Twelve plates were taken by this method for the iron lines from λ 5300–λ 5500 and from λ 4200–λ 4500. Various temperature intervals were taken, usually those for which the pyrometer gave differences of about 500° C. The actual difference was probably greater than this on account of the readings being affected by the cloudy condition of the furnace interior at the high temperatures. Very good plates were obtained at 12 atmospheres with lines reversed in each photograph taken with a temperature difference of about 300° C. The reversals appeared to be perfectly continuous in the two spectra side by side. Larger temperature intervals gave the low temperature lines unreversed, and the maximum difference should have been given for 20 atmospheres with a temperature interval of at least 500°. This gave an absolute displacement of about 0.1 Å for both lines, and no difference in position could be detected between bright low temperature lines and the same lines reversed at high temperature. A difference in displacement inversely proportional to the absolute temperature would amount to about 20 per cent under these conditions and although the quality of the lines makes one hesitate to say that there is certainly no difference, it can be said that a difference of this magnitude should have been perceptible in a visual comparison made in this way. Higher pressures were tried, up to 30 atmospheres, but difficulties attendant on the increased pressure prevented the obtaining of photographs satisfactory for comparison at both high and low temperatures.

Evidence offered by the structure of reversed lines bears on the effect of temperature difference. All of the lines whose measurements are given in this paper, when reversed at all, show reversals nearly if not quite symmetrical up to the highest pressures observed, 24 atmospheres in the case of the iron lines. An increased displacement for the low temperature line given by the cooler parts of the furnace tube should make the line as a whole unsymmetrically reversed, the portion to the violet of the absorption line being the wider. Lines having this appearance are rare in any source, the arc usually giving strong widening to the red when lines reverse unsymmetrically, while, for the lines considered in this paper, the symmetry of the arc reversals agrees with those shown by the furnace. A condition which is conceivable but highly improbable may be mentioned. If the emission line widened under pressure unsymmetrically toward the red and if at the same time the absorption line had a larger displacement owing to the lower temperature of the vapor producing it, the widening of the emission line and the increased displacement of the absorption line might keep pace and preserve the appearance of symmetrical reversal. There is no reason to believe that this actually takes place. Such lines when unreversed under pressure should show strong widening toward the red. The iron lines of Table II have been obtained in the furnace unreversed at several pressures. They also appear usually unreversed in the arc under pressure and much widened when iron terminals are used, but this widening remains nearly symmetrical both in furnace and arc.

If a decrease of displacement with increasing temperature were found really to exist, the widening of the lines, which seems to accompany increased pressure in all sources which have been observed, would be expected to remain, and we should have the condition in solar and stellar spectra that lines widened but not distinctly displaced might indicate high pressures which produced little or no displacement by reason of the high celestial temperatures involved. The evidence presented by the furnace experiments, however, is against a dependence of the pressure displacement on the temperature of the source.

2. *Effect of different compressed gases.*—It was noted in the dis-

cussion of Table II, that two good plates for the furnace in carbon dioxide at 8 atmospheres gave consistently larger displacements than were observed for compressed air, the difference of the means amounting to about 10 per cent. This difference is not certainly beyond possible errors of measurement, but the measurements by different observers agreed very closely and the difference is larger than was obtained between photographs of similar quality for any other conditions of the furnace. Photographs of poorer quality for the iron lines of Table I and the titanium lines of Table III, each at 8 atmospheres in CO_2, failed to show a decided difference from corresponding photographs for air. Higher pressures with carbon dioxide gave poor results on account of the large amount of white oxide which was generated.

The possibility of carbon dioxide giving larger pressure displacements than air on account of its higher dielectric constant was discussed in my preliminary paper.[1] The present results show that this can have no important bearing on the difference between furnace and arc displacements, since the greater part of the furnace work has been done with air. Rossi[2] has recently tested this question for the arc by obtaining the displacements of a number of iron lines in air and in carbon dioxide at 15, 30, and 50 atmospheres. The mean displacements for the two gases agreed closely.

It is worthy of note in this connection that, judging from my experiments with the furnace, the carbon dioxide is probably largely turned to carbon monoxide before it reaches the region where maximum radiation is taking place, and the same is probably true to a certain extent for the arc. As the dielectric constant of carbon monoxide is but slightly greater than that of air, being much less than for carbon dioxide, but little difference in displacement is to be expected through this agency. The furnace should, however, be more sensitive than the arc to any influence which the compressed gas can exert, since such a gas is thoroughly mixed with the metallic vapor and brought to the same temperature.

3. *Low vapor-density.*—Table V gives a summary of the displacements for the furnace at 8 atmospheres under various conditions,

[1] *Loc. cit.*

[2] *Philosophical Magazine* (6), **21**, 499, 1911.

the furnace tube being used without jacket in order to eliminate any effect due to vapors given off by the jacketing materials in the presence of oxygen. Compressed air was used throughout. On account of the tube rapidly becoming thinner when fully exposed to the air, some difficulty was found in keeping the temperature approximately constant, which affected the clearness of the reversals. The plates were thus not so satisfactory for measurement as those for which the jacket was used, and the deviations of small size from the values given in Table II are probably to be ascribed to this cause. From one to four good plates were measured for each condition summarized in Table V.

TABLE V

PRESSURE DISPLACEMENTS FOR IRON AT 8 ATMOSPHERES UNDER VARYING CONDITIONS OF THE FURNACE

λ	Tube 20 or 25 cm Long				5 or 6.4 cm Tube
	From Table II	Small Amount of *Fe*	*Fe* with *Ca* and *NaCl*	Small Amount of *Fe* with *Ca* and *NaCl*	
5269.723....	.....	0.044	0.036	0.038	0.037
5328.236....	0.043	0.043	0.044	0.041	0.039
5371.734....	0.041	0.043	0.040	0.041	0.038
5397.344....	0.042	0.044	0.040	0.045	0.045
5405.989....	0.041	0.043	0.037	0.041	0.039
5429.911....	0.044	0.044	0.040	0.040	0.042
5434.740....	0.036	0.044	0.033	0.037	0.039
5447.130....	0.047	0.042	0.038	0.042	0.038
5455.834....	0.040	0.042	0.035	.....	0.039
Mean.......	0.0418	0.0432	0.0381	0.0406	0.0396

Plates taken without the jacket for the regular size of tubes and usual amount of iron gave means very close to those of Table II; so the first column of displacements is taken directly from Table II and used for comparison with the values for the special conditions in which no jacket was employed.

A very little iron (about 0.05 gram as compared with 2 grams or more generally used) gave lines unreversed, but of good quality for measurement. The displacements for the several lines agreed very closely as is seen in the third column of Table V, the mean being close to the general mean of Table II. The furnace thus

confirms the conclusion that has been drawn by a number of observers of the arc under pressure that displacement is not dependent on the quantity of the metallic vapor present.

4. *Effect of foreign vapors.*—In a long column of vapor such as that given by the furnace tube, irregular refraction effects are possible, which may under certain conditions give a displacement of the lines to the red. In the usual operation of the furnace, with the jacket about the tube, the spectrum showed that a considerable amount of calcium and sodium was present, presumably given off by the carborundum and graphite, and in order to see if the displacement was affected thereby, an attempt was made to increase the effect by introducing a large quantity of metallic calcium and sodium chloride with the iron. To insure maximum effect during the pressure exposure, these were put into the tube, by removing the window-holder, after the first comparison exposure in vacuum had been made. A brilliant banded spectrum from the calcium, together with very widely reversed D lines, attested the presence of a dense vapor from these substances, especially in the earlier stages of the furnace run. Some of the photographs under these conditions were rather difficult to measure, the reversals appearing as if an unsteady distribution of the vapor had existed during the exposure, which may have been due to disturbances other than refraction. I am not prepared to say that anomalous dispersion does not enter in some degree when a mixture of this sort is present in the tube, but such effect as there is on the displacements is in the direction of lower values; so that the generally high values of the furnace as compared to the arc are not explained by an influence of this sort. Tubes with 25 cm heated between the contact blocks were used for some of these tests. The mean displacements obtained are given in the fourth and fifth columns of Table V, the latter column giving the results when a very little iron was used with the calcium and salt. The deviation of the means from the value in column two, in view of the character of the lines, is not large enough to indicate a real effect on the displacement.

5. *The use of short tubes.*—The method of using tubes of about one-fourth the regular length was described on p. 186. If the length of the column of vapor were an essential factor in determining the

displacement, a decided difference should have appeared for these tubes. The vapor-density was kept closely comparable with that for the long tubes by using a quantity of iron proportional to the length of the tube. The results are given in the final column of Table V. A difference of only 5 per cent appears between this mean and that in the second column, so that the length of the tube can be regarded as without decided effect on the displacement.

COMPARISON WITH THE ZEEMAN EFFECT

A comparison of the pressure displacements with the separations produced by a magnetic field for the lines considered in the present paper offers little evidence in support of a close connection between the two phenomena.

The magnetic separations for the iron and titanium lines have been published by the writer[1] and those for vanadium by Mr. Babcock.[2]

The iron lines of Table I show triplet separation in the magnetic field with the exception of $\lambda\lambda$ 4251 and 4294 which are complex. The remaining lines, including λ 4415 which may have more than three components, have the following values of the separation divided by the square of the wave-length for a field of 16,000 gausses:

λ	$\Delta\lambda/\lambda^2$	λ	$\Delta\lambda/\lambda^2$
4271.934	1.868	4404.927	1.720
4308.081	1.724	4415.293	1.734
4325.939	1.309	4427.482	2.194
4376.107	2.214	4461.818	2.185
4383.720	1.727		

These measurements for the separations are all of high weight. $\lambda\lambda$ 4376, 4427, and 4462 are seen to have magnetic separations of equal magnitude, this separation being distinctly larger than those of the remaining triplets in the list. Their pressure displacements in the furnace, however, are about half as large as those of the other

[1] *Papers from the Mount Wilson Solar Observatory*, Vol. II, Pt. 1; *Carnegie Institution Publication* No. 153, 1912.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 55; *Astrophysical Journal*, **34**, 209, 1911.

lines. The relative effects of pressure and of the magnetic field are thus opposite for the two groups of triplets in this list.

The iron lines of Table II, which agree among themselves as to pressure displacement, show a variety of magnetic separations, all of them complex with the exception of λ 5434.740, which is unaffected by the magnetic field. There is therefore no clear basis for comparison of the two effects. The case is similar for the titanium lines of Table III, the displacements being of about equal magnitude, while the magnetic separation varies from an unaffected line (λ 4295.914) to those having from 8 to 12 components.

The vanadium lines of Table IV show two groups of four lines each (λ 4330–λ 4353 and λ 4577–λ 4594) which have about half of the average displacement. These 8 lines are all given as showing triplet separation in the magnetic field, as do 25 other lines in Table IV. The average value of $\Delta\lambda/\lambda^2$ for 20,000 gausses is 1.81 for the 8 small-shift lines and 2.55 for the 25 lines having larger displacements. The effects of pressure and of the magnetic field are thus in the same direction for these groups.

The data here given do not materially alter the general situation as to the relation of the pressure and Zeeman effects, since the writer has shown[1] that no close correspondence exists between the effects for individual lines or even small groups, a general agreement as to relative magnitude becoming apparent only when the means of large numbers of lines are considered.

THE RELATION OF DISPLACEMENT TO WAVE-LENGTH

One or two points concerning the iron lines of Tables I and II may be referred to here, though the number of lines is small for a discussion of the relation to the wave-length. The average displacement per atmosphere of the lines from λ 5300 to λ 5500 is but little larger than for the lines showing the larger shifts in the region λ 4200 to λ 4500. The displacements of the three lines in Table I showing small shifts, however, are related to the displacements of the lines in Table II very nearly as the third power of the wave-length, a relation which was found by Gale and Adams[2] to hold for

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 46; *Astrophysical Journal*, 31, 433, 1910.

[2] *Loc. cit.*

the mean displacements of iron lines in the arc through a long range of the spectrum. The ratio of the cubes of the mean wave-lengths for the two regions would require a displacement of 0.023 Å at 8 atmospheres for $\lambda\lambda$ 4376, 4427, and 4462, a value slightly higher than that given for these lines in Table I.

The chief significance of this result, in the writer's opinion, lies in the fact that the three blue lines are very similar to the lines of Table II in their response to temperature excitation of the furnace. At low temperatures they are relatively strong as compared to the other lines in Table I, and the similarity holds for their appearance in the arc, where these low temperature lines are usually sharp and reverse with difficulty. The unstarred lines of Table I are less comparable in general behavior with those of Table II, since they are strong under all conditions and are susceptible to pronounced widening and reversal in the arc. The lack of distinct change in the displacements of such lines with the wave-length seems less significant than the approximate third-power variation for lines showing the same sort of response to temperature.

THE DIFFERENCE OF FURNACE AND ARC DISPLACEMENTS CONSIDERED IN CONNECTION WITH WIDENING PHENOMENA

It is apparent in Tables I to IV that there is a consistent difference, usually of about 80 per cent, between the displacements per atmosphere of groups of lines given by the furnace and arc which as a rule are fully comparable as to accuracy of measurement in the two sets of spectra. Naturally there are individual cases, some of which have been noted, where the quality of the lines for measurement is better in one source than in the other, but this can scarcely exert any effect on the general result. The precautions taken to make instrumental conditions as much alike as possible for furnace and arc observations made in this laboratory have been described. As a guard against personal differences in measurement, Miss Lasby, who took part in the reduction of the pressure-arc photographs, has kindly measured several of the furnace plates, with the result that no systematic difference in values can be ascribed to the method of measurement. The possible influence of unsymmetrical widening has been carefully considered, but this appears to explain but little

of the effect, largely because most of the lines listed here do not seem to widen unsymmetrically in any source. The reversals have occasionally been obtained very narrow compared to the total width of the line for pressures of 16 atmospheres or more (see for example λ 5371 in No. 6 of Plate XII). In such cases a widening of the reversal toward the red which could affect the measurement would be accompanied by a very pronounced dissymmetry of the line as a whole.

The various modifications of the furnace which have been tried have proved ineffective in producing a distinct difference in the mean displacement of the lines. For most of the differences between the radiation conditions of furnace and arc, however, it is difficult to bridge fully the gap between the two sources.

It is now desired to call attention to a difference in the structure of furnace and arc lines which I believe may furnish the key to their different displacements. I have always observed in furnace spectra a certain "softness" in the appearance of the lines. As compared to arc lines photographed on the same plate, the furnace lines have a more uniform intensity over their width, so that for the same width in the negative the furnace lines are less dense than those of the arc. This indicates a flatter intensity-curve for the furnace lines, a smoothing down of the central maximum, which seems to be different in character and more fundamental than a widening produced by changes in the quantity of vapor present.

The furnace, except when used for pressure experiments, has generally been operated in a partial vacuum to avoid oxidation, and even this condition, which favors the narrowness of furnace lines as compared to those given by the arc in open air, showed this relatively large width of lines in the furnace spectrum. In order to make a closer comparison of the widening in furnace and arc, a special set of photographs was made for the iron and titanium lines whose pressure displacements are studied in this paper. The furnace was excited at atmospheric pressure, the outlet valve being left open. Then a number of exposures were made on the same plate with the arc in open air for currents ranging from 2 to 20 amperes, the exposure times being graduated to make the various arc spectra of about equal intensity. A change in current seems

to be the most effective means of increasing the widening of arc lines when plenty of the substance is ready to be vaporized, as when metallic terminals are used. As we cannot get at the relative quantity of vapor per unit volume in the furnace and arc under given conditions, the two sources are best made comparable by having the metal abundant in each, as was done by placing a large amount of powdered iron or titanium in the furnace tube and using as arc terminals iron rods or carbons, of which the lower held a large quantity of titanium carbide.

Plate XIII shows portions of the iron and titanium spectrum containing the lines of Tables II and III as given by the furnace at atmospheric pressure and by the arc in air at 20, 10, and 2 amperes. Width of slit and all adjustments of the spectrograph were unchanged during each series and the negative was copied on a single plate, effects of this sort being very sensitive to differences in photographic contrast. The first set of spectra shows the iron arc at 20 amperes giving about the same degree of reversal as the furnace, $\lambda\lambda$ 5269, 5328, and 5371 being reversed. None of the lines are reversed in the arc at 10 amperes or less. It is seen that the furnace lines are wider than those of the arc even for 20 amperes, while the negative shows the arc lines to be slightly the blacker. Another negative, not so favorable for reproduction, shows a furnace spectrum having lines of the same width as arc lines which are twice as black.

The lower part of Plate XIII shows the two portions of the titanium spectrum measured in Table III. The enlargements are made from the same negative, the furnace spectrum at atmospheric pressure, together with arc spectra at 20, 10, and 2 amperes, being arranged as for the iron series above. Several of the stronger lines show the beginnings of reversal in the furnace, such lines being distinctly reversed in the arc at 20 amperes, but not for 10 amperes. The greater width of the furnace lines in proportion to their density when compared to the arc lines is very distinct. The appearance of the furnace lines is approached most nearly by the arc at 20 amperes, but even then the widening conditions in the arc are not as strong as in the furnace.

The furnace was not used at atmospheric pressure for vanadium,

but the difference in structure of furnace and arc lines is shown by comparing the arc spectrum of vanadium in Plate XI with the vacuum furnace photograph immediately below it, used as comparison for that at 8 atmospheres pressure, arc and furnace exposures being on the same plate. Even in vacuum, the furnace gives broad lines, easily reversed, in contrast to the narrow, dense structure of the corresponding lines in the arc.

It would seem, therefore, that we have consistent experimental evidence of a tendency of the furnace to produce lines wider in proportion to their density than are given by the arc, at least for the moderate currents usually employed in pressure-arc work. The application to the displacement question is that *if the radiation of the furnace is such as to give lines which are wide in proportion to their density, then since widening and displacement are inseparably bound together when pressure is acting, the furnace lines, being more susceptible to the widening influence, should also respond more readily to the displacing action of pressure.* Again it must be borne in mind that this widening refers to a change in the intensity-curve for a given line, not to the general strengthening and proportional widening which results from greater vapor-density.

It is not clear why the radiation conditions of the furnace should give lines of large absolute width, and the theory of widening, so far as worked out, gives little aid in the explanation; but experiment shows the action of the furnace to be in this direction. It is evident, however, that the conditions of emission and absorption in the furnace tube will account for some features in the appearance of the lines. Reversal certainly takes place easily in the furnace for most of the lines considered in this paper. It follows that such lines when unreversed will have the peak of the emission-curve flattened to a greater or less extent by absorption. An approach to uniform density across the width of the furnace line would then result, especially as there is some variation in the condition of the vapor during the exposure time required for these large scale photographs. This relation of emission and absorption, together with a width of line approached in the arc only for high current-strength, form the distinguishing characteristics of the furnace lines.

It seems to me probable that such other apparent variations of pressure displacement in different sources as have been observed may well be based on this ability of a source to give widened lines when such widening depends upon features of the source other than vapor-density. Gale and Adams[1] obtained consistently larger mean displacements for a number of lines in the titanium spark than were given by the arc. We have again the experimental fact that lines are wider in the spark than in the arc. Gale and Adams also note that these titanium lines reverse more widely in the spark, though usually symmetrically. The decidedly larger displacements of enhanced lines given by the spark may be based on the same cause, since for such lines the widening influence of the spark discharge has a maximum effect.

The mechanism by which the furnace gives relatively wide lines must be quite different from that acting in the spark, where the widening depends upon the disruptiveness of the discharge and the lines can be made very narrow by the use of self-induction in the circuit.

According to the above hypothesis, any light-source whose radiation is such as to give widened lines should give relatively large pressure displacements. A very high-current arc should give larger displacements than an arc with low current. Also, it should be possible to vary spark displacements by gradually taking out self-induction and increasing the capacity. No systematic experiments on these points have been carried out, and in any case comparative measurements would be difficult by reason of the large difference in the character of the lines produced by very diverse conditions of the same source. For sources widely different in nature, as are the arc and the furnace, differences in radiation can exert their full effect and still lines can be obtained in the spectrum of each whose measurements are fully comparable.

SUMMARY

The leading results of this investigation are as follows:

1. Sufficient material has been collected to give measurements of fairly high weight for the displacements of certain groups of lines in the iron, titanium, and vanadium spectra.

[1] *Op. cit.*, p. 41.

2. The measured displacements of iron lines for pressures from 1 to 24 atmospheres and for titanium and vanadium lines for 8 and 16 atmospheres as compared to vacuum show a close proportionality between displacement and pressure for these ranges.

3. The pressure effect in absorption has been developed as a useful method for certain kinds of lines, giving displacements in general of the same magnitude as for emission lines.

4. Temperature differences of at least 500° C. for a pressure of 20 atmospheres have failed to show a definite variation of displacement with the temperature.

5. Variations in quantity of vapor present and in length of tube, also the addition of foreign vapors, have not appeared to affect the furnace displacements.

6. Some additional data have been secured regarding the degree of correspondence between displacement and magnetic separation.

7. A few iron lines affected similarly by temperature-changes are compared in regard to change of displacement with wave-length.

8. The furnace has in general given displacements much larger than those of the arc, and a special study has been made of the structure of furnace and arc lines which it is believed may contribute to an explanation of this difference.

I am indebted to Miss Sheldon for regular assistance in measuring the photographs, also to Miss Lasby for check measurements on several plates.

Mount Wilson Solar Observatory
February 1912

ON THE MEASUREMENTS OF THE ZEEMAN EFFECT[1]

By A. COTTON

In the *Astrophysical Journal* of December 1911 (**34**, 212), J. E. Purvis calls attention to the fact that the absolute values of the Zeeman effect for four chromium lines given by himself and by Miller, W. Hartmann, and by Babcock, are not at all in agreement. I propose to show that this disagreement arises chiefly from the measurement of the magnetic fields used, and to call the attention of physicists to the precautions which must be taken in order to render the measurements made in different laboratories comparable with one another.

Let me say first of all that Purvis utilizes Miller's measurements in accepting the value of the field as 23,850 units, given in the author's paper. But Miller had never measured directly the magnetic fields that he used. Like several other physicists, Moore, Jack, etc., he calculated the value of the fields that he used on the basis of the magnetic separation of the series lines studied by Runge and Paschen.

Now, in their beautiful pieces of work[2] these scientists had not proposed to make absolute measurements themselves. A measurement by Färber on the blue lines of zinc had led them to estimate provisionally at 23,850 units the field which they used for the study of the lines of mercury and which was afterward taken as a comparison field by several physicists.

P. Weiss and I published in 1907[3] the results of an absolute measurement of the Zeeman effect for these lines. The magnetic fields used had been measured with care by two distinct methods; the plates were measured separately by the two collaborators, care being taken to make the measurements on different parts of the lines studied so as to avoid the influence of the grain of the photographic plates. The result obtained differed notably from that of

[1] Translated from advance proof sheets from *Journal de physique*, sent by the author.

[2] Runge and Paschen, *Astrophysical Journal*, **15**, 235, 1902; **15**, 333, 1902; **16**, 123, 1902.

[3] *Journal de physique* (4), **6**, 429, 1907.

Färber: we have explained the variation, which is more than 3 per cent, by examining critically the method adopted by Färber for measuring the fields; the bismuth spiral, which he used as intermediary, gives a quickly determined rough value of the field, but would require, in order to obtain precise results, precautions that are not taken.

The result of our measures was as follows: Let us call $\delta(\lambda)$ the difference between the outer components of these lines separated by the field, for example, the difference between the outer components of the pure triplet given by the line 4680; we have, using the electromagnetic C.G.S. units, that is, expressing λ and $\delta(\lambda)$ in centimeters and H in gausses:

$$\frac{\delta(\lambda)}{H\lambda^2}=1.875\times 10^{-4}$$

in place of 1.813 given by Färber.

Our result was completely confirmed by the absolute measurements made at Tübingen by Mlle. A. Stettenheimer and by Gmelin.[1] The latter, who also measured the magnetic fields by two distinct methods, gives finally the value:

$$\frac{\delta(\lambda)}{H\lambda^2}=1.878\times 10^{-4},$$

which differs from ours by only about two thousandths.

On the other hand, we may add that the following fact brings equally well an indirect confirmation to these measurements. When we published our work we assumed with Runge that the pure triplet of the line 4680 had double the separation of the normal triplet predicted by Lorentz' elementary theory. Our measurements on the blue lines of zinc led, however, on this assumption, to a value of the ratio e/m of the charge to the mass of an electron, smaller by 6 per cent than the value then accepted. From the preceding figures we may in fact deduce:

Weiss and Cotton.......... $\frac{e}{m}=1.767\times 10^7$

Gmelin.................... $\frac{e}{m}=1.771\times 10^7$

[1] *Annalen der Physik*, 28, 1079, 1909.

But subsequently the direct measurements of the value of e/m for cathode corpuscles of small velocities have shown that it was the value deduced from the Zeeman phenomenon that was correct.[1] Here are the results of recent measurements:

Classen[2]	$\frac{e}{m}=1.773\times 10^7$
Bücherer-Kurt Wolz[3]	$\frac{e}{m}=1.767\times 10^7$
Malassez[4]	$\frac{e}{m}=1.77\times 10^7$
Bestelmeyer[5]	$\frac{e}{m}=1.75\times 10^7$

Today, then, we know more accurately the fields used by Runge and Paschen; in their research on the mercury spectrum, the field (the provisional value of which was 23,850) was in reality very near 23,000 gausses:

Weiss and Cotton	23,060
Gmelin	23,010

The other field (the value of which was assigned as 31,000), to which are referred the results derived by the same authors for the spark spectra, ought to be reduced in the same proportion, which would bring it down to about 29,900 gausses:

Weiss and Cotton	29,975
Gmelin	29,910

When the results given by Miller, Jack, etc., are compared with measurements where the magnetic field was determined directly, it is necessary, then, to remember that the numbers given refer to a field of 23,000 gausses and not of 23,850. For the same reason the results given by B. E. Moore[6] refer to a field of 23,600 gausses and not to a field of 24,450. Moore himself had taken care to

[1] Weiss et Cotton, *Comptes Rendus*, **147**, 968, 1908.

[2] *Physikalische Zeitschrift*, **9**, 768, 1908.

[3] *Annalen der Physik*, **30**, 288, 1909.

[4] *Annales de Chimie et de Physique*, **23**, 424, 1911.

[5] *Physikalische Zeitschrift*, **12**, 974, 1911.

[6] *Astrophysical Journal*, **28**, 8, 1908; **30**, 143, 1909; **32**, 385, 1911.

point out that later absolute measurements would make it possible to increase the precision of the value.

If we make this correction for Miller's results, which Purvis cites, we find that they approach those found by Babcock. But this correction only accentuates the difference between the results of Purvis and those of Miller. I had already noticed this difference, and called attention to the fact that the measurements of Purvis, which are fully stated elsewhere[1] and are very interesting, were not directly utilizable excepting as relative values, because the value of the magnetic field adopted (39,980) is certainly much too high. As Purvis, in his note, states that he has no doubt of the absolute value of this field, without giving the details of the method which he used to measure it, I will point out the way in which it can be calculated indirectly, starting from different measurements made by Purvis on his plates.

1. In his work on the lines of the elements *Pb*, *Sn*, *Sb*, *Bi*, and *Au*, Purvis has had occasion to measure the magnetic separations of the two lines λ 3383 of silver and λ 3274 of copper which behave like the D_1 line of sodium. Comparing the distance of the four components of these quadruplets with those found by Runge and Paschen, I obtain for the value of the field 30,800 instead of 39,980.

2. In his work on the lines of the elements *Ti*, *Cr*, and *Mn*, Purvis has measured the pure triplet given by the line λ 4274.9. This triplet is given by Miller as having double the normal separations; Dufour (unpublished measurement) working independently has verified this result.[2] Assuming this, I calculate for Purvis' field 29,800.

3. In the same work Purvis gives the results of several measurements on the lines of zinc, magnesium, and cadmium (second subordinate series). Purvis does not state expressly that the current was regulated to the same value as in the rest of the work. Assuming that it was, I find 29,600 approximately, using the absolute measurements previously stated for these lines.

[1] *Cambridge Transactions*, **20**, 193, 1906; *Proceedings Cambridge*, **13**, 82, 325, 354, 1906; *Proceedings Cambridge*, **14**, 43, 217, 1907.

[2] This line constitutes a part of a natural triplet of chromium which we find again in the ultra-violet. I wish to call attention here to the fact that the two other lines which inclose it do not give, as was supposed, pure triplets (Babcock, *Astrophysical Journal*, **33**, 382, 1911).

4. Finally, it is not only for the four lines of chromium cited by Purvis that the results vary considerably from those of Miller. For the other triplets of chromium and for the triplets of manganese, the same systematic disagreement is found. Comparing the values of the separations given for 20 triplets of these two substances, the ratio between the field used by Purvis and the one (23,000) to which Miller's results are referred, may be estimated. Thus I find again for Purvis' field 29,900: the relative values of the chromium lines alone would give only 28,500. Assuming this last value for the field, the results would agree with those of Miller and Babcock.

It is seen from this why, in the table of several measurements on the Zeeman effect that I prepared for the tables of the Société française de physique,[1] I stated that the results given by Purvis are obtained not with a field in the neighborhood of 40,000 but of about 30,000 only. Moreover, to obtain in the interspace used by Purvis (pole-pieces terminated by disks 7 mm in diameter, 4 mm apart) a field of 40,000, it would be necessary with the best iron obtainable to have a large instrument like the Weiss electromagnet, and Purvis would certainly have indicated the diameter of the cores in this case.

When we verify in this way systematic differences between the results obtained by two different observers, we are led to suspect that it is the measurement of the magnetic field that is its cause. Differences as great as in the case of Purvis are never found; but there are other examples where the differences in the values of $\frac{\delta(\lambda)}{H\lambda^2}$ reach 5 per cent or even 10 per cent. Thus the results of Hartmann on chromium are smaller than the others; it is quite probable that the direct measurement of the field (made by an induction method) explains here again the variations from the results of other physicists. In fact, the comparison of Hartmann's figures with those just obtained at the Zeeman laboratory, by Mme. I. M. Graftdijk on another spectrum, that of nickel, leads to the same conclusion: the magnetic fields given by Hartmann are a little too large.

[1] A. Cotton, *Le radium*, 8, 42, 1911.

How may we avoid in the future similar difficulties in the comparisons—difficulties which take away part of their value from the results of measurements that require much time and patience? It would suffice to put greater care on the determination of the magnetic fields. This measurement can be made indirectly, utilizing the measurements on the zinc lines for which the absolute measurements have furnished results practically identical; it is this process that is actually employed at the Zeeman laboratory;[1] it is also the one that Stefan Rybar[2] employed in some recent work at Göttingen. This optical measure of the field is, besides, the best for studying the spark spectra obtained with electrodes of ferromagnetic metals, the presence of which necessarily affects the fields studied. Besides, it can be rather rapidly used, and the plates employed are suitable for precise measurements, if we are careful to render the lines very fine with a suitable self-induction connected in the discharge circuit (Hemsalech). The 4680 zinc line, which gives a pure triplet, is the best line to use.

It is not the only one that can be used; the measurements of Runge and Paschen, those we made at Zurich, and those made at Tübingen agree in showing that for other series lines the Zeeman effect varies proportionally to the field and can be used to measure it. But it is essential to reduce as much as possible the number of intermediary lines that serve finally to refer the measurement to the measurements properly called absolute. Each one of these comparisons introduces a slight uncertainty; should we, for example, use the simple relations established by Runge and Paschen, or take careful account of the very small variations from these simple laws which these physicists themselves find in the different measurements? From this point of view it would be desirable that the physicists who employ this process indicate with precision, in their articles, which lines they have used and what (in Ångström units, for example) are the variations actually measured on the plates used to study the field. In this manner we should later be able to render still more precise the results calculated for this field,

[1] Mme. Bild-Van Meurs, *Proceedings Amsterdam*, **11**, 223, 1908 (thèse d'Amsterdam); I. M. Graftdijk (thèse d'Amsterdam, December, 1911).

[2] *Physikalische Zeitschrift*, **12**, 889, 1911 (thèse de Budapest, 1911).

profiting by the more complete data that the succeeding researches will bring to the exact knowledge of the ratio between the magnetic separations of the different lines and the absolute values themselves.

The example of Purvis shows clearly that the direct measurement of the field can lead to errors when only one method has been employed to make this measurement in absolute value. A check is always necessary to avoid errors which can then escape even a very good physicist. An absolute measurement is, besides, fairly difficult, since one should verify or calibrate the apparatus which is used. The balance which in particular P. Weiss and I used, a further improved model of which P. Sève[1] described recently, gives rapidly, when we have a verified ammeter, a very precise value of the field, provided the interspace between the iron pieces is sufficiently wide. But we employ more often for the study of the Zeeman effect pole-pieces too small to allow the direct use of this instrument. It is then necessary to use the balance to calibrate, in a very uniform field, the combination of a small coil connected to a ballistic galvanometer or to a Grassot fluxmeter.[2] This last apparatus, once calibrated, is very convenient[3] and permits the frequent verification of the field which is being used. There is here a final precaution, necessary especially in researches extending over long periods of time, to which I think it useful to call attention. Usually the experimenter limits himself to measuring once for all the field for different values of the magnetizing current, and seeks afterward to restore the current to the same value in the magneto-optical measurements, properly so called. This process is theoretically legitimate, since for the strong inductions which are used, the errors coming from the "previous history" of the electromagnet are not appreciable. But it must not be forgotten that it

[1] *Comptes Rendus*, **150**, 1309, 1910. We shall find in Sève's paper, now in press, details on the use of this instrument, constructed by Pellin.

[2] Grassot, *Journal de physique*, **3**, 696, 1904 (apparatus constructed by the company for the manufacture of meters).

[3] Another method, suggested by Faraday, would also permit comparing a magnetic field, by a very rapid and sure method, with a standard field; we should use the properties of diamagnetic or paramagnetic crystals, measuring simply, for example, the periods of oscillation. I hope to come back in the future to this method which M. Sève and I have commenced to study.

is assumed that during the interval the ammeter has not been affected, this instrument does not contain magnets or springs that can be modified with time, and that it is also assumed that the coils of the electromagnet have remained well insulated.

An article by H. D. Babcock, which appeared in the December 1911 number of the *Astrophysical Journal* (**34**, 288), shows also that the preceding remarks are not without significance, and accentuates again the interest there would be in knowing better the absolute values of the separations measured in the different laboratories. Babcock was led by his own measurements on chromium and on vanadium,[1] and by the measurements of King[2] on titanium and on iron, made also at the Pasadena laboratory, to remark that the pure triplets of the non-series lines, which vary, as we know, between large limits, are not, however, distributed at random, but appear to group themselves about certain favored values.

Babcock did not know that I had myself called attention to this fact, and presented curves representing the distribution of the displacements according to their own size.[3] He finds again results analogous to mine, but our results are not identical, because Babcock used, without correcting them, the values of the magnetic field given in the papers of Miller and of Moore. Hence the agreement, which he points out between the mean value of the separations near the principal maximum and the value of the normal separation, appears to me altogether accidental. It would in reality be necessary for 8 of the 13 spectra studied to increase by 3 per cent the values of all the separations referred to the unit field.

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[1] *Contributions from the Mount Wilson Solar Observatory* Nos. 52 and 55; *Astrophysical Journal*, **33**, 217, and **34**, 209, 1911. The field is measured by a bismuth spiral.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 56; *Astrophysical Journal*, **34**, 225, 1911.

[3] Société française de physique, séance du 7 mai, 1909, *Bulletin des séances*, fasc. 4, p. 55, 1909. Reductions of two of these curves will be found in *Le radium*, **8**, 42, 1911. Mme. Graftdijk has just published a more complete study, from this point of view, of the iron spectra (138 triplets), and has published also similar curves for nickel (163 triplets) and for cobalt (59 triplets). Mme. Graftdijk finds also that the first maximum occurs for values exceeding notably the value for the normal state. She finds also for iron, but not for nickel, a maximum for a separation in the neighborhood of 3/2 of that of the normal state.

STANDARD WAVE-LENGTHS IN THE ARC SPECTRUM OF IRON, REDUCED TO THE INTERNATIONAL UNIT

I. FROM λ 4282 TO λ 5324

By F. GOOS

According to the plan of the International Solar Union[1] the secondary[2] international standards in the arc spectrum of iron are to be supplemented by tertiary standards, from 5 to 10 Ångström units apart, obtained from interpolation by means of a grating. To obtain the highest possible accuracy this work should, of course, be done only with the largest and best concave gratings; and yet in view of the fact that many observers and different methods are needed, I decided to take up the problem with a comparatively small plane grating.

1. THE PHOTOGRAPHIC APPARATUS

Through the courtesy of Professor Ames, to whom I take this opportunity of expressing my sincere thanks, I obtained a very beautiful two-inch plane Rowland grating, ruled with 7000 lines to the inch. The grating was mounted in the autocollimating fashion devised by Littrow, and in the fifth and sixth orders gave good, bright images. The objective, by Hilger, consists of two elements cemented together and achromatized in such a way as to give a color-curve which is practically a straight line from λ 4200 to λ 8000; the focal length increases from 194 cm in the violet to 197 cm in the red. The radii of curvature of the front and back surfaces of the lens were so chosen that the images of the slit reflected from these surfaces were real and were located about midway between slit and objective.

This renders it possible by a slight inclination of the objective to elevate one image and depress the other so that their glare no longer strikes the photographic plate, which is covered except for a narrow band on the side toward the objective. With this lens

[1] *Astrophysical Journal*, **32**, 259–260, 1910.

[2] *Ibid.*, **32**, 215–216, 1910; **33**, 85, 1911.

the dispersion is such that in the sixth order 1 Ångström is represented by 0.36 mm.

The slit lies 12 cm above the middle of the plate-holder; the supports for the slit, plate-holder, objective, and grating are made of mahogany and rest upon two strong brass rings. The whole apparatus is inclosed in a long, slender, light-tight wooden case so that for half an hour at a time the inside temperature can be held constant to within half a degree.

The electrodes are ordinary iron rods from 8 to 9 mm in diameter, carrying a current of 6 or 7 amperes, sometimes on the 110-volt circuit, sometimes on the 220-volt circuit. A "condensing lens" of 19 cm focal length is used in such a way that only the middle part of the arc is used, and so that the grating is illuminated with perfect uniformity.

2. THE PHOTOGRAPHS

The spectra of the sixth order were employed for the photographs, thus requiring the use of two light-filters, one to cut out the higher orders and one the lower orders. For this purpose the gelatin sheets of Wratten & Wainright served admirably. In the blue and violet, the ordinary fast plate of different makes was employed; but for the longer wave-lengths the panchromatic plates of the firm above named. In the region of λ 4200 the exposure was about half a minute; at λ 4500, one minute; at λ 4800, two minutes; at λ 5100, three and a half minutes; at λ 5400, five minutes. Throughout the entire work, the width of the slit was $\frac{1}{50}$ of a millimeter; the inclination of the plate toward the normal varied from 5 to 8 degrees.

One of the sixth-order spectra, the one employed for the earlier photographs, gave good images with a fine sharp line in the middle upon which one could set the reading microscope very accurately. The limit of resolution in the violet was 0.10 Å. The available field amounted to approximately 7 cm or 200 Å. I soon, however, came to prefer the other spectrum of the sixth order, which gave lines a little wider, but of perfectly uniform density; the resolving power was a little smaller, only 0.13 Å., in the violet; the field was here a little larger, about 10 cm, that is, 280 Å.; so that in general from ten to eleven of the secondary iron standards were found upon

a plate, a matter of the highest importance in the evaluation of standards.

The striking difference between the slit-images of the right and left sixth order, which is also observed between the right and left of lower orders, probably finds its explanation in some asymmetry in the groove on the grating. Since each region of the spectrum is photographed some seven or eight times, the plates are so set that the wave-length in the middle of each plate is advanced by 35 Å. In this manner every line appears in from seven to eight different places on different plates, and each secondary standard is gradually displaced from one end, through the middle, to the other end of the plate; so that in the computation of the wave-lengths a little later, a smoother result is obtained.

For the determination of wave-lengths between λ 4282 and λ 5324 thirty-seven plates were taken, covering the following ranges:

No. of Plate	From λ	To λ	No. of Plate	From λ	To λ
1	4148	4353	20	4737	5002
2	4148	4376	21	4754	5012
3	4191	4376	22	4790	5050
4	4191	4427	23	4824	5083
5	4234	4467	24	4860	5127
6	4282	4467	25	4903	5167
7	4282	4531	26	4919	5192
8	4315	4495	27	4966	5233
9	4353	4531	28	5002	5267
10	4427	4593	29	5050	5302
11	4427	4647	30	5083	5324
12	4467	4707	31	5110	5371
13	4467	4737	32	5167	5406
14	4531	4790	33	5167	5435
15	4548	4790	34	5192	5456
16	4593	4860	35	5233	5498
17	4603	4878	36	5267	5507
18	4647	4919	37	5302	5570
19	4691	4966			

3. THE MEASUREMENTS AND THE DETERMINATION OF WAVE-LENGTHS

The measurements were made by means of a Töpfer micrometer, the screw of which had a pitch[1] of one-half mm. In place of the ordinary eyepiece I used the Zeiss binocular designed by Abbé. By the addition of an auxiliary objective this was transformed into

[1] F. Goos, *Zeitschrift für Instrumentenkunde*, **31**, 52, 1911.

a regular binocular microscope, with which one can directly observe the image of the spectral line in the focal plane of the ordinary objective. The image produced by the complete system is erect. The use of both eyes not only enables one to set on the line with much greater ease, but also prevents weariness of the eyes even after a long series of measures. Each line is measured eight times on each plate, four times in position "I"—longer wave-lengths to the right—and four times in position "II"—longer wave-lengths to the left. In each of these positions two settings are made with a right-handed rotation of the screw, and two with a left-handed rotation.

As a dispersion formula I have used

$$\lambda = a + bx + cx^2$$

where λ is the wave-length; x, the reading of the screw; and a, b, c, are constants. (1 rev. = 0.5 mm.)

The constants for each plate were determined from three secondary standards, one at the left end, one in the middle, and one at the right end.

In general, $b = +1.4$; $c = -0.0001$.

But this simple formula does not perfectly represent the other secondary standards on the plate. These outstanding differences were corrected graphically by means of a correction-curve. In the earlier plates (sixth order on the right) there were some seven or eight of these standards; in the case of the other plates (sixth order on the left) as many as ten or eleven of the standards were shown on a single plate; so that these curves could be drawn with great accuracy and the standards adjusted among themselves. These correction-curves have somewhat the form of a sine curve with an amplitude of about 0.02 Å.

4. RESULTS

The following table contains the wave-lengths of 184 lines, derived from 1292 single measures. Each line has in general been measured seven times. As mentioned above, the measurements are distributed over thirty-seven plates. Certain weak lines were missing on some of the plates; but no line is given in the table which has not been measured at least three times. The third

λ	Intensity	No. of Plates	Mean Error	I. A.	λ−I. A.	D_s	D_z	Remarks
4282.408	5	7	±0.001	.408	0.000	+0.004	0.000	
94.130	6	7	2			+ 3	+ 3	Kayser gives the intensity as 4
99.247	6	7	1			+ 3	+ 1	
4307.912	8	6	4			+ 2		
15.089	5	8	1	.089	0	0	− 1	
25.764	8 *R*	8	5			0	+ 1	
37.052	4	8	2			+ 1	− 2	
52.741	4	9	1	.741	0	+ 3	− 2	
58.510	2	6	3			+ 4	+ 6	
67.584	3	8	2			+ 5		
67.905	2	8	2			+ 5		
69.778	3	8	1			+ 6	+ 6	
75.934	4	8	1	.934	0	+ 6	− 2	
83.551	10 *R*	6	4			+ 6	− 7	
88.420	3	6	1			+ 6		
90.956	2	5	3			+ 6	0	
4404.755	8 *R*	6	2			+ 5	− 3	
07.716	3	6	1			+ 5		
08.420	3	6	1			+ 5		
15.129	7	6	2			+ 5	− 1	
22.574	3	6	1			+ 5	+ 7	
27.314	4	8	1	.314	0	+ 5	− 5	
30.623	3	7	3			+ 5	0	
33.215	2 *u*	6	3			+ 5		
42.349	4	7	1			+ 5	+ 1	
43.197	3	7	2			+ 5		
47.725	4	7	2			+ 5	− 3	
54.384	3	7	2			+ 5	− 3	
59.127	4	7	2			+ 5	+ 2	
61.660	4	7	1			+ 5	+ 3	
66.555	4	9	1	.556	− 1	+ 5	− 3	
69.396	3 *u*	7	2			+ 5	+ 9	
76.024	4 *ur*	6	1			+ 5		
82.173	3	5	3			+ 5	+ 5	
82.270	4	5	5			+ 5	− 1	
84.241	3	7	2			+ 5		
89.747	2	7	2			+ 5	− 2	
90.085	2	6	3			+ 5	− 8	
94.573	4	7	1	.572	+ 1	+ 4	− 2	
4514.196	1	4	4			+ 2	+ 2	
17.543	1	4	3			+ 2	+ 2	
25.155	3	6	3			0	+ 3	
28.622	5	6	1			− 1	+ 1	Eversheim's interferometer value is 4528.622
31.154	3	7	1	.155	− 1	− 1	− 4	
47.854	3	6	1	.853	+ 1	− 2	− 3	
52.552	1	6	3			− 2		
56.125	3 *u*	6	3			− 1	− 1	
74.732	1	6	3			+ 2	+ 1	
81.541	2	6	4			+ 3	+ 11	
87.138	1	6	2			+ 4	− 1	
92.660	3	7	1	.658	+ 2	+ 5	− 1	
98.144	2	6	1			+ 6	+ 2	
4602.017	1	6	1			+ 6	+ 7	
02.945	4	7	1	.947	− 2	+ 6	− 6	
11.298	3	6	2			+ 7	− 4	

λ	Intensity	No. of Plates	Mean Error	I. A.	λ−I. A.	D_s	D_z	Remarks
4613.236	2	5	±0.003		0.000	+0.007	+0.004	
19.300	3	6	3			+8	−3	
25.073	3	6	2			+8	−1	
32.915	3	6	2			+8	−10	
37.528	3	6	2			+8	+3	
38.026	3	6	1			+8	−1	
47.439	4	7	1	.439		+8	−5	
54.514	3	6	2			+8	+11	
54.645	3	6	4			+8	−2	
67.457	4	7	2			+5	−5	
68.157	4	7	3			+5	+4	
73.178	3 *u*	7	6			+4	+7	
78.856	4	7	1	.855*	+1	+3	−1	
83.575	1	7	3			+2	−7	
91.414	3	8	1	.417	−3	+1	−2	
4707.289	4	8	1	.288	+1	−2	+2	
10.281	3	7	2			−2	+3	
21.000	1	4	5			−3	+4	
27.425	2 *u*	7	2			−4	−9	*Fe, Mn*
33.587	2	7	2			−4	−1	
36.789	4	8	1	.786	+3	−4	+6	
41.529	2	7	2			−4	+3	
45.792	2 *u*	7	3			−4	−6	
54.046	3	8	2	.047*	−1	−3	0	*Mn*
57.572	1	6	3			−3	−1	
62.372	3	8	2			−3	+4	*Mn*
65.861	2	6	3			−2		*Mn*
66.420	2	7	2			−2	0	*Mn*
72.812	2	8	2			−2	−2	
83.437	3	8	2			−1	+2	*Mn*
86.810	2	8	2			−1	0	
89.654	3	9	1	.657	−3	−1	+4	
98.273	1	3	2			0	−5	
4800.648	1	7	4			0	−5	
02.885	1	7	2			0	−1	
07.734	1	3	4			0	−1	
23.526	4	8	1	.522*	+4	0	+3	*Mn*
32.734	1	4	7			0	0	
39.546	1	8	3			0	0	
43.161	1 *u*	7	5			0	0	
55.690	1	4	2			−1	−2	
59.756	4	9	1	.758	−2	−1	+3	
63.665	1	5	6			−2	+3	
71.329	5	8	2			−2	−2	
72.155	4	8	1			−2	+2	
78.226	4	8	1	.225	+1	−2	−1	
81.718	1	6	4			−2	−4	
82.160	1	6	3			−2	−4	
85.441	2 *u*	6	3			−2	−2	
90.769	5	7	1			−2	0	
91.510	6	7	1			−2	+2	
4903.325	4	8	1	.325	0	−2	+4	
10.035	1	7	4			−2	+6	
19.006	5	9	1	.007	−1	−2	−6	
20.518	7	8	1			−2	−3	

λ	Intensity	No. of Plates	Mean Error	I. A.	λ−I. A.	D_s	D_z	Remarks
4924.768	2	8	±0.002		0.000	−0.002	−0.007	
38.196	1	7	4			−2	+3	
38.827	3 *u*	8	2			−2	−1	
39.679	2	8	1			−2		
40.406	2	8	3			−2	0	
57.306	4	6	3			−2	+5	
57.613	5	6	2			−2	+6	
66.105	3	9	1	.104	+1	−2	+2	
73.114	2	7	2			−2		
78.619	1 *ur*	6	6			−2	+1	
82.541	3 *u*	8	3			−2	+14	
85.269	2	8	3			−2	−3	
85.564	2	8	3			−2	−3	
94.131	2	8	3			−2	−6	
5001.882	3	9	1	.881	+1	−2	0	
05.732	3	8	2			−2	+2	
06.137	4	8	2			−2	+3	
12.070	3	8	1	.073	−3	−2	+2	
14.961	2	7	1			−2	+4	
22.249	2	7	1			−3	−6	
28.126	1	7	3			−3	−7	
41.072	2	7	2			−4	−3	
41.758	3	7	2			−4	+4	
49.828	3	8	1	.827	+1	−4	+7	
51.640	3	7	1			−4	+4	
68.786	3	7	2			−4	+4	
74.722	3 *u*	7	2			−4	−14	
79.226	3	7	2			−4	−1	
79.741	2	7	1			−4	−6	
83.346	2	8	1	.344	+2	−4	+4	
96.999	1	5	4			−3	+2	
98.696	3	7	2			−2		
5105.547	2	7	3			−1		*Cu.* Interferometer value of Fabry-Buisson 5105.543
07.468	2	6	2			−1	+7	
07.643	2	6	2			−1	−6	
10.411	3	8	1	.415	−4	0	−1	
23.732	2	8	2			+2	+1	
27.367	2	8	1	.364*	+3	+2	−1	
(33.646)	3 *u*	7	3			+3	−(32)	Kayser gives the intensity as 1
39.278	3	6	3			+3	+7	
39.493	4	6	3			+3	+8	
42.934	2	7	3			+4	−4	
50.847	2	7	3			+4	+2	
51.916	2	6	3			+4	−3	
(62.347)	3 *ur*	7	3			+4	+(33)	
67.490	5	9	2	.492	−2	+4	−2	
71.602	4	8	2			+4		
91.469	4	8	1			+4	−12	Eversheim by the interferometer obtains 5191.473
92.364	4	9	1	.363	+1	+4	−2	
94.951	3	8	2			+4	+1	
98.720	1	8	2			+4	−1	
5202.341	2	8	3			+4	−4	
08.614	2	7	3			+3	−8	
15.197	2	8	2			+3	+1	
16.279	3	8	2			+3	+1	

λ	Intensity	No. of Plates	Mean Error	I. A.	λ−I. A.	D_s	D_z	Remarks
5217.409	2	8	±0.002		+0.000	+0.003	+0.003	
26.873	3	8	2			+ 2	− 3	
27.194	4	8	2			+ 2	+ 7	
29.862	2	8	2			+ 2	+ 17	
32.955	5	9	1	.957	− 2	+ 2	− 7	
35.382	1	8	2			+ 1		
42.495	1	8	2			+ 1	− 1	
50.644	1	8	3			0		
63.318	2	8	2			− 1	0	
66.570	4	9	1	.569	+ 1	− 1	− 3	
69.532	6	8	1			− 1	− 2	
70.353	5	8	1			− 1	0	
73.178	2	8	3			− 1	+ 3	
73.371	1	5	4			− 1	− 6	
81.808	3	8	1			− 1	+ 3	
83.635	3	8	1			− 1	+ 1	
5302.316	2	9	1	.315	+ 1	0	+ 2	
07.364	1	7	3			+ 1	− 1	
24.195	4	8	1	.196	− 1	+ 2	− 4	

column gives the number of plates upon which the line has been measured. Column 2 gives the intensities. These adhere as nearly as possible to Kayser's[1] system and were estimated either with the naked eye or with a low-power eyepiece; they indicate, therefore, mainly the visibility of the line without regard to its breadth or blackness. The intensity of the weakest lines is denoted by 1; that of the strongest, by 10.

In general the numerical values agree well with those of Kayser; only in the case of the lines λ 4294 and λ 5134 is there a deviation as great as two units. Lines which are not sharp are so marked in this column, where the following nomenclature is employed:

u = hazy;
ur = hazy toward the red;
uv = hazy toward the violet.

R denotes that the line is easily reversed. Column 4 gives the "mean error" of the wave-lengths computed by taking the average value from the different plates, giving equal weight to all the measures.

Taking the average for the entire series, this mean error, ϵ amounts to ±0.0022 Å.; hence the "probable error," r, is

[1] H. Kayser, *Astrophysical Journal*, **32**, 217, 1910; *Zeitschrift für wissenschaftliche Photographie*, **9**, 173, 1911.

±0.0015 Å. A mean error of ±0.005 occurs four times; one of ±0.006 occurs three times; one of ±0.007 occurs once; these have to do mostly with lines which are not sharp or with lines which were measured only four or five times.

Besides the iron lines are included some which appear as impurities; among these are seven manganese lines and one of copper.

Column 5 gives the values of the thirty-six secondary standards from which the other wave-lengths are determined by interpolation. Besides the standards established by international agreement, I have employed four others which rest upon the interferometer measurements of Fabry and Buisson,[1] and Eversheim;[2] these are indicated by asterisks.

The agreement with standards is excellent throughout. The deviations which are given in column 6 never exceed 0.004 Å., and reach this value only in two cases out of thirty-six. The sum of the positive deviations amounts to +0.025 Å.; that of the negative, to −0.026 Å.; so that the average deviation is $\frac{0.051}{36} =$ ±0.0014 Å. It is, therefore, perhaps fair to assume that the wave-lengths of these lines are accurate to 0.001 Å. It is, however, a rather striking fact that the larger deviations occur precisely in those lines for which the three interferometer measures, by which the secondary standards are determined, agree very well.

The probable error mentioned above, ±0.0015 Å., as an average for the entire series, naturally gives no information except regarding the accuracy of wave-lengths determined by my particular instrument. Whether my measures are actually correct to within the limits assigned can be answered only by comparison with measures obtained on other apparatus. The only results yet available for purposes of such a comparison are those of Kayser. His values show striking systematic differences with respect to mine, differences which need explanation. By means of some of his spectrograms, which he has kindly placed at my disposal, I think I have found[3] that these deviations are to be explained by the fact that

[1] *Trans. International Union for Solar Research*, **2**, 138, 1908.

[2] *Annalen der Physik* (4), **30**, 815, 1909.

[3] *Zeitschrift für wissenschaftliche Photographie*, **10**, 200, 1911.

Kayser has used a simple linear interpolation between each two of the secondary standards and then averaged the results. The larger deviations which thus occur between individual values point, in Kayser's opinion, to irregularities in the system of secondary standards.

But I have found the improved (smoothed-out) values of the secondary normals given by Kayser to be only partially confirmed.

In order to compare Kayser's wave-lengths with mine, it is first necessary to place in evidence the systematic deviations. Column 7, headed "D_s," gives these differences, in the sense of Goos-Kayser, determined graphically. We observe that the variations at λ 4630 reach the value $+0.008$ Å., while the deviations on the negative side are not so considerable and at λ 4740 and λ 5060 are only -0.04 Å.

I have neglected those small differences—amounting at most to 0.002 Å.—which result from the fact that Kayser has used standards whose values depend not only upon the measurements of Fabry-Buisson and Eversheim but also upon those given by Pfund[1] in 1908, reduced, however, to the value 6438.4696 for the red cadmium line, while the international secondary standards established later depend upon Pfund's[2] more recent work.

Column 8, headed "D_z," contains the accidental differences which remain after correcting for the systematic differences.

Kayser and I have observed in common 163 lines. The sum of the positive deviations amounts to $+0.284$ Å., the negative to -0.285 Å. The average deviation is therefore $\frac{\pm 0.569}{163} = \pm 0.0035$ Å.

Computed from the above the probable error of a single observed difference between my measures and those of Kayser is approximately $\pm 0.0035 \times 0.845 = \pm 0.0030$ Å.; giving equal weight to each observer, the probable error of a single wave-length determination for each observer becomes ± 0.0021 Å. This value is larger than that computed above from the internal agreement of my own measurements, namely, ± 0.0015 Å.

[1] *Astrophysical Journal*, **28**, 197, 1908.

[2] *Ibid.*, **32**, 215, 1910.

It is evident also that if the weight is properly distributed, there are still other sources of error to be looked for in the method itself; perhaps one's estimation of the center of gravity of a line is different in the case of a spectrogram such as that of Kayser, which has a dispersion three or four times as great as mine.

In fourteen cases the accidental deviation is greater than 0.008 Å. Concerning these the following is to be noted:

The lines λ	4469.4	deviation	+0.009	are marked as hazy
	4727.4	"	−0.009	
	4982.5	"	+0.014	
	5074.7	"	−0.014	
The lines λ	4490.1	"	−0.008	are components of close doubles
	4654.5	"	+0.011	
	5139.5	"	+0.008	
The lines λ	4581.5	"	+0.011	are sharp lines in which the deviation is not immediately explicable
	4632.9	"	−0.010	
	5191.5	"	−0.012	
	5208.6	"	−0.008	
	5229.9	"	+0.017	

In the case of the line λ 4581 the mean error is rather great—+0.004; the line λ 5191, given by Eversheim's interferometer measurement as 5191.0473, shows also a deviation of −0.004 Å. from my measurements. The line 5229.9 was measured by me eight times, with a mean error of +0.002, the individual measures being

	5229.852
	858
	861
	860
	868
	870
	862
	865
Mean	.862

The difference between this and Kayser's value of 5229.845, after a correction for systematic differences, is very striking. The

lines 5133.6, with a deviation of −0.032 Å., and 5162.3, with a deviation of +0.033, must be at once thrown out. They are both very hazy. I have placed them in brackets in the table and have omitted them from consideration in deriving the probable error. They are not at all available for the purpose of standards.

The measurement of close doubles naturally offers some difficulty and it is apparent that my distances turned out to be smaller than those of Kayser. In the following table are collected under the headings "G" and "K" the distances of all doubles less than 0.410 Å.

Double	G.	K.	G.−K.
	Å.	Å.	Å.
5005.7–6.1	0.405	0.404	+0.001
4489.7–90.1	0.338	0.344	−0.006
5226.9–7.2	0.313	0.311	+0.002
4957.3–7.6	0.307	0.306	+0.001
4985.3–5.6	0.295	0.295	0.000
5139.3–9.5	0.215	0.214	+0.001
5273.2–3.4	0.193	0.202	−0.009
5107.5–6	0.175	0.188	−0.013
4654.5–4.6	0.131	0.144	−0.013
4482.2–2.3	0.097	0.103	−0.006

It is to be hoped that these doubtful cases may soon be cleared up by measurements in the iron spectrum made by other observers. I am expecting soon to extend these measurements toward the red as far as to λ 6495.

Hamburg, Physikalisches Staatslaboratorium
December 20, 1911

PRELIMINARY STATEMENT OF THE EARLY OBSERVATIONS OF *NOVA GEMINORUM* NO. 2, MADE AT THE OBSERVATORY OF THE UNIVERSITY OF MICHIGAN

By R. H. CURTISS

A brief résumé of the results obtained prior to March 23 by observation of *Nova Geminorum* No. 2 at the University of Michigan is contained in the following statement.

All estimates of magnitude were obtained by comparison with neighboring stars, according to Argelander's method. The spectra were made with a single-prism spectrograph, and with one exception in the photographic region.

March 13. At 11^h Central Standard Time, Mag. = 3.9.
Exposures: $9^h\ 4^m$ C.S.T. on Seed 23.
$9^h\ 27^m$ C.S.T. on Seed Lantern Slide.
$10^h\ 2^m$ C.S.T. on Seed Lantern Slide.
$11^h\ 0^m$ C.S.T. on Seed 23.
$11^h\ 40^m$ C.S.T. on Seed 23.

On all these plates the continuous spectrum resembles that of *Altair*, with alternating wide regions of emission and absorption. The comparison with *Altair* extends only to the character of the spectrum, however, and not to the position of the lines. Few narrow dark lines are to be found in the spectrum. Nearly all the prominent lines of Type F 5 are faint or absent. No certain trace of such strong F 5 lines as λ 4481 and λ 4549 can be found in the *Nova*, but the strong blend at λ 4172 seems to be represented. There is also strong absorption in the *Nova* in the neighborhood of the G band, possibly partly due to titanium. Probably other identifications with lines of type F 5 might be made, but such lines are difficult and apparently not numerous. On the other hand there are absorption lines in the *Nova*, not certainly associated with the bright lines, which are weak or absent in *Procyon*.

The lines $H\delta$, $H\gamma$, $H\beta$, λ 5016, λ 4922, and λ 4472 of helium, H and K of calcium, are all strong lines with complex structure. A heavy broad absorption component (in λ 4922 clearly double, in

$H\delta$ multiple) is strongly displaced toward the violet. Wide emission is clearly present on the red edge of these lines and in case of the calcium lines, sharp dark reversals, and in the hydrogen and helium lines probably weak dark reversals, all indicating a small velocity of recession, are present in this emission. Emission is present also on the violet edge of the absorption components of some of these lines, but is weaker and less extensive than that on the edge of greater wave-length.

The displacement of the absorption groups of the principal lines for this date follows.

Lines:	*Ca:* K,	H;	*He:* λ 4471,	λ 4922,	λ 5016;	*H:* $H\delta$,	$H\gamma$,	$H\beta$
Disp. in Å:	−7.80,	−7.70,	−8.60,	−4.50,	−3.50,	−5.80,	−6.50,	−8.15.

This displacement increases with the square of the wave-length for the hydrogen lines, differs for different elements, and is quite different for helium and the so-called parhelium.

The sharp reversals of the calcium lines indicate a velocity of +5 km ± 3 km.

March 15. At 6^h C.S.T. Mag. = 4.2; 12^h, Mag. = 4.9.
Exposures: 10^h 0^m C.S.T. on Seed 23.
11^h 45^m C.S.T. on Seed 23.

The continuous spectrum is similar to that of March 13, but with contrasts much increased. λ 4481 is now visible as a diffuse line, but λ 4549 of *Ti–Fe* cannot be certainly seen. λ 4172 is strong and accompanied by emission. Well-marked absorption lines have developed at λ 4363, λ 4611, λ 4670. In the case of the complex lines, K, H, and $H\epsilon$, $H\delta$, $H\gamma$, λ 4472, $H\beta$, λ 4922, and λ 5016, the absorption lines have about the same intensity as on the previous plates, but are displaced about one Ångström farther toward the violet. The emission on the edge of greater wave-length has increased greatly, and shows a strong maximum near the normal position of the line with evidences of reversal. The velocity from the H and K reversals is +10 kilometers.

March 16. At 9^h C.S.T., Mag. less than 5.5.

Unsuccessful exposure through heavy sky shows only bright lines.

March 17. At 9^h C.S.T. Mag. = 5.7.
Exposures: 9^h 25^m Seed 30.
11^h 15^m Seed 30.

No marked changes in the continuous spectrum are shown, but a less marked alternation of bright and dark maxima is seen. Bright lines are relatively much stronger, with the strong displaced absorption lines of hydrogen and helium much weaker or absent. The position of the edge of the strong emission lines toward the red remains unchanged, while the edge toward the violet has extended greatly (ten Ångströms in the case of $H\beta$) covering the dark lines of two nights before. A strong emission maximum has appeared near the red edge of all hydrogen and helium emission lines mentioned above, with the possible exception of λ 4472. Other regions uncertainly bright on the plates of March 13 are now much stronger, as for example λ 4230, λ 4590, and λ 4610–4670. There is no emission which can certainly be traced to the lines of nebulum except possibly at λ 5007. The emission at λ 4472 is now very weak. Velocity from the H and K reversals $= +9$ kilometers.

March 18. At 8^h C.S.T. Mag. = 5.2.
Exposure: $8^h\ 27^m$ Seed 30.

Continuous spectrum shows little change. The emission lines are now slightly narrower than on the previous night, and with much sharper edges generally marked by absorption. Width of the $H\beta$ line about 35 Å. The violet edges of the emission lines have grown brighter, forming a companion emission maximum with the strong nucleus near the red edge of these lines. Velocity from H and K reversals $= +6$ kilometers.

March 19. At 10^h C.S.T. Mag. = 5.1.
Exposures: $10^h\ 0^m$ Seed 30.
$11^h\ 18^m$ Seed 30.

Continuous spectrum little changed. There is now little resemblance to the spectrum of *Altair*. Several fairly well-defined narrow absorption lines are present. Strong emission lines are still contracting, with absorption at edges coming out more strongly. Two distinct emission maxima are clearly defined near either edge of the emission lines as on the previous night. There is no emission that can certainly be traced to the nebular lines λ4363, λ4686, and λ4960. Velocity from H and K reversals $= +7$ kilometers.

March 22. At 8^h C.S.T. Mag. = 4.9; at 11^h Mag. = 4.7.
Exposures: $8^h\ 48^m$ Seed 30.
$11^h\ 0^m$ Seed 23 bathed in Pinachrome.

On the slower (Seed 23) plate, alternate bright and dark maxima in the continuous spectrum are clearly shown and in many cases the maxima of emission of March 22 correspond to the maxima of absorption of March 13. The spectrum seems to have been reversed in a little more than a week. This finds explanation if it be assumed that complex lines resembling the strong lines of hydrogen and helium are distributed well over large regions of the spectrum.

The changes since March 19, in the strong lines of hydrogen and helium, are very marked. The strong emission band remains of the same width, bordered by well-defined absorption. On the edge toward the violet this absorption is widely double for the hydrogen and helium lines, the component of shorter wave-length being the stronger for $H\beta$, $H\gamma$, $H\delta$, and $H\zeta$, and the weaker for the helium lines. In the case of $H\gamma$, at least, emission extends faintly over the absorption lines on both edges of the strong bright line. The emission lines show the emission maxima near their edges noted previously, with several dark lines in the region between. In the region from λ 5000 to λ 6600, a number of complex bright lines are shown. The D lines have the same general structure as the H and K lines of calcium. A second strong absorption line on the side of shorter wave-lengths may be due to D_3 or may be a second component of the dark edge of the sodium emission. There is no certain indication of nebulum emission between λ 3880 and λ 5020.

Velocity from D_1, D_2, H, and K reversals = +13 kilometers.

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SLIT-WIDTH CORRECTIONS IN SPECTRO-PHOTOMETRY AND A NEW FORM OF VARIABLE SECTORED DISK

By EDWARD P. HYDE

PART I. SLIT-WIDTH CORRECTIONS IN SPECTRO-PHOTOMETRY

A. INTRODUCTION

The spectro-photometer, in its various forms, has been widely employed as a precise instrument in studying relative energy distributions within the limits of the visible spectrum. As formerly used, except in the case of polarization instruments, the settings on the spectro-photometer were made by varying the width of one of the collimator slits. Later the errors which attend this practice, even when the Vierordt double slit is employed, were studied,[1] and this study led to the more general use of the rotating sectored disk in one or another form. But although the spectro-photometer has been widely employed as a precise instrument, only quite recently has any attention been paid to the errors which enter on account of the impurity of the spectrum due to the finite slit-widths of the telescope and of the two collimators. The corresponding error in the measurement of energy distribution by the use of the infra-red spectrometer and linear bolometer or thermopile was evaluated

[1] Capps, *Astrophysical Journal*, **11**, 25, 1900.

by Runge[1] some years ago. The neglect of this error in spectro-photometric measurements may have arisen from the conception that the error would be small because the measurements are relative—the energy distribution of one spectrum being measured in terms of that of another somewhat similar spectrum. This fact does in general reduce the magnitude of the error, but on the other hand in spectro-photometric comparisons the sensibility-curve of the eye enters to produce an added complication.

In connection with an investigation by the author[2] and others of the radiating properties of metals, spectro-photometric measurements were used in determining the relative energy distributions in the spectra of two sources. The question arose as to the possible errors incident to the assumption that the ordinates of the observed luminosity-curves of two continuous spectra, as of a black body at two different temperatures, bear the same ratios to one another throughout the visible spectrum as the ordinates of the corresponding pure energy-curves. A theoretical investigation of the problem was begun, but before its completion the same question was discussed by Nichols and Merritt[3] in connection with their work on phosphorescence. Inasmuch as their method of treatment, consistent with their object in view, was somewhat different from that of the present author and since this investigation led incidentally to the design of a new form of variable sectored disk, the author[4] presented a brief report of his work to the Physical Society. For similar reasons it seems expedient to publish a more detailed account of the investigation, and to describe more in detail the new form of sectored disk adapted to spectro-photometric measurements.

B. THEORY OF SLIT-WIDTH CORRECTIONS

In the following discussion it is assumed that the substitution method of measurement is employed. Thus (Fig. 1) a constant comparison source is placed before the slit S' of one collimator, and

[1] *Annalen der Physik*, **60**, 712, 1897.

[2] Hyde, Cady, and Middlekauff, *Trans. Ill. Eng. Soc.*, **4**, 334, 1909. Hyde, *Jour. of Franklin Inst.*, **169**, 439, 1910.

[3] *Physical Review*, **30**, 328, 1910.

[4] *Ibid.*, **31**, 183, 1910.

the two sources whose spectra are to be intercompared are successively placed before the slit S of the second collimator.[1] Through the slit T of the telescope the two spectra formed by the prism P are seen juxtaposed in the compound field of view of the Lummer-Brodhun contrast prism L.

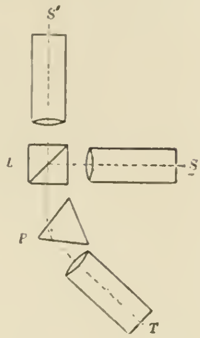

Fig. 1.—Diagrammatic sketch of spectrophotometer.

In spectro-photometric comparisons measurements of the relative *luminosities* of the *impure* spectra of two sources are made. The problem, therefore, is to determine from these measurements the relative curves of *energy* distribution in the *pure* spectra of the two sources. To this end we seek an expression of the intensity in the pure energy spectrum of a source at S (Fig. 1) in terms of the luminosity of its impure

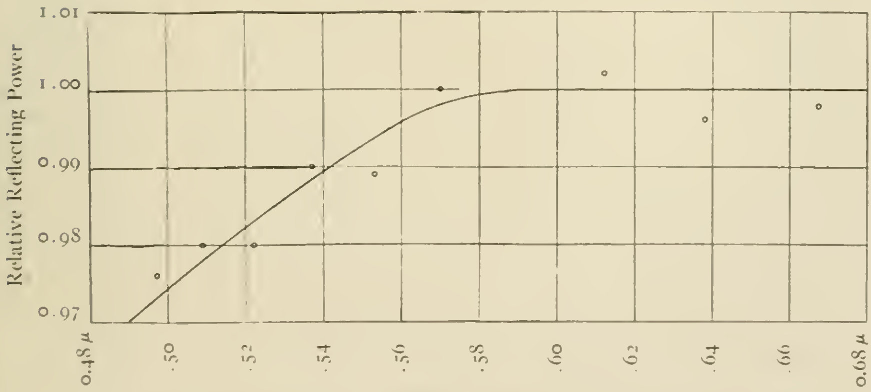

Fig. A.—Relative reflecting power of magnesium carbonate

[1] In practically all the work of the author the light from the source under investigation falls, not upon the slit directly, but upon a block of compressed magnesium carbonate which is mounted at an angle before the slit and which diffusely reflects the incident light into the collimator. In Fig. A are given the results of a study of the selective reflection of the magnesium carbonate block. It is seen that its reflection coefficient at the red end of the spectrum is about 3 per cent greater than that for blue light. If the substitution method is employed and the proper precaution is taken, there is no need to make any allowance for the selective reflection of the magnesium carbonate block.

spectrum formed by the prism P in the plane of the slit T of the telescope.

Let

a = slit-width of S measured in angular units on the divided circle of the telescope.[1]

a' = slit-width of S' measured in angular units on the divided circle of the telescope.

b = slit-width of T measured in the same angular unit.

$c=\dfrac{a+b}{2}$.

θ = angle of deviation of the ray considered.

$f(\theta)$ = intensity in the pure energy spectrum of a source.

$\psi(\theta)$ = sensibility of the eye for the pure spectrum.

$\phi(\theta)=\psi(\theta)f(\theta)$ = luminosity of the pure spectrum of a source.

$F(\theta)$ = luminosity of the impure spectrum of a source, represented in the present case by the luminous flux through the telescope slit, T.

We seek first to find an expression for $\phi(\theta)$ in terms of $F(\theta)$ and constants. It will then be a simple matter to determine $f(\theta)$ in terms of $F(\theta)$ if the sensibility-curve of the eye $[\psi(\theta)]$ is assumed as known. As will be shown later, however, this last step is not necessary, since in spectro-photometry relative measurements of two spectra are made, and if the substitution method is employed, and the comparison spectrum is maintained constant, the function $\psi(\theta)$ is not directly involved.

Expression of $F(\theta)$ as a function of $\phi(\theta)$.—The luminous flux through the slit T of the telescope from the source at S may be separated for theoretical considerations into two portions. The first portion corresponds to wave-lengths for which the images of S formed at the eyepiece slit T by the light of those wave-lengths fall entirely within the limits of T. The second portion consists of the remaining flux from S through T. Let v represent in magnitude a deviation $\Delta\theta$ in θ, where θ corresponds to the deviation of

[1] This slit-width thus measured will theoretically be slightly different in different regions of the spectrum owing to the magnification by the prism, which is equal to unity only at the angle of minimum deviation. If, however, the slit-widths of S are always measured in angular units on the divided circle of the telescope no error will result in the application of the following analysis.

the rays which form an image of S at the center of the slit T of the telescope. The two portions of the luminous flux through the slit T are represented by the first and second integrals, respectively, of the right-hand member of equation (1).[1]

Where $b>a$

$$F(\theta)=\int_0^{\frac{b-a}{2}}[\phi(\theta+v)+\phi(\theta-v)]dv+\int_{\frac{b-a}{2}}^{\frac{b+a}{2}}\frac{b+a-2v}{2a}[\phi(\theta+v)+\phi(\theta-v)]dv \qquad (1)$$

Expanding by Taylor's theorem the functions $\phi(\theta+v)$ and $\phi(\theta-v)$ there follows—

$$\left.\begin{aligned}F(\theta)=&\int_0^{\frac{b-a}{2}}\left[2\phi(\theta)+\frac{2v^2}{2!}\phi^{\mathrm{II}}(\theta)+\frac{2v^4}{4!}\phi^{\mathrm{IV}}(\theta)+\ \dots\ \right]dv+\\ &\int_{\frac{b-a}{2}}^{\frac{b+a}{2}}\frac{b+a-2v}{2a}\left[2\phi(\theta)+\frac{2v^2}{2!}\phi^{\mathrm{II}}(\theta)+\ \dots\ \right]dv\end{aligned}\right\} \qquad (2)$$

which on integration gives

$$F(\theta)=A\phi(\theta)+B\phi^{\mathrm{II}}(\theta)+C\phi^{\mathrm{IV}}(\theta)+D\phi^{\mathrm{VI}}(\theta)+\ \dots \qquad (3)$$

the desired expression where

$$\left.\begin{aligned}A&=\frac{1}{(2)^1\cdot 2!a}[(b+a)^2-(b-a)^2]\\ B&=\frac{1}{(2)^3\cdot 4!a}[(b+a)^4-(b-a)^4]\\ C&=\frac{1}{(2)^5\cdot 6!a}[(b+a)^6-(b-a)^6]\\ D&=\frac{1}{(2)^7\cdot 8!a}[(b+a)^8-(b-a)^8]\end{aligned}\right\} \qquad (4)$$

[1] In this analysis it is assumed that $b \geq a$, since there would probably be no reason why a should be larger than b in practice. Moreover, the corrections for a given value of $a+b$ and for a given ratio $\frac{b}{a}=k$ are precisely the same as those for $\frac{a}{b}=k$, if $a+b$ remains constant.

Expression of $\phi(\theta)$ in terms of $F(\theta)$.—To this end consider the function $F(\theta+c)$. In a manner similar to that followed above for the expression of $F(\theta)$ in terms of $\phi(\theta)$ there results

$$F(\theta+c)=A\phi(\theta+c)+B\phi^{\text{II}}(\theta+c)+C\phi^{\text{IV}}(\theta+c)+\ \ldots\ldots \tag{5}$$

where

$$\left.\begin{aligned}\phi(\theta+c)&=\phi(\theta)+c\phi^{\text{I}}(\theta)+\frac{c^2}{2!}\phi^{\text{II}}(\theta)+\frac{c^3}{3!}\phi^{\text{III}}(\theta)+\ \ldots\ldots\\ \phi^{\text{II}}(\theta+c)&=\phi^{\text{II}}(\theta)+c\phi^{\text{III}}(\theta)+\frac{c^2}{2!}\phi^{\text{IV}}(\theta)+\ \ldots\ldots\end{aligned}\right\} \tag{6}$$

Substitution of these values of $\phi(\theta+c)$, $\phi^{\text{II}}(\theta+c)$, etc., in equation (5) gives

$$\left.\begin{aligned}&F(\theta+c)=A(\phi\theta)+Ac\phi(\theta)+\left(\frac{Ac^2}{2!}+B\right)\phi^{\text{II}}(\theta)+\left(\frac{Ac^3}{3!}+Bc\right)\phi^{\text{III}}(\theta)+\\ &\left(\frac{Ac^4}{4!}+\frac{Bc^2}{2!}+c\right)\phi^{\text{IV}}(\theta)+\left(\frac{Ac^5}{5!}+\frac{Bc^3}{3!}+Cc\right)\phi^{\text{V}}(\theta)+\\ &\left(\frac{Ac^6}{6!}+\frac{Bc^4}{4!}+\frac{Cc^2}{2!}+D\right)\phi^{\text{VI}}(\theta)+\ \ldots\ldots\end{aligned}\right\} \tag{7}$$

Similarly,

$$F(\theta-c)=A\phi(\theta)-Ac\phi(\theta)+\left(\frac{Ac^2}{2!}+B\right)\phi^{\text{II}}(\theta)-\left(\frac{Ac^3}{3!}+Bc\right)\phi^{\text{III}}(\theta)+\ldots\ldots \tag{8}$$

If $\Delta^2F(\theta)$ is defined as follows

$$\Delta^2F(\theta)=F(\theta+c)+F(\theta-c)-2F(\theta),$$

there results from equations (3), (7), and (8)

$$\Delta^2F(\theta)=B'\phi^{\text{II}}(\theta)+C'\phi^{\text{IV}}(\theta)+D'\phi^{\text{VI}}(\theta)+\ \ldots\ldots \tag{9}$$

where

$$\left.\begin{aligned}B'&=\frac{2Ac^2}{2!}\\ C'&=2c^2\left(\frac{Ac^2}{4!}+\frac{B}{2!}\right)\\ D'&=2c^2\left(\frac{Ac^4}{6!}+\frac{Bc^2}{4!}+\frac{C}{2!}\right)\end{aligned}\right\} \tag{10}$$

From equation (9) it follows that

$$\Delta^2F(\theta+c)=B'\phi^{\text{II}}(\theta+c)+C'\phi^{\text{IV}}(\theta+c)+D'\phi^{\text{VI}}(\theta+c)+\ \ldots\ldots \tag{11}$$

Substituting for $\phi^{\mathrm{II}}(\theta+c)$, $\phi^{\mathrm{IV}}(\theta+c)$, etc., the values given in equations (6)

$$\left.\begin{aligned}\Delta^2F(\theta+c)=\frac{B'}{C'}\phi^{\mathrm{II}}(\theta)+\frac{B'c}{1!}\phi^{\mathrm{III}}(\theta)+\left(\frac{B'c^2}{2!}+\frac{C'}{1}\right)\phi^{\mathrm{IV}}(\theta)+&\\ \left(\frac{B'c^3}{3!}+\frac{C'c}{1!}\right)\phi^{\mathrm{V}}(\theta)+\ \ldots\ .&\end{aligned}\right\}\quad(12)$$

In a similar way

$$\left.\begin{aligned}\Delta^2F(\theta-c)=\frac{B'}{1}\phi^{\mathrm{II}}(\theta)-\frac{B'c}{1!}\phi^{\mathrm{III}}(\theta)+\left(\frac{B'c^2}{2!}+\frac{C'}{1}\right)\phi^{\mathrm{IV}}(\theta)-&\\ \left(\frac{B'c^3}{3!}+\frac{C'c}{1!}\right)\phi^{\mathrm{V}}(\theta)+\ \ldots\ .&\end{aligned}\right\}\quad(13)$$

Defining $\Delta^4F(\theta)$ as follows:

$$\Delta^4F(\theta)=\Delta^2F(\theta+c)+\Delta^2F(\theta-c)-2\Delta^2F(\theta)$$

there results by substitution from equations (9), (12), and (13)

$$\Delta^4F(\theta)=C''\phi^{\mathrm{IV}}(\theta)+D''\phi^{\mathrm{VI}}(\theta)+,\ \text{etc.}\qquad(14)$$

where

$$\left.\begin{aligned}C''&=2c^2\frac{B'}{2!}\\ D''&=2c^2\left(\frac{B'c^2}{4!}+\frac{C'}{2!}\right),\ \text{etc.}\end{aligned}\right\}\quad(15)$$

In a quite similar way expressions may be obtained for $\Delta^6F(\theta)$, $\Delta^8F(\theta)$, etc., in terms of $\phi^{\mathrm{VI}}(\theta)$, $\phi^{\mathrm{VIII}}(\theta)$, etc., and constants. It is then possible to evaluate $\phi^{\mathrm{II}}(\theta)$, $\phi^{\mathrm{IV}}(\theta)$, etc., of the right-hand member of equation (3) in terms of $\Delta^2F(\theta)$, $\Delta^4F(\theta)$, etc., with the following result [see equations (9) and (14)]:

$$\phi(\theta)=\frac{1}{A}[F(\theta)-K\Delta^2F(\theta)+L\Delta^4F(\theta)+\ \ldots\ .]\qquad(16)$$

where

$$\left.\begin{aligned}K&=\frac{B}{B'}\\ L&=\frac{BC'-B'C}{B'C''}\end{aligned}\right\}\quad(17)$$

[For the case where $b=a$ and where the sensibility-curve of the eye does not enter, so that $\psi(\theta)=\text{constant}=1$, there follows

$\phi(\theta)=f(\theta)$, and equation (16) reduces to Runge's expression for the correction in spectrometry with the linear bolometer or thermopile

$$af(\theta)=2\left\{\frac{F(\theta)}{2!}-\frac{1}{4!}\Delta^2F(\theta)+\frac{(2!)^2}{6!}\Delta^4F(\theta)-,\text{ etc.}\right\}] \tag{18}$$

Expression of $f(\theta)$ in terms of $F(\theta)$.—Thus far only the spectrum of a single source has been considered, and the analysis has led to an expression of the luminosity of the pure spectrum $[\phi(\theta)]$ in terms of the luminosity of the impure spectrum $[F(\theta)]$. From the definition $\phi(\theta)=\psi(\theta)f(\theta)$ it is seen that the determination of $f(\theta)$, the intensity in the pure energy spectrum, in terms of the measured $F(\theta)$, necessitates a knowledge of the sensibility function $\psi(\theta)$ under the conditions under which $F(\theta)$ was measured. The same result may be accomplished, however, in a simpler way.

In spectro-photometry one is concerned with the relative intensities of two spectra, rather than with the absolute intensity of one. If the substitution method of measurement is employed one determines the impure luminosity-curve of one source, represented by the function $F(\theta)$, in terms of the impure luminosity-curve $F_0(\theta)$ of some other source, which may for simplicity be called the standard source, through the medium of the impure luminosity-curve $F'(\theta)$ of some third comparison source. The pure energy-curve of the standard source, represented by $f_0(\theta)$, is presumably known, and from this it is desired to determine the corresponding function $f(\theta)$ of the test source. *If the comparison spectrum is maintained absolutely constant* throughout the intercomparison of the test and standard sources (which condition involves the constancy of the slit-widths b and a') then $\psi(\theta)=\psi_0(\theta)=\psi'(\theta)$, and so

$$\frac{\phi(\theta)}{\phi_0(\theta)}=\frac{f(\theta)}{f_0(\theta)} \tag{19}$$

from which

$$f(\theta)=f_0(\theta)\frac{\phi(\theta)}{\phi_0(\theta)} \tag{20}$$

Substituting in equation (2) for $\phi(\theta)$ and $\phi_0(\theta)$ their values as given in equation (16), and remembering [equation (4)] that $A=A_0=b$, there follows:

$$f(\theta)=f_0(\theta)\frac{F(\theta)-K\Delta^2F(\theta)+L\Delta^4F(\theta)+,\text{ etc.}}{F_0(\theta)-K_0\Delta^2F_0(\theta)+L_0\Delta^4F_0(\theta)+,\text{ etc.}} \tag{21}$$

But the actual quantities determined in the spectro-photometric measurements are

$$\left.\begin{aligned} R(\theta) &= \frac{F(\theta)}{F'(\theta)} \\ R_0(\theta) &= \frac{F_0(\theta)}{F'(\theta)} \end{aligned}\right\} \qquad (22)$$

so that

$$\left.\begin{aligned} F(\theta) &= R(\theta)F'(\theta) \\ F_0(\theta) &= R_0(\theta)F'(\theta) \end{aligned}\right\} \qquad (23)$$

Substituting these values in equation (21) there follows

$$f(\theta) = f_0(\theta)\frac{R(\theta)F'(\theta) - K\Delta^2[R(\theta)F'(\theta)] + L\Delta^4[R(\theta)F'(\theta)] +, \text{ etc.}}{R_0(\theta)F'(\theta) - K_0\Delta^2[R_0(\theta)F'(\theta)] + L_0\Delta^4[R_0(\theta)F'(\theta)] +, \text{ etc.}} \qquad (24)$$

where K and L, and K_0 and L_0 are given by equations (17), (15), (10), and (4) in terms of the constant slit-width b of the telescope and the slit-width of the collimator, respectively taken as a and a_0 for the test and standard sources.[1]

In equation (24) the function $f(\theta)$, giving the distribution of energy in the pure spectrum of the test source, is expressed in terms of known or measurable quantities. The function $f_0(\theta)$ for the standard source is presumably known; $R(\theta)$ and $R_0(\theta)$ are the ordinary spectro-photometric ratios where the substitution method is employed; $F'(\theta)$, the luminosity of the impure spectrum of the

[1] Various investigators have found indications of a lack of proportionality between the apparent flux of light through the collimator slit of a spectro-photometer and the width of the slit, even though the slit is of the Vierordt type having bilateral motion. From the theory developed in this paper such a result is to be expected for all wave-lengths except in those regions of the spectrum where the impure luminosity-curve is a straight line. But Murphy (*Astrophysical Journal*, 6, 9, 1897) finds lack of proportionality even at these wave-lengths, and ascribes the discrepancy to diffraction. In order to clear up this point very careful measurements were made at the wave-length $\lambda = 0.57\mu$ at which for our conditions the impure luminosity-curve is a straight line. An extended source (the illuminated magnesium carbonate block) was used, and readings were made for slit-widths ranging from 0.05 mm up to 0.4 mm, the observed luminosity being maintained constant by the variable sectored disk. Under these carefully prescribed conditions there was an apparent lack of proportionality if the drum readings were taken as giving the slit-width; but a careful calibration of the slit with a micrometer microscope showed the drum readings to be in error by such an amount that the proportionality was established when the corrected slit-widths were taken.

comparison source, remains to be determined. The curve representing this function may be obtained by a direct measurement, using either the spectro-photometer (at least this measurement can be made in the Schmidt & Haensch form of the Lummer-Brodhun spectro-photometer) or some other suitable instrument, or it may be computed from the known energy-curve of the source, together with the dispersion-curve of the spectro-photometer prism and the proper sensibility-curve of the eye. The requisite accuracy in the determination of $F'(\theta)$ will be discussed later in considering numerical results.

C. APPLICATION TO SPECTRO-PHOTOMETRIC MEASUREMENTS

In applying equation (24) to spectro-photometric measurements it is assumed that the energy distribution $f_0(\theta)$, in the pure spectrum of the so-called standard source, the luminosity distribution $F'(\theta)$ in the impure spectrum of the comparison source, and the luminosity ratios $R(\theta)$ and $R_0(\theta)$ are known continuous functions. It is further assumed that $R(\theta)$ and $R_0(\theta)$ were determined under the prescribed condition that $F'(\theta)$ remained constant throughout the measurements, this condition implicitly involving the constancy of the two slit-widths b and a'. Supposing the constants K, K_0, L, and L_0 to have been evaluated, the application of equation (24) is made as follows:

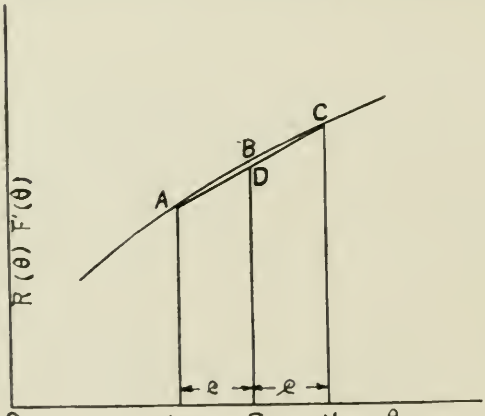

Fig. 2

Form the two functions $[R(\theta)F'(\theta)]$ and $[R_0(\theta)F'(\theta)]$ and plot them as functions of θ as illustrated in Fig. 2. If $\overline{BP}$ represents the value of this function for some value of θ given by OP, and if $\overline{AM}$

and $\overline{CN}$ represent values of the function corresponding to $\theta+c$ and $\theta-c$, where c is expressed in the same angular unit as θ, then

$$\Delta^2[R(\theta)F'(\theta)] = -2BD \tag{25}$$

In a similar way, from the curve representing the function $\Delta^2[R(\theta)F'(\theta)]$ the numerical values of $\Delta^4[R(\theta)F'(\theta)]$ are obtained. From the values of $\Delta^2[R(\theta)F'(\theta)]$ and $\Delta^4[R(\theta)F'(\theta)]$ the numerator of the right-hand member of equation (24) is computed for all values of θ; and in a quite similar way the denominator is evaluated. Hence $f(\theta)$ is known at once in terms of $f_0(\theta)$. If there were no slit-width correction equation (24) would be

$$f(\theta) = f_0(\theta)\frac{R(\theta)}{R_0(\theta)} \tag{26}$$

If we denote by δ the correction evaluated in the way outlined above, equation (24) may be written in this form:

$$f(\theta) = f_0(\theta)\frac{R(\theta)}{R_0(\theta)}(1+\delta) \tag{27}$$

D. NUMERICAL RESULTS FOR BLACK-BODY SPECTRA

As was stated in the introduction, the preceding theory was developed in order to determine with greater accuracy from spectro-photometric measurements the energy distribution in the spectrum of a black body at some temperature, say 2500° Abs., assuming as known the energy distribution in the spectrum of the black body at some other temperature, say 1500° Abs. The correction factors, δ, [equation (27)] have been evaluated for spectro-photometric intercomparisons of black-body spectra at different temperatures and under different conditions of collimator and telescope slit-width.

The results are plotted in Figs. 3 and 4. In Fig. 3 the abscissas are wave-lengths and the ordinates are values of δ expressed in per cent for the intercomparison of the spectrum of a black body at 1500° Abs. with that of a black body at the following temperatures: 1800° Abs. (curve a), 2000° Abs. (curve b), and 2500° Abs. (curve c), in each case under the slit-width condition that $a=b=0\overset{\circ}{.}1$ (=0.4 mm approximately). $f_0(\theta)$ and $R_0(\theta)$ refer to the black

body at 1500° Abs. and $f(\theta)$ gives the pure energy distribution for the black body at the higher temperatures. By reference to curve c it is seen at once that the relative intensity of energy emission in wave-lengths 0.5 μ and 0.66 μ in the case of a black body at 2500° Abs. as determined from spectro-photometric comparison with a

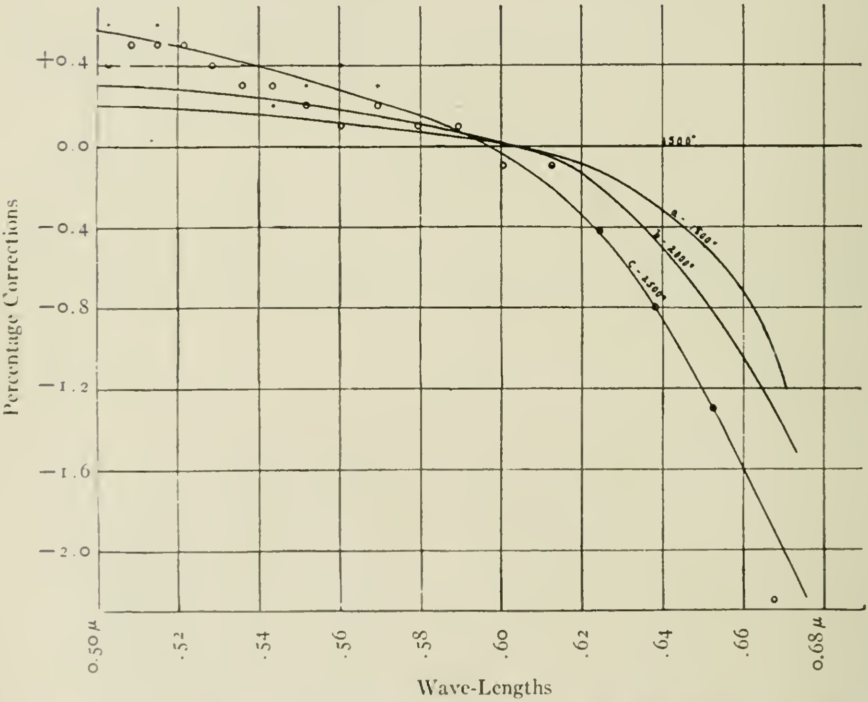

FIG. 3.—Percentage corrections to energy-curves of a black body at various temperatures as determined through comparison with that of a black body at 1500° Abs., when the collimator and telescope slits are each equal to 0°.1 (0.4 mm approximately).

+ Data based on visibility-curve of A. W. (Some of these defective on engraving.)
o Data based on visibility-curve of A. K.

black body at 1500° Abs. is in error by 2.2 per cent ($\delta = +0.5$ per cent at $\lambda = 0.5\,\mu$, and $\delta = -1.7$ at $\lambda = 0.66\,\mu$) and needs to be corrected by that amount. Similar but smaller errors enter for intercomparisons at smaller temperature differences but even when the temperature differences are only 400° or 500° the corrections are

appreciable and should be allowed for in accurate work where the attempt is made to keep the errors within one per cent.

The values of δ plotted in Fig. 3 are for the case where the collimator and telescope slit-widths are equal to each other and each is equal to $0°.1$ (=0.4 mm, approximately). As either slit-width is increased the values of δ also increase. Thus Fig. 4 shows

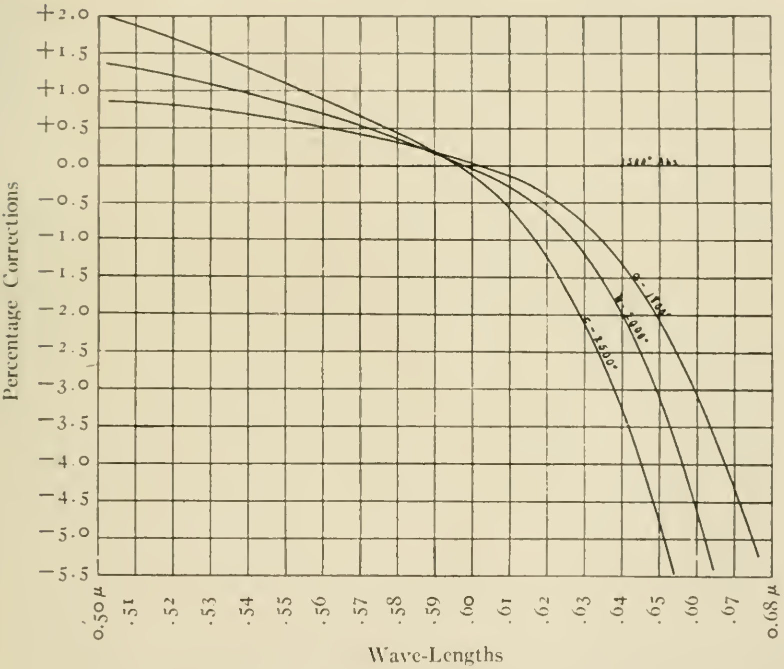

FIG. 4.—Percentage corrections to energy-curves of a black body at various temperatures as determined through comparison with that of a black body at 1500° Abs., when the collimator and telescope slits are each equal to $0°.2$ (0.8 mm approximately).

curves similar to those given in Fig. 3, except that the slit-widths are twice as wide, each equaling $0°.2$ or 0.8 mm, approximately. The values of δ, and hence the errors under these circumstances are about four times as large as the corresponding values of δ for $a=b=0°.1$. In intercomparing the spectra of two black bodies at 1500° and 2500° Abs. the relative errors between the two wave-lengths $\lambda=0.5\,\mu$ and $\lambda=0.66\,\mu$ amount to approximately 9 per

cent. Even when the temperature difference is only 200° or 300° the correction becomes appreciable in accurate work when the slit-widths are as large as 0.8 mm.

When $a+b$ remains constant, but the relative values of a and b change, the values of δ again undergo a change. Thus, consider

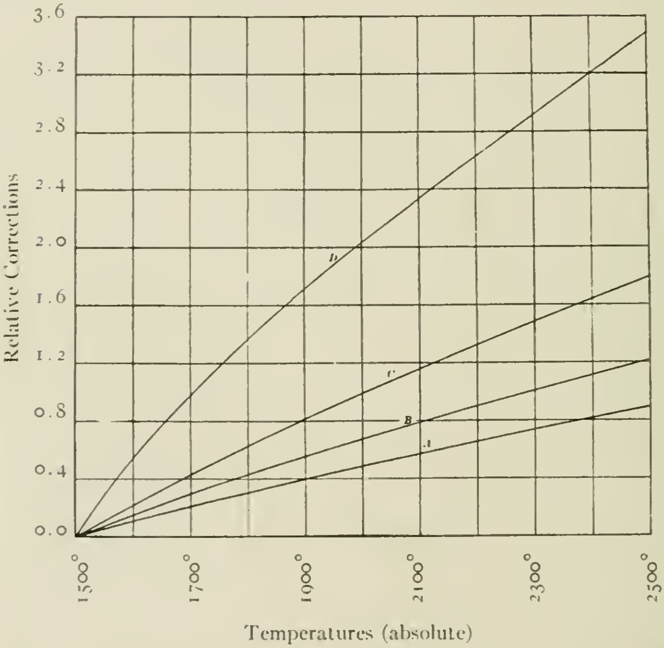

FIG. 5.—Relative corrections at $\lambda=0.625\ \mu$ and $\lambda=0.5\ \mu$ to the energy-curves of a black body at various temperatures obtained through comparison with that of a black body at 1500° Abs.

Curve A, $a+b=0.2$; $\frac{b}{a}=1$ Curve C, $a+b=0.2$; $\frac{b}{a}=\infty$

Curve B, $a+b=0.2$; $\frac{b}{a}=4$ Curve D, $a+b=0.4$; $\frac{b}{a}=1$

the errors in the relative emission at the two wave-lengths $0.625\ \mu$ and $0.5\ \mu$ for various temperatures and various values of a and b. Data of this nature are given in Fig. 5 in which the ordinates are $\delta_{0.625\mu}-\delta_{0.5\mu}$ and the abscissas are temperatures between which and 1500° Abs. the comparisons are made. Curves are plotted for the following cases:

Curve *A*; $a+b=0°.2$; $\frac{b}{a}=1$

Curve *B*; $a+b=0°.2$; $\frac{b}{a}=4$

Curve *C*; $a+b=0°.2$; $\frac{b}{a}=\infty$

Curve *D*; (for comparison) $a+b=0°.4$; $\frac{b}{a}=1$

It is thus seen that as the ratio $\frac{b}{a}$ differs from unity, $a+b$ remaining constant, the error increases until $\frac{b}{a}=\infty$ when the error is about twice that at $\frac{b}{a}=1$. Curve *D* for $\frac{b}{a}=1$ but $a+b=0°.4$ is given for comparison.

If the curves of Fig. 5 had been plotted for the two wave-lengths $0.66\,\mu$ and $0.5\,\mu$ instead of $0.625\,\mu$ and $0.5\,\mu$, the corrections would have been nearly twice as large (see Figs. 3 and 4), but the uncertainty in the corrections makes it advisable to avoid the extreme wave-lengths at the red end of the spectrum. In this region the rapid change in the sensibility-curve combined with the increasing energy and the relatively small dispersion makes the correction relatively large and difficult of evaluation.

From a consideration of the data given in the three preceding figures it is evident that quite appreciable errors may occur in the determination of the energy-curve of a black body at one temperature from the energy-curve of a black body at another temperature by means of spectro-photometric comparisons, since it is not unusual that the slit-widths of the collimator and telescope are of the order of magnitude of 0.4 to 0.8 mm.

It is to be noted that when the energy-curve of a black body at any temperature is determined from spectro-photometric comparison with a black body at a *lower* temperature the correction for impurity of spectrum is always such as to *increase* relatively the *blue* end of the computed energy-curve.

The question of the accuracy to which $F'(\theta)$ must be known was raised in an earlier part of the paper. An answer to this is found in curve *c*, Fig. 3. The points plotted thus, "o," were obtained on

the basis of one $F'(\theta)$ curve, and the points plotted thus, "+," were obtained on the basis of another somewhat different curve of $F'(\theta)$. These two curves of $F'(\theta)$ are shown in Fig. 6, and it is evident that the curve of $F'(\theta)$ need not be determined with extreme accuracy

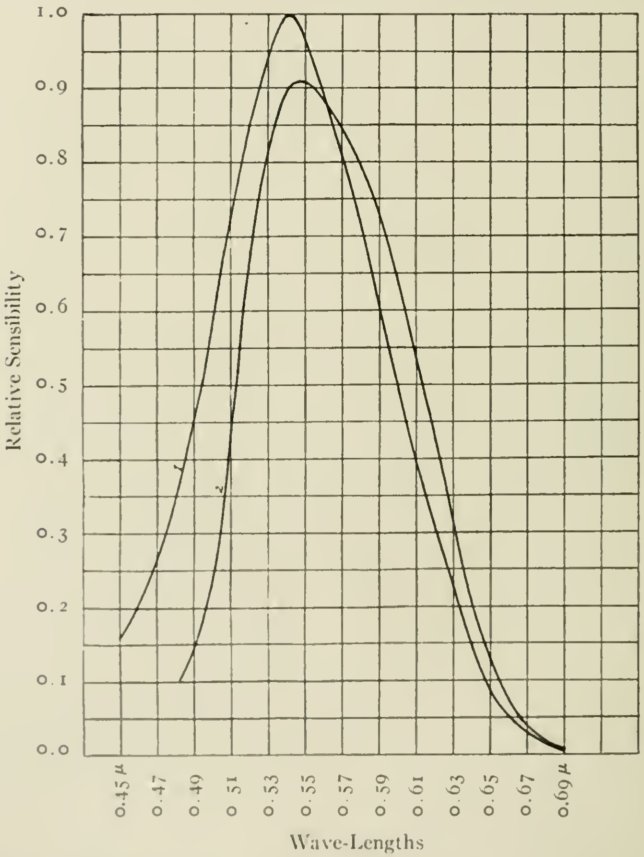

FIG. 6.—Sensibility-curves corresponding to luminosity distribution in spectrum of a black body at 1500° Abs.

Curve 1, Observer K. Curve 2, Observer W.

in order to compute the values of δ. The reason for this is to be found in the use of the same $F'(\theta)$ curve in both the numerator and denominator of equation (24). On the other hand, small errors in the two curves representing $R(\theta)$ and $R_0(\theta)$ render difficult the

determination of $\Delta^2[R(\theta)F'(\theta)]$, $\Delta^2[R_0(\theta)F'(\theta)]$, $\Delta^4[R(\theta)F'(\theta)]$, etc. The numerical data given in this paper were computed assuming the Planck equation for spectral energy distribution. In correcting the relative energy-curves of two incandescent solids with spectra similar to those of a black body, it is probable that higher accuracy

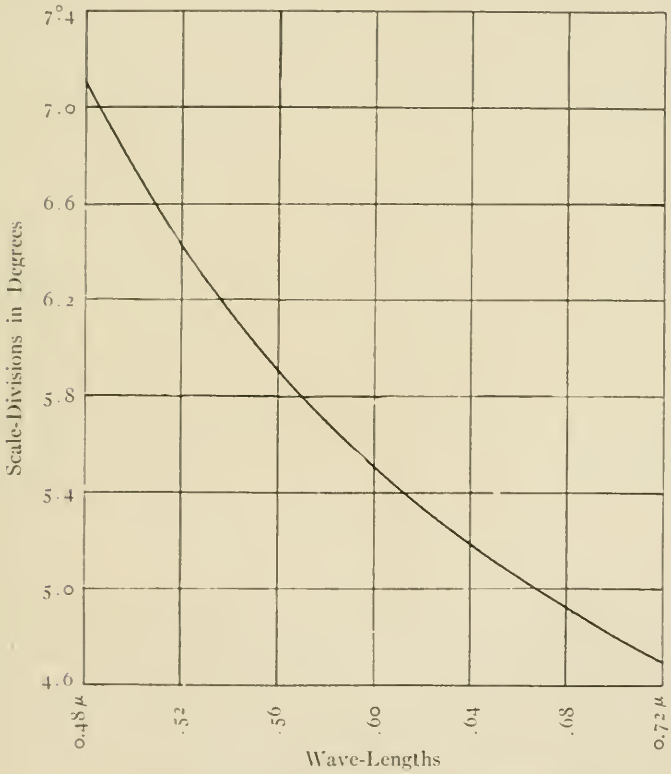

FIG. B.—Calibration-curve of dispersing prism

will result if the correction terms are approximated from the curves given in this paper for a black body, rather than if an attempt is made to compute them directly from the observed ratios $R(\theta)$ and $R_0(\theta)$.[1] Thus, if one incandescent solid has a luminosity-curve

[1] In order that anyone may be better able to determine the applicability of the numerical results given in this paper to his work the calibration-curve of the dispersing prism is given in Fig. B.

quite similar to that of a black body at 1800°, and another solid has a curve similar to that of a black body at 2300°, the corrections will be very nearly the same as those for two black bodies at 1800° and 2300°.

Although all the data given in the various curves of the different figures were computed rather than observed, the following experiment gives a general experimental confirmation of the deductions. If two black-body spectra, one corresponding to a black body at 2100° or 2200° Abs. and the other to a black body at 1700° or 1800° Abs., are intercompared by the substitution method, first, when the collimator and ocular slits are each 0.4 mm and then when they are each 0.8 mm, the comparison spectrum remaining constant, the spectro-photometric ratios under the two circumstances should be different by an amount readily discernible. Thus, if R_{λ_1} is the ratio of the luminosity of the black body at the high temperature to that at the low temperature for $\lambda_1=0.515\,\mu$ and for slit-width $a=b=0.4$ mm; R_{λ_2} the corresponding ratio for $\lambda_2=0.67\,\mu$; R'_{λ_1} the similar ratio for $\lambda_1=0.515\,\mu$, and $a=b=0.8$ mm; and R'_{λ_2} the corresponding ratio for $\lambda_2=0.67\,\mu$; then it should be found according to the theory that

$$\frac{R_{\lambda_1}}{R_{\lambda_2}}=\frac{R'_{\lambda_1}}{R'_{\lambda_2}}\ (1+4 \text{ per cent or } 5 \text{ per cent}).$$

For the two black-body spectra, that of a tungsten lamp at relatively high voltage, and that of a carbon lamp at relatively low voltage, were used, the voltages being chosen to approximate the stated conditions of temperature. The comparison spectrum was maintained constant both in quality and in intensity by making the comparison collimator slit 0.8 mm wide when the ocular slit was 0.4 mm, and 0.4 mm wide when the ocular slit was 0.8 mm. Under these conditions two different observers made measurements which showed the ratios $\frac{R_{\lambda_1}}{R_{\lambda_2}}$ and $\frac{R'_{\lambda_1}}{R'_{\lambda_2}}$ to be different in the direction and by the approximate amount indicated in the above equation.

E. APPLICATION OF SLIT-WIDTH CORRECTIONS IN TEMPERATURE MEASUREMENTS

It is sometimes convenient, as in the author's work on selective radiation, to compute the temperature at which a black body would have a given curve of energy distribution in the visible spectrum, in cases where this energy-curve has been obtained through spectro-photometric comparisons from a known distribution of energy, as of a black body at some known temperature. The magnitude of the error in the computed temperature owing to the error due to impurity of spectrum can readily be evaluated. From Wien's Law

$$J=\frac{C_1}{\lambda^a e^{\frac{C_2}{\lambda T}}} \tag{28}$$

there is obtained on differentiation

$$\frac{dJ}{J}=\frac{C_2}{\lambda T}\quad\frac{dT}{T} \tag{29}$$

For a given percentage change in temperature, $\frac{dT}{T}$, the relative change in intensity at λ_1 compared with the change at λ_2 is given by the equation

$$\left(\frac{dJ}{J}\right)_{\lambda_1}-\left(\frac{dJ}{J}\right)_{\lambda_2}=\frac{C_2}{T}\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)\frac{dT}{T}\,. \tag{30}$$

Conversely, if the relative change at λ_1 compared with that at λ_2 is known, i.e., if $\left(\frac{dJ}{J}\right)_{\lambda_1}-\left(\frac{dJ}{J}\right)_{\lambda_2}$ is known, the corresponding change in temperature can be computed. Applied to the slit-width errors

$$\left(\frac{dJ}{J}\right)_{\lambda_1}-\left(\frac{dJ}{J}\right)_{\lambda_2}=\delta_{\lambda_1}-\delta_{\lambda_2} \tag{31}$$

the relative errors at the two wave-lengths λ_1 and λ_2. Hence the percentage error in temperature owing to the slit-width errors is given by the equation

$$\frac{dT}{T}=\frac{\lambda_1\times\lambda_2}{\lambda_2-\lambda_1}\cdot\frac{T}{C_2}(\delta_{\lambda_1}-\delta_{\lambda_2})\,. \tag{32}$$

Thus, suppose λ_1 and λ_2 are taken as $0.5\,\mu$ and $0.66\,\mu$ respectively, and that the intercomparison is made between the spectrum of a

black body at 1500° Abs. and that of a black body at 2500° Abs., the slit-widths being $a=b=0°.2$. From Fig. 4, the values of δ are $\delta_{0.5}=+2.0$ per cent and $\delta_{0.66}=-7.0$ per cent so that $\delta_{\lambda_1}-\delta_{\lambda_2}=9.0$ per cent. Moreover, $C_2=14{,}500$ so that equation (32) becomes

$$\frac{dT}{T}=\frac{0.66\times 0.5}{0.66-0.50}\times\frac{2500}{14500}\times 0.09=0.032\,, \tag{33}$$

or 80° in 2500°, a quite appreciable quantity. Hence if the temperature of a black body at 2500° were computed from the spectrophotometric ratios obtained through intercomparison under the stated conditions with a black body at 1500°, it would be found to be 2420° instead of 2500° unless the corrections for slit-width were applied.

It is to be noted that the temperature correction is always in such a direction as to increase the observed difference. Thus, the true temperature is higher than that observed from spectrophotometric comparison with a black body at a lower temperature, and conversely the true temperature is lower than that observed from spectro-photometric comparison with a black body at a higher temperature.

F. SUMMARY

1. Owing to the impurity of the spectrum due to the finite widths of the collimator and ocular slits, it is necessary to correct the relative luminosity-curves of two sources compared in a spectrophotometer in order to determine the true relative energy-curves of the sources.

2. This slit-width correction is most readily applied when the substitution method of measurement is employed, and the comparison spectrum is maintained constant.

3. The correction is less as the slit-widths are less and for any given sum of the two slit-widths, $a+b$, the correction is least where $a=b$.

4. Applied to spectra of the black-body type, the correction is appreciable in accurate work even though the temperature difference between the two sources is only a few hundred degrees, and the slit-widths are quite favorable ($a=b=0.4$ mm). The tem-

perature correction is always in such a direction as to increase the observed difference of temperature.

5. In determining the temperature of a black body at about 2500°, from spectro-photometric comparison with a black body of known temperature of about 1500°, an error of 80° may result if the slit-widths are $a=b=0.8$ mm and the wave-lengths at which the comparison is made are $\lambda_1=0.5\,\mu$ and $\lambda_2=0.66\,\mu$.

PART II. A NEW FORM OF VARIABLE ROTATING SECTORED DISK FOR USE IN SPECTRO-PHOTOMETRY

A. INTRODUCTION

It has been recognized for a long time that in spectro-photometric comparisons, as well as in ordinary photometric measurements, more accurate results in general are attainable if the method of substitution is employed. Any dissymetry in the instrument or any personal idiosyncrasy of the observer is much less productive of error when the two spectra to be compared are formed successively by precisely the same optical system, and are viewed in precisely the same relation to the comparison spectrum. But heretofore apparently no attention has been paid to the necessity of maintaining the comparison spectrum absolutely constant throughout the measurements. It has apparently been a matter of indifference whether the variable sectored disk is placed on the side of the comparison source or on the other side, where the two sources whose spectra are to be compared are successively mounted.

The theory developed in the preceding paragraphs of this paper indicates, however, the necessity of maintaining the comparison spectrum absolutely constant throughout the measurements. The correction for impurity of spectrum in its simplest application involves the luminosity-curve of one of the sources, and further assumes that the luminosity of the comparison spectrum remains constant throughout the measurements. If the luminosity-curve of the comparison source is determined once for all—a rather tedious task under any circumstances—it may be used in much of the spectro-photometric work involving continuous spectra of the general type given by a black body at various temperatures.

If the comparison spectrum is to be maintained constant, however, it becomes necessary to secure the photometric balance by varying the intensity of the test source. If a variable rotating sectored disk is used for this purpose,[1] it must be capable of accurate adjustment over a wide range of openings, particularly at small openings. One is ordinarily more interested in obtaining accurate luminosity values in the region of the spectrum where the luminosity is relatively high. If the sectored disk is used on the comparison side the readings of relatively high luminosity of the test source are made with the sectored disk set at a large angular opening, whereas the readings of relatively low luminosity are obtained by reducing the sectored disk opening to somewhat smaller values. If, on the other hand, the sectored disk is used on the test side the reverse is the case. It hence becomes necessary at times to determine accurately very small transmissions.

The two types of variable sectored disk available for accurate work are the elaborate Lummer-Brodhun sector and the very ingenious sector used in the Brodhun "Strassenphotometer." The former is quite complicated and expensive, whereas the latter is comparatively simple and cheap. But in both forms of disks as regularly designed it is most difficult to make accurate settings at small angular openings because very small angular motions at small openings produce relatively large changes in transmission. Thus, an angular motion of 1° corresponds to a change of 1 per cent in transmission when the total opening is 100°, whereas the same angular motion corresponds to a change of 10 per cent in transmission when the total opening is 10°. On this account it is evidently difficult to make accurate settings at small transmissions of the disk even if the accuracy of the sector openings is not questioned.

As has been explained, when measurements are being made with due consideration of the corrections for impure spectrum, it is frequently necessary to measure small transmissions with high accuracy. It therefore seemed desirable to devise, if possible, a new

[1] The assumption is made that the variable rotating sectored disk gives in general the best results of any of the various methods that are employed for accurately measuring the intensity corresponding to a balance in spectro-photometric comparisons.

form of variable rotating sectored disk which should meet all the requirements demanded in the most accurate spectro-photometric comparisons. Such a disk has been designed and constructed, and has now been used for a considerable time with entire satisfaction.

B. DESCRIPTION OF NEW DISK

The principle on which the new form of disk has been designed is the utilization of openings whose edges are not radial. In all other disks which have been used[1] the angular openings are the same at all radial distances from the center. This is necessary if the beam of light to be intercepted is wide, as in ordinary photometry. But when the width of the beam is reduced to the width of the ordinary slit of a spectro-photometer, the necessity of radial sectors is eliminated. The form of sector which has been found very satisfactory in the new disk is that shown in Fig. 7. At the radial distance a, the total angular opening is approximately 324°, corresponding to a transmission of 90 per cent. At a radial distance b the transmission is zero. Between these two limits all intermediate transmissions are obtained. If then, in some way the disk is made to move back and forth along the line $\overline{CD}$, the slit $\overline{SS}$ of the spectro-photometer collimator can be made to come at any radial distance and hence at any percentage transmission of the disk. This motion is accomplished by mounting the disk-bearings on a carriage which is moved along ways by means of a screw which is supplied with the necessary scales for reading the position of the disk. The

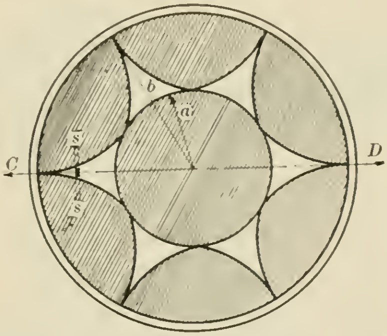

FIG. 7.—Variable sectored disk for spectro-photometer.

[1] Brace devised a rotating sector to be used in calibrating the Vierordt slit of his spectro-photometer which had a number of different angular openings at different radial distances. The changes in angle were made in steps, however, so that for a given step of the disk, say between radii r_3 and r_4, the edges were straight and radial, though the angular opening was not as large as for the next step, between radii r_4 and r_5.

ways are mounted on a casting which is rigidly attached to the base of the spectro-photometer. A photograph of the complete apparatus is shown in Fig. 8.

In an early form of the disk the edges of the sector openings were straight lines, but it was found on computation that the transmission gradient for different radial distances was quite non-

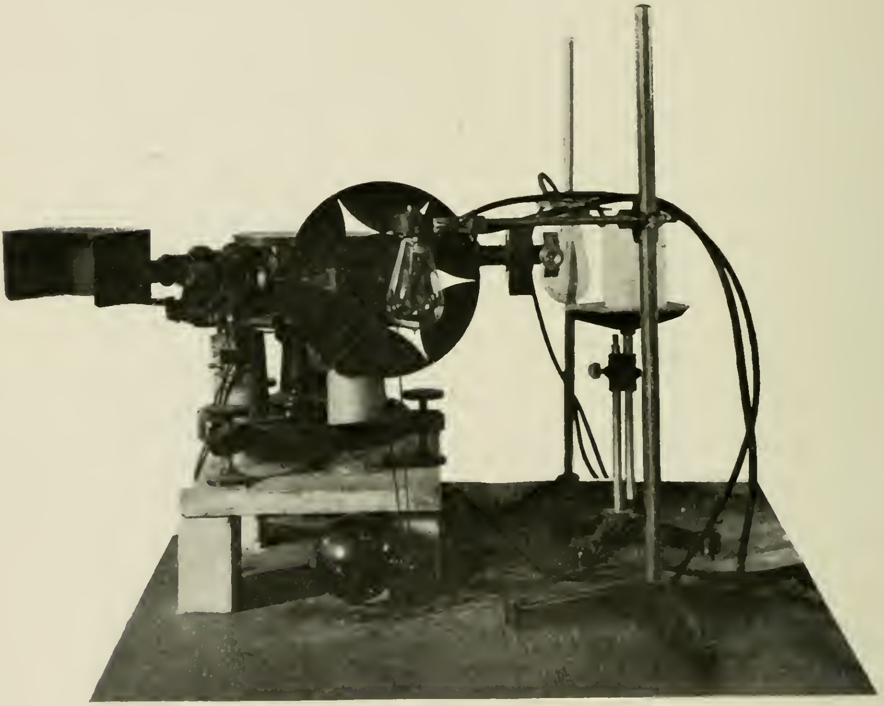

Fig. 8.—The complete apparatus

uniform, particularly near the periphery of the disk, when a slight linear motion of the disk produced a much larger percentage change in transmission than at smaller radial distances.

Since it is desirable, as explained previously, to have the transmission gradient in this region as nearly as possible the same as that in the center of the disk a modified design was sought. Of course it is possible, theoretically, to construct a curve for the edges of the sectors such that the transmission gradient shall be absolutely

constant, but this extreme condition is not necessary. It was found by trial that if the edges of the disk are made arcs of a circle of properly chosen radius the desired end can be approximately attained.

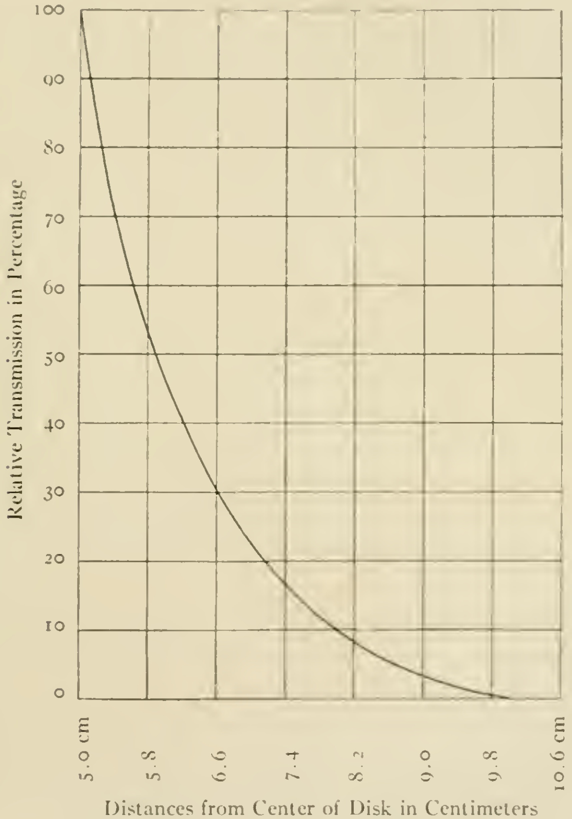

FIG. 9.—Computed transmission-curve for variable sectored disk

In the disk which is shown in the photograph the radius a is 5 cm and the radius b is 10 cm. For this disk it was found by computation that the most satisfactory results are obtained when the edges of the sectors are arcs of a circle of 10 cm radius. The computed transmission-curve for this disk is shown in Fig. 9 in which the abscissas are linear distances measured from the center of the disk and the ordinates are relative transmissions in terms of the

maximum transmission taken as unity. The curve in which the interest centers, however, is that giving the percentage change in transmission at different relative transmissions corresponding to a linear motion of the disk of some given amount, say 0.1 mm. Such a curve is shown in Fig. 10.

From this curve it is seen that near the center of the disk, i.e., for radial distances of about 6 or 7 cm, a linear motion of 0.1 mm corresponds to a percentage change in transmission of approximately 0.75 per cent. This relative transmission gradient remains fairly constant over a large range of distance or over a range of relative transmissions from about 10 per cent to 100 per cent. At very low transmissions, however, the transmission gradient becomes somewhat steeper, so that at a relative transmission of 5 per cent a

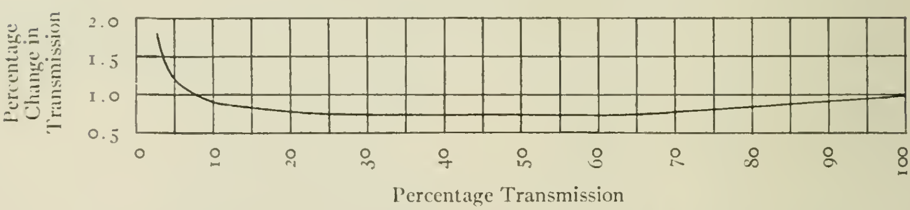

FIG. 10.—Percentage change in transmission corresponding to a linear displacement of 0.1 millimeter.

linear motion of 0.1 mm produces a change in transmission of 1.3 per cent (i.e., 1.3 per cent of 5 per cent) and at 2 per cent transmission it produces a change of 1.7 or 1.8 per cent (i.e., 1.7 or 1.8 per cent of 2 per cent). In other words, throughout the entire range of transmission, from 2 per cent to 100 per cent (relative values) the relative transmission gradient remains of the same order of magnitude, the extreme values being in the ratio of about 2:1. To appreciate the significance of these figures it is only necessary to note that for a corresponding range of relative transmissions for the ordinary types of disks the corresponding ratio of the extreme transmission gradients is about 50:1.

The data given in the above curves were computed on the assumption that the disk was constructed precisely according to specifications. It may be of interest to those who might desire to construct a similar disk to know how the disk was actually con-

structed, and to what accuracy it fulfilled the specifications. The construction of the disk was made possible by the application of a device previously employed in our laboratory in the construction of ordinary disks with sectors having radial edges. This device consists in making the edges separately and mounting them subsequently on the body of the disk which has had cut in it openings slightly larger than those desired in the finished disk. The separate edges can be ground straight or made of any desired shape and then screwed on to the disk body, using a templet in mounting them so as to secure the greatest uniformity and accuracy.

This scheme was employed in constructing the variable sector. The body of the disk was made of aluminium, and the edges were turned out of hard brass, great care being exercised to keep the ends of the individual edges sharp and free from burr, in order that the two edges of each sector could be mounted accurately in contact on the disk body. This precaution is most important. The edges were beveled to prevent reflection, blackened, and then mounted with the aid of a specially designed templet. The completed disk was then attached to a shaft mounted with ball bearings on a carriage, which is movable along a V-shaped track by a carefully ground screw. The position of the disk is read off on a millimeter scale in conjunction with a micrometer attached to the screw-head. The disk is moved back and forth slowly in making a setting by means of the knurled screw-head; but for rapid motion a small crank attached to the shaft of a gear-wheel is provided. The position of the disk can be read to 0.01 mm, and the backlash is negligibly small. The disk is driven by means of a pulley and belt from a small motor placed in a stationary position on the table under the disk.

C. CALIBRATION OF DISK

The disk was calibrated in position by comparison with a number of standardized sectored disks of the ordinary type with fixed openings. Taking the scale-reading of the variable sector for equal transmission with one of the standard disks, say a 90° disk, the scale-readings for other transmissions were obtained, and these were compared by reference to Fig. 9, with the readings that would

have been obtained if the disk had conformed precisely to the specifications. The differences between the observed calibration-curve and the computed curve were found to be too small to be shown definitely on the scale to which the curve in Fig. 9 is drawn. Over about one-half the scale near the middle the differences between the observed and the computed curves were less than 1 per cent, and only at the ends of the scale, i.e., at extreme transmissions, were the errors as large as 1.5 or 2 per cent. Expressed in another way, from the maximum transmission of the disk, which we may call 100 per cent, to a transmission of 5 per cent, on the same relative scale, the difference between the observed and computed curves does not at any point exceed 1.5 per cent of the transmission at that point. The calibration was carried down to 2 per cent or 3 per cent transmission. Even at this minimum transmission the correction was found to be only 2 per cent or 3 per cent of the transmission at this point. The mechanical accuracy obtained by Mr. Würth, the mechanician of the laboratory, in the construction of the disk is thus seen to be very great when one considers the actual magnitude of the sector openings at the low transmissions. Of course even the small errors observed are allowed for in plotting the calibration-curve on a sufficiently large scale, as has been done for our own use.

D. POSSIBLE SOURCES OF ERROR

It is evident at once that a disk of this form is adapted only to those cases where the beam of light to be intercepted is of small dimensions compared with the dimensions of the disk. This is particularly true regarding the width of the beam, but even though the width may be small, one must be careful to see also that the height of the beam is not excessively great. In the case of the ordinary spectro-photometer the height of the collimator slit need give no concern; but the width of the slit needs some further consideration. However narrow the slit for all finite widths, the transmission through one side will be slightly different from that through the other, owing to the transmission gradient of the disk. The question naturally arises as to the effect of this on the indications

of the disk in practice, particularly as the slit-width may necessarily undergo change.

In considering this question it will be assumed that if a variable slit is used on the collimator it will be of the Vierordt double variety, so that the center of the slit may be considered fixed. It is then necessary to consider two cases. If the variable sector is mounted at some distance from the plane of the collimator slit the effect of the transmission gradient of the disk will be, primarily, to produce a non-uniform field, the degree of non-uniformity increasing as the disk is placed farther and farther from the slit. If the disk is mounted at any great distance from the slit, therefore, errors may arise in the indications of the disk, even though one may be able to make some kind of a setting notwithstanding the non-uniformity of the field.

If, on the other hand, the variable disk is placed as close as possible to the plane of the slit, the effect of the transmission gradient will be, not to produce a non-uniform field, but rather to permit relatively more energy of some wave-lengths and relatively less energy of other wave-lengths to penetrate the ocular slit, as compared with the energy of that wave-length in which the image of the collimator slit is formed in the center of the ocular slit. This effect would theoretically affect the luminosity values obtained by the use of the sectored disk, but under all practical conditions the error is negligibly small. Thus, by actual computation it was found that for a collimator slit of 0.4 mm and an ocular slit of 1.0 mm the error in the case of a black-body spectrum is of the order of magnitude of 0.1 per cent. By simple considerations it can readily be seen that under all ordinary conditions the error due to the transmission gradient of the disk is negligibly small. It is seen, therefore, that no appreciable error will result if the precaution is taken to mount the variable sectored disk as close as possible to the plane of the collimator slit. This distance in the case of our own instrument is only 3 or 4 mm.

The only other errors which might possibly enter are that due to back-lash in the motion of the carriage along the screw, and that due to a possible lateral shift of the collimator slit with reference to the disk mounting. It has already been stated that the

former error is negligibly small in the instrument in use in this laboratory. The latter error is mentioned, not because it demands any particular consideration, but rather because it suggests a slight modification in the construction of the Lummer-Brodhun spectro-photometer. In our instrument of this type, made by Franz Schmidt & Haensch, the collimator farther from the telescope is rigidly mounted on the base, but the nearer collimator is adjustable, though without scale for measuring the angular displacements. This flexibility is provided presumably to permit bringing the two spectra in exact juxtaposition. It is useful, however, in measuring the luminosity-curve of a source, though its value in this regard would be greatly enhanced if the position of the movable collimator could be read off a scale.

Another decided improvement would result if the present stationary and movable collimators were interchanged. It is necessary to have the variable sector mounted before the collimator nearer the telescope, but it would be much better if this collimator were rigidly attached to the base of the instrument, and the other collimator were adjustable with an attached index and divided circle to give its position. It would then be much easier to determine a luminosity-curve using the variable sectored disk in making the settings.

E. ADVANTAGES OF NEW TYPE OF DISK

The various features of this new type of disk which should be emphasized as distinct advantages over one or another of the rotating sectored disks at present available, or as valuable characteristics of all satisfactory variable sectors, may be briefly summarized as follows:

1. No selectivity in the transmission.

2. A range of transmission from above 85 per cent to almost 0 per cent. The maximum transmission obtained in any other variable sector within the author's knowledge does not exceed 50 per cent.

3. Continuous and easy adjustment over the entire range of transmission.

4. Applicability in infra-red spectrometry. The absence of

glass which absorbs strongly in the deep infra-red makes this disk peculiarly suitable for measurements of this kind.

5. A nearly uniform relative transmission gradient so that a given angular displacement of the screw-head operating the disk produces approximately the same percentage change in transmission throughout the entire range, except for slight variations at very small transmissions.

6. Because of the properties stated in the preceding paragraph the disk is particularly adapted to spectro-photometric measurements to which the correction for impurity of spectrum is to be applied.

In conclusion the author desires to express his indebtedness to various members of this laboratory for valuable services rendered in connection with the investigation. He is particularly indebted to Dr. C. F. Lorenz and Mr. F. E. Cady for much assistance in the calibration of the variable sectored disk, and to Dr. A. G. Worthing for most of the observations and computations in connection with the slit-width corrections.

Physical Laboratory
National Electric Lamp Association
Cleveland, Ohio
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PRESSURE-SHIFT OF SPECTRAL LINES

By W. J. HUMPHREYS

An interesting paper by Sanford[1] has recently appeared with the above title. It is particularly interesting and valuable because of the relations he finds to exist between certain properties of the alkali elements and the shifts of their spectral lines—relations from which he concludes: "Taken all together, there accordingly seems to be a considerable amount of evidence to show that the pressure-shift of spectral lines is related to just those properties of the atom which seem to depend upon the specific inductive capacity."

It was long ago pointed out by the author[2] that: "The shift of similar lines is a periodic function of atomic weight, and consequently may be compared with any other property of the elements which itself is a periodic function of their atomic weights." If this statement is really true, and observations seem still to support it, it follows that the pressure-shift is related to many, possibly all, other properties of the atoms. It is related, not only to just (precisely) "those properties of the atom which seem to depend upon the specific inductive capacity," but also and equally well to just (precisely) all other properties of the atom which, like the above, are periodic functions of the atomic weight. There is therefore no surprise in the relations referred to; in fact three of the six given were mentioned in the paper cited above, but it is important to know as many relations as possible and exactly what they are. Nevertheless, in spite of their importance, it is probable that all such relations are quite too general to show definitely what it is that produces the pressure-shift of spectral lines. Some other test must be found.

The theory that assumes the pressure-change in wave-length to depend upon change in the specific inductive capacity is capable of further development and possibly of a crucial test. But there

[1] *Astrophysical Journal*, **35**, 1, 1912.

[2] *Ibid.*, **6**, 226, 1897.

is at least one serious objection that clings to every form in which this theory has been presented: it does not account for any spreading of the line toward the side of shorter wave-lengths.

Sanford recognizes this difficulty, but attempts to obviate it on the assumption that air molecules weaken the fields of metallic molecules. "If this point of view is correct," he says, "it would seem that the greatest shift toward the red should be due to those atoms in the center of the arc, while the greatest shift toward the violet should be due to atoms around the outer surface."

But many lines give narrow well-defined reversals (due to absorption of light from the center of the arc by material in the outer layers) strongly shifted toward the red, with the rest of the line quite symmetrically spread to either side of the reversal. Maximum emission from the center of the arc and maximum absorption by the material surrounding it seem to coincide in the case of symmetrical lines, whatever the pressure. Fine non-reversed lines of this type, due to traces of the substance in the arc, appear always, under the same pressure, to have the same wave-length as their reversals, though the one must be due essentially to conditions within the arc and the other to conditions in the region immediately surrounding it; nor does this relation of fine lines to their reversals, or even the total shift greatly, if at all, depend upon the material of the electrodes. Hence it appears that the suggested way out of the difficulty is not sufficient.

At one place in his article Sanford says: "All three of the abovementioned theories [the forced vibration theory, the specific induction theory, and the induction or magnetic theory] make the change in the period of the vibrating mechanism depend upon the proximity of atoms of its own kind, since the greater the atmospheric pressure around the arc the denser must be the metallic vapor in the interior of the arc."

This statement is true enough when the correct interpretation is put upon the expression "its own kind." It is not true, seemingly, that the displacements of iron lines or copper lines, for example, are dependent upon the density of iron or of copper vapor respectively, since lines due to impurities in the arc, as already explained, are displaced equally with the same lines under the same pressure,

whatever the quantity of material present. "Same kind," therefore, should be understood as meaning same in only a very broad sense. According to the magnetic theory it means only that the atoms have individual magnetic fields, and it has nothing to do with what particular element or elements they consist of.

Sanford holds that the magnetic theory of the pressure-shift may be tested by its relation to the magnetic properties of the elements, which properties he says are already known. On applying this test he naturally finds the theory to fail, since the pressure-shifts of the lines due to such strongly magnetic substances as iron, nickel, and cobalt are less than the shifts of lines due to many of the non-magnetic elements.

The author admits of course all these facts, but he holds that they are not conclusive, nor even applicable. It is not the magnetic properties of *cold masses* of the pure elements that are here needed—presumably the ones Sanford had in mind—but the magnetic properties of their *atoms when luminous*.

Now the Zeeman effect shows that the luminous atoms of every element so far examined have magnetic fields of their own. How else could they be acted upon by an outside magnetic field? But it does not show that the iron atom, for instance, has any stronger magnetic field of its own than has a copper atom, and hence we cannot infer, on the magnetic theory, that it would be any more effective than the copper atom in producing pressure-shifts of spectral lines.

Another objection urged against the magnetic theory is the improbability of the powerful magnetic field it demands for the interior of the luminous particle. Frankly, this at first does seem staggering, but to some of us perhaps not one whit more so than does the enormous store of energy experiment demonstrates is locked up within the atoms of many elements—probably within the atoms of all elements. Nor is it any more staggering than the fact that a single atom of iron can give out thousands of independent vibrations at the same time; nor half so staggering as the further fact that it would take one hundred million people something like three hundred years to count the number of complete vibrations produced by an atom of iron in a single second of time. And even

this probably is far too small an estimate, for it leaves out of account the numerous satellites and other complexities of the individual lines; nor is the iron atom by any means the busiest atom known!

It is also interesting to know that the strength of the magnetic fields of the luminous atoms required to account for pressure-shift is of the same order of magnitude as that demanded for them by the theory of Ritz,[1] which, though of subsequent date, was developed from a different standpoint—a theory that explains many phenomena, and, in its general form, has received numerous adherents.

It is true that in details there is not a one-to-one connection between pressure-shift and the Zeeman effect; but in the general phenomena there is a close relation between them. Line spectra show both effects; band spectra, with but few known exceptions, show neither; both increase, in general, with increase of wavelength; both differ in magnitude with different elements, and also with different lines of the same element.

The Saturnian atom accounts for the general phenomena of the Zeeman effect, and it accounts too for the general phenomena of pressure-shift, nor does it demand that in every particular there shall be a one-to-one relation between the two phenomena. Atoms with weak fields, for instance, should show large magnetic separation, but not a correspondingly large pressure displacement due to their mutual interaction. But the complexity of the Zeeman phenomena is not yet fully accounted for by any theory; the structure and motions of the atom are not known. This much however seems certain:

a) The luminous particle, being acted upon by an outside magnetic field, has a magnetic field of its own.

b) Since the luminous particle is affected by an outside magnetic field it must also be affected by the magnetic fields of its neighbors.

On the whole the mutually attracting faces of such particles obviously will be closer together and produce a greater effect than the mutually repelling faces, and therefore, while the lines will be spread in both directions, their maximum intensities will be shifted toward the red to an extent that must increase with increase of pressure.

[1] A. Cotton, *Le Radium*, 8, 451, 1911.

But is this magnetic effect, which seems to be in the right direction and almost inevitable, of sufficient magnitude? The smallness of the Zeeman separation seems to indicate that it is, though there are many details, both experimental and theoretical, yet to be filled in before the magnetic or any other theory of the pressure displacement of the spectral lines can be fully tested.

In the meantime the author is extremely gratified to know that his tentative theory, which he is ready to modify or discard as the facts may direct, is fulfilling the true function of a theory: stimulating investigation and thereby helping to increase our store of knowledge.

U.S. Weather Bureau
Washington, D.C.
March 1912

ON "EARTH LIGHT," OR THE BRIGHTNESS, EXCLUSIVE OF STAR LIGHT, OF THE MIDNIGHT SKY

By W. J. HUMPHREYS

An attempt was made a few years ago by Professor Newcomb[1] to determine the total light of all the stars. The results were far from being in accord with expectations, especially in respect to the relative brightness of the galactic and non-galactic portions of the sky.

A provisional explanation of the difficulty was made by referring the total light to two sources: direct star light, and indirect or atmospherically diffused star light; but this did not prove to be a sufficient explanation—it did not fully account for the observed phenomena. "On the whole," Newcomb says, "it seems either that my observations are wholly at fault—erroneous by an amount which I should find it difficult to account for—or we must materially modify our conclusions from the combination of star gauges with the existing photometric estimates of star light."

More recently this whole subject has been investigated most carefully by Yntema,[2] who reaches the following conclusions:

1. "The light of the sky at night is composed of two parts, one reaching us directly from the stars, the other resulting from processes in the atmosphere."

2. "The latter, termed 'earth light,' is only partly due to the diffused star light. It seems probable that the rest, wholly or in part, is due to a permanent aurora."

Yntema also finds that:

a) "The general brightness of the sky, being measured by the relative brightness of the North Pole, is variable during the same night."

b) "It is varying from night to night."

c) "The observed brightness increases toward the horizon."

[1] *Astrophysical Journal*, **14**, 297, 1901.

[2] *On the Brightness of the Sky and the Total Amount of Starlight.* Groningen: Gebroeders Hoitsema, 1909.

Yntema's observations were made at Borger, Holland, August 1907—May 1908.

Still more recently, in August 1910, Abbot,[1] using an instrument similar to one of those used by Yntema—a Kapteyn sky photometer—made measurements, from the top of Mount Whitney, Cal., on two successive nights of the relative brightness of the night sky; and also of the total light per square degree in terms of a first-magnitude star.

The results obtained by Abbot on Mount Whitney at an elevation of 4420 meters agree in general with those of Yntema, which were obtained practically at sea level. They are, however, smaller in the ratio of 7 to 10, approximately.

The phenomenon of "earth light" seems therefore to be a thing of the high atmosphere, and to be a very general occurrence both as to time and place. Its variability and its increase with zenith distance seem to preclude attributing it wholly to any celestial or combination of celestial sources, such as diffused star light, reflection and scattering of sun light by meteoric particles and dust, or anything in the nature of the zodiacal light or the *Gegenschein*. On the other hand, Yntema's suggestion that it may be due wholly or in part to a permanent aurora—whatever the cause of this in turn—appears very plausible, and especially so since the green "auroral line," λ 5770, may be seen on almost any dark clear night in any part of the sky.[2] That it is a phenomenon essentially of the upper atmosphere, auroral or what not, seems probable.

If it is wholly of auroral origin it would appear that it should, in general, be brightest in those regions where ordinary auroras are brightest, and faintest in equatorial regions. This question, however, is not definitely answerable from the published data.

Apart from the permanent aurora there is one other source of light—perhaps to some extent of the permanent aurora itself—the possible effect of which it is the specific purpose of this paper to consider.

The source in question is the continual bombardment of the outer atmosphere by material of meteoric origin; including under

[1] *Annual Report Smithsonian Institution*, 1911, p. 64.

[2] W. W. Campbell, *Astrophysical Journal*, **2**, 162, 1895.

this term all particles picked up by the earth in its orbital motion, whether of cosmical, solar, or any other origin. So far as this produces light at all it will be through a considerable depth of the outer rare atmosphere, and hence, in general, must appear somewhat brighter with increase of zenith distance. It should also be nearly independent of any attainable altitude, and therefore, in both particulars, is in accord with observations.

For simplicity of numerical calculations it will be assumed that the "earth light" is of both constant and uniform intensity—the same over all parts of the sky, invariable, and continuous. Obviously these conditions could be met, so far as intensity of light reaching an observer at the surface of the earth is concerned, by a properly illuminated white-mat screen concentric with and some distance above the surface of the earth.

Further, the total amount of light given off by a self-luminous layer of the atmosphere, since it must radiate equally both outward and inward, would be *twice* that diffusely reflected by an equally bright white-mat surface of the same area. Now the amount of "earth light" per square degree is found by Yntema to average, roughly, one-tenth the light from a star of the first magnitude.

This furnishes a means of also comparing the brightness of "earth light" with that of the moon, as follows. When full the moon is equal to a star of -11.77 magnitude, or the equivalent of 120,000 stars of the first magnitude, and covers about 0.2 of a square degree. Hence the full moon is 6×10^6 times brighter than the sky would be if illuminated by "earth light" alone.

It is now possible, and also essential to certain energy calculations to follow, to measure the brightness of "earth light" in terms of meter candles. The brightness of the full moon is the same as that of a white-mat surface illuminated by a 1200 candle-power light at one meter's distance,[1] or, in symbols:

Brightness of full moon $=1200$ m.c. (meter candles).

Therefore

Brightness of "earth light" $=2\times10^{-4}$ m.c.

Now the average velocity of meteoric matter, as it enters the atmosphere, seems to be about the "parabolic velocity" due to the

[1] *Circular of the Bureau of Standards*, No. 28, p. 7, 1911.

sun's attraction at the earth's distance, or roughly 42 kilometers per second. Also the number of such particles appears rapidly to increase with decrease of size, and presumably the number of those that are far too small to produce visible streaks is vastly greater than the number of all combined that are large enough to be individually seen.

Further, any object, however small, entering the atmosphere with so great a velocity is in the condition of being acted upon by a flame of very great temperature—a temperature quite independent of the density of the atmosphere, assuming it of constant composition. Even if the atmosphere were all hydrogen, the entrance of an object into it with the velocity of 42 kilometers per second would produce the same effect as submitting it to equally dense hydrogen at the temperature (computed) of 142,000° C. If the atmosphere were oxygen the computed temperature would be 2,266,000° C. In either case both the velocity and resulting temperature (computed) are far beyond anything of the kind dealt with in the laboratory, except in the case of electrons, α particles and the like; and quite sufficient, as both theory[1] and experiment[2] indicate, to produce abundant ionization.

This then may be, at least in part, the source of the so-called permanent aurora—the origin of the necessary ionization, assuming the green auroral line to be due to electrical discharges.

It is well known that the amount of light that a solid body gives off increases very rapidly with increase of temperature. Hence meteors may be intrinsically brighter than any known artificial source. At any rate they have the general appearance of stars in rapid motion as their popular name, "shooting stars," indicates. It will therefore be assumed that the distribution of the total energy between heat and light is the same for meteors as it is for the sun. But the solar constant is about 1.92 calories per square centimeter per minute, and this gives, on the surface of the earth, when the sun is overhead, an illumination of 10^5 m.c.[3]

[1] P. G. Nutting, *Astrophysical Journal*, **21**, 400, 1905.

[2] A. Wehnelt, *Phys. Zeitschr.*, **9**, 134, 1908; O. W. Richardson, *Phil. Trans.*, A, **201**, 497, 1903; and others.

[3] *Circular of the Bureau of Standards*, No. 28, p. 7, 1911.

As seen above, the brightness of "earth light" is 2×10^{-4} m.c., and therefore normal zenith sunshine is 5×10^{8} times brighter than "earth light," and delivers 25×10^{7} times as much energy per square centimeter as is given out from both sides combined per square centimeter of the effective self-luminous shell or surface to which the "earth light" is due.

Hence the total energy used, according to the above assumptions in producing the "earth light" is

$$4\pi R^2 \times \frac{1.92}{25\times 10^7} \text{ calories per minute,}$$

in which R is the radius of the earth in centimeters, or

$$27\times 10^{15} \text{ ergs per second, roughly.}$$

Assuming this energy to be furnished by M grams of matter moving with the velocity of 42 kilometers per second we have,

$$\tfrac{1}{2}MV^2 = 27\times 10^{15}$$

or

$$M = 3\times 10^3, \text{ roughly.}$$

While this is nearly 300 times the estimated amount of material in visible meteors, it is less than three times the amount Young[1] assumes as allowable, and, so far as there is any present means of knowing, may be even less than the actual amount of meteoric dust caught up per second by the earth's atmosphere. Indeed it is relatively so small that it would take something like two hundred million years for it to increase the radius of the earth a single centimeter!

But, as stated above, probably a good deal of ionization is produced by the swiftly moving meteoric dust. If such ionization is produced it follows that the radiations thus excited, like those due to ordinary electric discharges in gases, may be largely concentrated in the visible region of the spectrum. Hence the above calculated amount of meteoric matter, 3 kilograms per second, may be more than is actually necessary to generate by the two processes combined, high temperature and electrical discharges, the observed amount of "earth light." In this connection it should be remembered that Yntema's values are distinctly larger than those obtained by most others[2] who have worked on this subject,

[1] *General Astronomy*, p. 475.

[2] C. Fabry, *Astrophysical Journal*, **31**, 394, 1910.

and that therefore the computed amount of energy and meteoric material may be excessive, or, at least, much above the average.

If "earth light" is wholly due, directly or indirectly, or even measurably due to the bombardment of the upper atmosphere by meteoric dust, then it should be brighter, presumably, during the later hours of the night, when the sky overhead is more nearly on the forward side of the earth in its orbital motion. Unfortunately, however, the data at hand are not sufficient for the application of this check to the above theory as to the origin of "earth light." Many more observations are needed for the complete understanding of this faint, but apparently continuous, light; and the above roughly quantitative examination of one probable source of at least some of the light is offered in the hope that it may help to narrow the problem, and even indicate one or two special points to be examined.

CONCLUSION

Numerical calculations indicate that it is within the bounds of reason to assume "earth light" somehow due to bombardment of the outer atmosphere by fine meteoric material.

U.S. Weather Bureau
Washington, D.C.
March 1912

ON A POSSIBLE ORIGIN OF THE SPECTRUM LINES NEAR THE POLES OF A METALLIC ARC

By R. ROSSI

It has been noted by several observers[1] that spark lines which appear in arc spectra between metallic electrodes are often stronger near the tips of the poles than near the middle; and the term "polar lines" was introduced by Duffield to distinguish them from the lines which appear at the center of either arc or spark and which he denoted by "median lines."

Like all spark lines in arcs, the polar lines are stronger and more numerous in the ultra-violet regions of the spectrum; and are usually sharp, while the median lines are often nebulous. As to their intensity at the two tips of the electrodes, the observations so far obtained are discordant. Fowler, working with an iron arc in the green region, found them stronger near the positive pole than near the negative; Duffield, in an iron arc in the ultra-violet, got them of equal intensity at the two tips, while Barnes, experimenting on a magnesium line, found it stronger near the cathode.

Several explanations have been offered by various authors to account for the origin of polar lines. Duffield in summarizing them shows that temperature as the cause for their production can be rejected; further, that although pressure and density may take some part in the phenomenon they can but doubtfully account for it. There are only the electrical conditions of the arc in its various parts left to be considered as a possible cause for the production of such lines.

The most striking features of the distribution of electricity in an electric arc are the very steep potential gradients near the poles and the very slight fall of potential along its central part. In a carbon arc there is a large potential fall near the anode (about 35 volts), a smaller one near the cathode (about 9 volts), and a gentle potential gradient along its center.

[1] J. Barnes, *Astrophysical Journal*, **21**, 74, 1905; A. Fowler, *Monthly Notices R.A.S.*, **67**, 154, 1907; W. G. Duffield, *Astrophysical Journal*, **27**, 260, 1908; P. G. Nutting, *ibid.*, **28**, 66, 1908.

The rapid potential drops near the poles extend over a very short distance, amounting to a small fraction of the length of the arc, so that, although the potential gradient of an electric arc is often spoken of as about 80 volts per centimeter, in some parts of the arc it may be of the order of as much as 1000 volts per centimeter, i.e., almost comparable with that of a spark. It is known[1] that when the current is increased in a carbon arc a decrease of the potential difference between the poles takes place. For high currents, however, and small arcs this decrease is more prominent at the negative pole than at the positive, the potential gradient in the center remaining practically unaffected. Hence if the polar lines were due to potential gradients and if a metallic arc behaves like a carbon one, by taking photographs of ultra-violet spectra first with small currents and then with large currents, a slight enhancement of the polar lines near the positive pole relative to the negative ought to be noticed in the second case.

Photographs of an iron arc fed by currents varying from 2.5 to 18 amperes were taken in the first order of a $21\frac{1}{2}$-foot concave grating, the astigmatism in that region being not very troublesome. These first photographs proved encouraging, the ends of the polar lines near the positive pole being usually stronger than the ends near the negative at high currents, while they were practically always of equal intensity for small currents. The effect was not quite so marked when the positive pole was below, some convection currents probably taking place in the arc. (Something similar has been noted by Huff[2] in a carbon arc.) In the case of the metallic arcs of iron, nickel, and copper it was subsequently found that when the positive pole is below, the arc is less steady, less luminous, and uses slightly less current than when the positive terminal is above. It will be shown later that when some electrical investigations were made on these arcs, some changes also took place in the electrical distribution according as the positive pole was above or below.

For the sake of getting rid of the astigmatism completely, the taking of the grating photographs was discontinued and a Fuess

[1] Mrs. Ayrton, *The Electric Arc*, chap. vii.

[2] *Astrophysical Journal*, 28, 59, 1908.

quartz spectrograph used instead. A large number of photographs were taken with an iron arc, with the positive pole above and below, for arcs of different lengths (2.5 to 7 mm) and different currents (2 to 18 amperes). The exposure was always begun after the arc had been started. For the majority of cases especially at high currents the lines were stronger near the positive pole. The strong portion began at the tip of the line, extended over a short length, and then the intensity decreased abruptly. It was subsequently found that this strong part of the line was due to the end of the red-hot globule or drop of molten iron oxide (according as the positive pole is below or above) from which the arc starts at the positive terminal. By screening the slit from the image of the arc and projecting on it the molten globule only, it was found that the spectrum yielded was a faint continuous one, but superimposed on it was a strong iron spectrum. (This probably accounts for the lines being stronger near the positive pole in the grating photographs, the astigmatism masking the cause of the phenomenon.) On this account a very large number of photographs had to be discarded and observations taken only on the few which had not been contaminated by this globule spectrum. Nickel also showed in a few cases the same effect but never to such a degree as iron, while the copper photographs were only very rarely affected by it.

The copper photographs were rather interesting (Plate XIV) inasmuch as some lines appeared only near the negative electrode, and it was only when the positive pole was below and very high currents were used that they faintly appeared at the positive electrode. They are essentially spark lines and do not appear in Kayser and Runge's or Exner and Haschek's wave-length tables for arc spectra; their relative intensities in the arc are practically identical with those in the spark. The following is a list of the most prominent ones, the wave-lengths of which were taken from the latter authors' tables for spark spectra.

2376.43	2526.79	2666.52
2403.51	2529.50	2689.56
2424.62	2545.02	2701.21
2473.55	2590.75	2703.42
2485.99	2599.03	2713.76
2506.51	2600.49	

Altogether 32 good photographs were taken for iron, 14 for nickel, and 24 for copper, for currents varying from 2 to 18 amperes. It was found that the polar lines were usually equally intense at the two electrodes, but that on some plates differences could be noted between their intensity at the two poles.

On summarizing the results, however, no certain connection could be established between their relative intensity and current-strength. It was also noted whether the width as well as the intensity of the lines near the poles varied with the current, as a change of potential gradient might alter the velocity of the luminous particle and thus affect any possible line-of-sight motion. It was found, however, that as a rule the lines were wider at the end where they were stronger, and this was probably due to mere photographic effect.

When some electrical measurements on the potential differences near the poles of metallic arcs were taken it was found that these discordant results might still be in part explained on the assumption that the polar lines are due to the strong potential gradients near the terminals. In the case of the two metals tried (iron and copper), the potential differences near the two poles were almost equal and varied but little and somewhat irregularly by varying the current; so that in this respect these two metallic arcs differ greatly from a carbon one.

The experimental method adopted was practically the same as that used by Mrs. Ayrton[1] and other workers for carbon arcs. A thin carbon rod carried by a special holder and sharpened to a long point was introduced in the metallic arc and the potential difference between it and either pole measured with a moving coil galvanometer (290 ohms resistance) with one megohm in series and a shunt of 20 ohms, the deflection on the scale being about 4 mm per volt. The iron and copper poles were 1.25 cm in diameter, the carbon electrode 4 mm, but owing to its sharp point the portion of it in the arc was never more than 1.5 mm in diameter. By having the exploring carbon first as close as possible to one pole, then to the other, and measuring the two potential differences in each case it was ascertained that the potential difference at the

[1] *The Electric Arc*, chap. vii.

center of the arc was of the order of about 2 volts, the main fall occurring near the poles. Hence, in all subsequent experiments, the exploring carbon was fixed at the center of the arc and the potential differences measured between it and the two poles. Owing to the unsteadiness of the iron arc and the tendency of the poles to melt, the measurements were very hard to take, especially at high currents and when the positive pole was below. The figures given below for iron have therefore to be taken as only very approximate. The readings for copper were taken with comparative ease, the copper arc being almost as steady as a carbon one, and the figures for copper ought to be correct within 10 per cent or less. The following tables are the summaries of the results for the two metals studied. The lengths and currents of the arcs were chosen so as to obtain more or less the same conditions at which most of the photographs of the spectra were taken.

IRON

Current in Amperes	Length in mm	Potential Differences in Volts at + Pole	Potential Differences in Volts at − Pole
Positive Pole Above.			
3.5	3.5	22.9	19.2
6.0	5.0	24.5	24.2
18.0	5.0	17.0	20.5
Positive Pole Below			
3	3.5	27.6	21.1
5	5	24.7	17.0
8	7	28.3	19.7
15	6	22.9	17.0

COPPER

Current in Amperes	Length in mm	Potential Differences in Volts at + Pole	Potential Differences in Volts at − Pole
Positive Pole Above			
4	4	27.6	20.6
4	6	23.9	26.6
10	6	19.9	19.4
10	8.5	23.5	21.6
16	8.5	19.4	19.0
Positive Pole Below			
4	4	25.4	24.2
4	6	29.0	27.5
10	6	24.2	20.2
10	8.5	25.7	21.4
16	8.5	21.6	21.6

Although these experiments do not prove that the polar lines are due to steep potential gradients, they do not disprove that hypothesis. The practically equal intensity of the lines at the two poles, and the occasional slight irregular variation of intensity for changes of current and inversion of the polarity of the electrodes, are very similar to the behavior of the potential gradients in the

arc under the same conditions. This view is also in accord with the experiments of Crew[1] and of Nutting,[2] who consider potential gradient as the main cause for the production of spark lines in arc spectra. The latter found spark and polar lines very prominent in arc spectra at 4000 volts and 0.05 amperes.

The polar lines may of course be due to temperature; temperature and potential gradient in an electric arc being probably linked as they are in vacuum tube phenomena, numerous researches[3] in that direction having shown that they practically vary at the same rate. A point in favor of this view seems to be the fact remarked by Sir J. J. Thomson[4] that however the current be increased in a carbon arc the temperature of the crater remains constant. As stated before, the experiments of Mrs. Ayrton show that by increasing the current in the carbon arc the potential gradient near the cathode is more decreased than the one near the anode, the latter being little affected.

J. Hartmann's[5] chief objection to temperature as the cause of production of the magnesium polar line at λ 4481 studied by him and which he obtained much stronger in low-current than in high-current arcs was his assumption that the temperature in a large-current arc is higher than in a low-current arc. This view was deduced from the larger tendency of the electrodes to melt with a high current; but a larger mass of incandescent vapor at a lower temperature (which probably is the case in a high-current and low-potential arc) would certainly produce the same effect, especially on such low melting-point electrodes as magnesium. It is true that the work of Hale, Adams, and Gale[6] tends to show that a 30-ampere arc is hotter than a 2-ampere one; but in their experiments the difference of potential between the poles was kept approximately constant and further they dealt only with median arc

[1] *Astrophysical Journal*, **20**, 274, 1904.

[2] *Ibid*, **28**, 66, 1908.

[3] Sir J. J. Thomson, *Electricity through Gases*, chaps. xvi and xvii.

[4] *Ibid.*, chap. xviii.

[5] *Astrophysical Journal*, **17**, 270, 1903.

[6] *Ibid.*, **24**, 185, 1906.

lines and not polar lines; i.e., their considerations were probably applied to the central parts of the arc.

The only ground on which Duffield rejected temperature was that the lines were of the same intensity at the two poles which he assumed to be at different temperature. The present work, however, shows that the potential gradients are practically the same at the two electrodes and the temperature presumably the same.

The temperature hypothesis might also in part explain the fact that polar lines are more numerous and stronger in the ultra-violet than in any other region of the spectrum; for an increase of temperature of the emitting vapor (such as in passing from the center to the region near the poles of an arc) might be expected to be accompanied by an increase of the relative intensity of the more refrangible lines. This is in accord with some bolometric experiments by Pflüger[1] who has shown that in metallic sparks (where the vapor is at a high temperature) the maximum of energy is in the remote ultra-violet.

It would therefore appear from the above evidences that temperature or eventually potential gradient can afford a plausible explanation for the production of polar lines in arc spectra, the probable close relationship between these two causes rendering for the present impossible to ascertain which of them plays the most important part in the phenomenon.

I take this opportunity to thank Professor Rutherford for placing the necessary apparatus at my disposal.

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[1] *Annalen der Physik*, **13**, 890, 1904.

ON THE SPECTRUM OF *P CYGNI*

By EDWIN B. FROST

This previously unrecorded star was discovered by Janson on August 18, 1600. It was observed by Kepler two years later when it was of the third magnitude, and it remained visible to the eye until 1621. Cassini found it to be of the third magnitude for a short time in 1655, and after another less-marked rise in 1665 it slowly declined in brightness. Since 1677 the light of the star has apparently been very nearly constant, at magnitude 5.0.

As is shown in Plate XIV, the striking features of the spectrum are the intense, broad, bright lines of hydrogen and helium and the relatively narrow adjacent dark lines which fringe the margins toward the violet and thus appear to have a very large displacement toward shorter wave-lengths.

This paper, the principal observational facts of which were communicated to a meeting of the Astronomical and Astrophysical Society[1] in 1905, is intended to show (1) that the spectrum has not varied appreciably in recent years; (2) that the displacements of the dark lines are spurious, being due merely to the fact that all but the edges of the dark lines are obscured on the photographic plate by the intense bright lines; and (3) that the radial velocity of the dark-line system is probably about −82 km per second, that of the bright-line system about −12 km. An explanation of the difference of 70 km in these two values is not obvious; and when spectrographic observations covering a much longer period of years have been made, this difference may perhaps be found variable. The velocity of the dark-line system, after correction for the solar motion, is about −65 km (approach), which is unusually large, but not exceptional for a steady radial velocity of a helium star.

The spectrograms have been taken with a dispersion of one prism, and are as given in the following list:

[1] *Publications of the Astronomical and Astrophysical Society of America*, 1, 244.

JOURNAL OF OBSERVATIONS

Number	Date	G.M.T.	Duration	Hour Angle	Quality	Observers
IB 348	1904 May 21	18^h23^m	60^m	E 3^h48^m	V.g.	F.B.S.
351	May 31	20 55	58	E 0 37	V.g.	F.S.
356	June 10	20 05	70	E 0 48	Too strong	F.S.
554	1905 June 23	20 20	60	W 0 25	Too strong	F.S.
583	Sept. 18	14 57	72	W 0 36	V.g.	F.S.
813	1906 July 23	19 36	60	W 1 30	V.g.	B.S.
842	Sept. 10	15 33	60	W 0 45	Weak	F.S.
1814	1908 Nov. 2	12 13	70	W 0 52	G.	L.
3049	1912 April 22	21 20	80	E 2 45	G.	G.S.
IA 11	1909 Sept. 27	15 18	155	W 1 28	Fair	F.L.S.

B = Barrett; F. = Frost; G. = Gingrich; L. = Lee; S. = Sullivan.

The last spectrogram in the list, IA 11, was obtained with the instrument altered to include the red region, on a Cramer "Spectrum" plate. No. 3049 was secured after this manuscript had been written, on a Seed "L-Ortho" plate. It extends farther into green as well as into ultra-violet than most of the other plates of the series.

The principal lines observable on these plates are now given according to elements:

Hydrogen (with both bright and dark components): ϵ, λ 3970; δ, λ 4102; γ, λ 4340; β, λ 4861; α, λ 6563.

Helium (both bright and dark): $\lambda\lambda$ 3889, 3964, 4026, 4121, 4143, 4388, 4472, 4713, 4923, 5016, 5876 (D_3), 6678. Helium lines missing or very faint: $\lambda\lambda$ 4009 (dark only), 4024, 4169, 4438 (very faint), 5048 (dark component only visible).

Oxygen (both bright and dark): $\lambda\lambda$ 4349, 4351, 4367, 4591, 4649 (dark), 4661, 4676.

Nitrogen (bright and dark, unless otherwise indicated): $\lambda\lambda$ 3995, 4601, 4607, 4614 (d.), 4622, 4631, 4643.

Calcium (dark only): K, λ 3933.

Silicon (dark only): $\lambda\lambda$ 4553, 4567, 4574.

Unidentified lines: $\lambda\lambda$ 4002 (d.), 4004 (d.), 4252 (d.), 4419 (br.), 4570 (d.), 5055 (br.), 5043 (d.), 5123 (d.) and 5127 (br.), 5152 (d.) and 5155 (br.), 5189 (v. ft.), 5240 (br.), 5285 (d.), 5408 (d.), 5664 (d.) *N?*, 5676 (d.) *N?*

In the Harvard Revision (*Annals*, **50**) and in **56**, 183, the type

assigned to *P Cygni* is B4p—probably it is the only case where B4 occurs. In Harvard *Annals*, **56**, 171, we find it classed as "B1 with hydrogen lines bright." This latter assignment agrees better with the appearance of our spectra as indicated by the above list of lines.

The spectrum strongly resembles that of other novae at certain stages of their development, but with marked differences: the bright lines, and still more so the dark lines, are very much narrower in *P Cygni* than in novae; the bright lines are only slightly displaced and do not on these plates[1] show distinct components or maxima; they are not visible at H or at K; the bright band of novae at λ 4640 is not conspicuous beyond its fellows of oxygen and nitrogen in *P Cygni*. The dark lines, too, are single in *P Cygni* and their displacement, while large for an ordinary star, is small for a nova: numerous enhanced metallic lines are often seen in a nova, but few in *P Cygni*.

The evidence that the displacements of the absorption lines are spurious and merely due to the concealment of the greater part of the lines by the overlying bright line is most readily shown in the following tables, which indicate very different values of the velocity of approach for the different lines and different elements. The value of the displacement of the dark lines is given in Ångström

TABLE I

DISPLACEMENTS AND VELOCITIES FOR HYDROGEN LINES

Plate	$H\delta$			$H\gamma$			$H\beta$		
	Dark		Bright	Dark		Bright	Dark		Bright
	Å	Km	Km	Å	Km	Km	Å	Km	Km
348	−2.39 =	−175	−14	−2.78 =	−192	−20	−3.52 =	−217	− 6
351	−2.25	−165	−11	−2.66	−184	−17	−3.54	−218	−10
356	−2.45	−179	− 4	−2.74	−189	− 9	−3.42	−211	− 8
554	−2.20	−161	− 7	−2.56	−177	+ 7	−3.88	−239	−10
583	...	...	...	−2.71	−187	− 5	......	.....	....
813	...	...	...	−2.98	−205	−28	−3.75	−231	−15
842	...	...	...	−2.65	−183	−14	......	.....	....
1814	−2.87	−210	− 4	−3.03	−209	−10	−3.66	−226	−21
3049	−2.12	−155	− 2	−2.46	−171	− 7	−3.47	−214	+ 5
Means	−2.38 =	−174	− 7	−2.73 =	−189	−11	−3.61 =	−223	− 9

Mean for 22 measures of bright *H* lines = −9.5 km.

[1] On Plate 583 there are suggestions of duplicity for bright $H\gamma$ and bright λ 4388.

units, to which are added the corresponding values as velocities, although it is not believed that this should be interpreted as due to velocity; but for the emission lines, the displacement is expressed only in km per second, as it would appear to be a genuine Doppler effect. Correction for the velocity of the earth has of course been made for each individual value.

Considering the hydrogen lines first, we note that in the mean the displacement of the dark component at $H\beta$ is -3.6 Å, as compared with -2.8 Å for $H\gamma$ and -2.4 Å for $H\delta$: if this were to be regarded as properly convertible into velocity, that derived for $H\beta$ would be greater by nearly 50 km than that for $H\delta$. This amount is obviously much in excess of even the large accidental errors unavoidable in the measurement of such broad lines; and at once negatives the possibility that these are Doppler effects.

TABLE II

DISPLACEMENTS AND VELOCITIES FOR HELIUM LINES

Plate	λ 4388.100 Dark		Bright	λ 4471.676 Dark		Bright	λ 4713.308 Dark		Bright
	Å	Km	Km	Å	Km	Km	Å	Km	Km
348.....	−1.90 =	−130	−34	−2.46 =	−165	−23	−2.22 =	−141	....
351.....	−1.76	−120	−14	−2.49	−167	−18	−2.09	−133	−17
356.....	−1.86	−127	....	−2.53	−170	0:	−2.53	−161	....
554.....	−2.02	−138	....	−2.41	−162	−19	−2.22	−141	−11
583.....	−1.68	−115	+ 4:	−2.49	−167	− 8	......	.....	....
813.....	−2.01	−137	− 6	−2.53	−170	−11	......	.....	....
842.....	−1.79	−122	....	−2.44	−164	−12	......	.....	....
1814.....	−1.61	−110	−24	−2.52	−169	−24	−2.34	−149	−24
3049.....	−1.98	−135	+ 2:	−2.27	−152	−12	−2.34	−149	−28
Means .	−1.85 =	−126	−12	−2.46 =	−165	−14	−2.29 =	−146	−20

Mean for the 19 measures of bright *He* lines = −14.7 km.

Per contra, the displacements of the centers of the bright lines are in excellent accordance, with a mean velocity of 10 km of approach. This is a very reasonable value of the velocity of a helium star; it is confirmed by the velocity derived from the bright components of the three principal helium lines, which will be seen from Table II to be −15 km (this small difference of 5 km has no significance in settings on the centers of such very broad lines). On one plate the displacements of the stronger lines of

oxygen and nitrogen in the region λ 4600–4700 were measured. The values of the velocity for the bright components are again entirely consistent with those above. We therefore conclude that the measures upon the centers of the bright lines yield the radial velocity of the body or envelope from which they originate, except as possibly modified by pressure.

Reverting to the dark components, we note in Table II that there is similarly a consistent difference in displacement among themselves for the helium lines, but without any apparent relation to the wave-length. The mean of all velocities of the dark components of the three hydrogen lines would be more than 50 km greater than the similar mean for the three helium lines, a condition wholly inconsistent with any interpretation of their displacements as a Doppler effect.

TABLE III

VELOCITIES FOR SILICON LINES (DARK ONLY)

Plate	λ 4552.636	λ 4567.897	λ 4574.791	Mean
	Km	Km	Km	Km
348	−74	−91	−85	−83
351	−82	−75	−72	−76
356	−74	−86	−66:	−75
554	−97	−67	−71	−78
583	−82	−98	−82	−87
813	−85	−76	−78	−80
842	−106	−87	−95	−96
1814	−92	−86	−72:	−83
3049	−82	−84	−81	−82
Means	−86	−83	−78	82

This is still more strikingly brought out when we pass to the silicon lines, which are dark, without visible emission components on these plates.[1] Table III shows the evidence (which was noted eight years ago, as the first of the plates were measured) that we have here a very different radial velocity, practically identical[2]

[1] An occasional suspicion of a bright component to *Si* 4553 is indicated in the notes: No. 583: "Probably no bright"; No. 3049: Settings were made on "Is it a bright fringe?" It yielded a value of +8 km. In many cases it was recorded that no bright fringes were visible.

[2] It would be more appropriate in measurements on such stars as novae, and in general on those having very broad lines, to employ for velocity a unit of 10 kilometers; that unit, the "myriameter," has been so little used, however, that its employment in special cases might lead to confusion.

for the three lines and apparently constant during the eight years covered by the plates, at about −82 km. As we see no bright components which might be masking all but the more refrangible positions of the dark lines, we are not justified in making any other assumption than that we obtain from the three silicon lines the true velocity in the line of sight of the body or envelope responsible for the absorption lines.

It remains to see what would be the true breadth of the dark lines, on this assumption that the actual velocity for the dark components is −82 km per second, and that their apparently narrow character is due to their being for the most part covered up by the broad overlying bright components. On the spectrograms best

TABLE IV

WIDTHS IN ÅNGSTRÖMS OF CERTAIN LINES

Plate	$H\delta$		$H\gamma$		$H\beta$		He λ 4472	
	Bright	Dark	Bright	Dark	Bright	Dark	Bright	Dark
348	...	...	3.0	...	3.9	...	...	...
351	2.0	1.6	3.2	1.5	4.2	2.0	2.6	1.6
554	2.2	1.4	3.5	0.9	3.8	2.8	2.6	1.3
813	...	...	2.9	2.0	4.0	2.4	2.4	1.8
842	...	...	2.8	1.9	...	...	2.0	1.9
1814	2.0	1.0	2.9	1.0	4.4	2.0	1.5	1.5
3049	2.3	1.9	2.8	1.7	5.2	1.6	1.9	1.9
Means	2.1	1.5	3.0	1.5	4.3	2.0	2.2	1.6

suited for this purpose, I made settings on the edges of both bright and dark components, whence were derived the widths of the lines, as given in Table IV. Taking the relative difference in velocity for the centers of the bright lines and the concealed centers of the dark lines to be 70 km, we examine the data to find whether the margins toward red of the dark lines ought to project beyond the margins of the bright components. It can readily be shown that they should thus project only when the width of the visible portion of the dark component is greater than twice the relative displacement. The displacement corresponding to 70 km is as follows for the lines concerned: $H\delta$, 0.96 Å; $H\gamma$, 1.01 Å; $H\beta$, 1.13 Å; He, λ 4472, 1.04 Å. Hence there should be second dark components

on the red-ward margins of the bright lines only when the visible width of the dark components exceed twice these values, or from 1.92 Å for $H\delta$ to 2.26 Å for $H\beta$. It will be seen from the table that the mean widths of the dark lines are well under this limit. There is but one individual exception, for $H\beta$ on No. 813, and here the setting on the vague edges of the bright line is doubtless at fault. Careful examination of the spectrograms does not show any dark margins on the edges toward red, so that this evidence, although in a sense negative, tends to confirm the interpretation we have given.

It will be seen from Table IV that the widths for the bright hydrogen lines, δ, γ, and β, are in about the ratio 2:3:4. Hence it is to be expected that the broadest lines will give the largest spurious displacement to their dark components, as was shown to be the case in Table I. The bright lines are much less broad than is usually the case for an active nova.

If we regard the spectrum of *P Cygni* to be due to a single body having a radial velocity of approach of 82 km, then the bright components are displaced toward the red by the equivalent of 82–13 km, or 1.0 Å at $H\gamma$, 1.1 Å at $H\beta$. We have no adequate data as to the quantitative effect of pressure upon the displacement of lines of permanent gases like hydrogen and helium; but if of the order of the shifts for metallic lines, this would imply a pressure of over 200 atmospheres, effective in the portion of the star emitting the bright lines, but not in the reversing layer. While this seems highly improbable, the phenomena of temporary stars are generally so improbable that they would a priori be regarded as impossible.

My results are in excellent agreement with those obtained by Belopolsky[1] from spectrograms taken on September 21 and 22, 1899. The special purpose of his article was to call attention to the identity of numerous lines (chiefly between λ 4601 and λ 4651) with those of air lines (*N* and *O*) in his comparison spectrum, which had not been previously established. Belopolsky gives substantially the same displacements for the hydrogen lines β and γ that I have tabulated above, which is additional evidence of the absence of any considerable change in the spectrum in nearly 13 years. He attributes the difference in displacement for the dark $H\beta$ and $H\gamma$ to

[1] *Astrophysical Journal*, 10, 319, 1899; *Astronomische Nachrichten*, 151, 37, 1899.

photographic irradiation, if I correctly interpret his words: "This is certainly to be explained by silver precipitation of the bright lines overlying the dark lines at their edges, so that the observer makes his settings on the edge of the bright line, and not on the dark line." Photographic irradiation evidently does complicate the measurements on such spectra, tending to narrow the visible dark components, besides giving an undue width to the strong bright lines.

It is unfortunate that only one of our spectrograms (that last secured) is strong enough at K to give us information as to that interesting line. It appears to have no bright component, being a quite narrow dark line. It yielded a velocity of 0 km, thus differing widely from the dark silicon lines. Such behavior of the K line is found so frequently in spectra of type B that it is hardly abnormal, although as yet unexplained.

Paul W. Merrill has recently[1] published a note on the spectrum of *P Cygni*, from which it appears that bright components of the silicon lines are visible on the three-prism spectrograms taken at Mt. Hamilton, and that these bright lines have a displacement toward the red of 0.26 Å as compared with the bright lines of *H* and *He*. I am somewhat surprised that these bright *Si* components should not be more conspicuous on our single-prism plates than on these three-prism plates, but it is evidently not the case. The effect of invisible bright components on my measures of these dark lines would be to make the negative displacements too great, so that the velocity of −82 km inferred from these lines might be too large. Mr. Merrill's velocities for the bright hydrogen and helium lines are practically in agreement with those given here.

My interest in this spectrum has been largely due to the hope that some of the complications in the spectrum of *β Lyrae* might be explained if we assigned to one of the components a character like that of *P Cygni*. There are many points of similarity in the spectra of the two stars, and if an oscillating system of lines, due to the orbital revolution of a second body, were superposed upon the spectrum of *P Cygni*, it would reproduce some of the interesting features of *β Lyrae*.

YERKES OBSERVATORY
April 1912

[1] *Lick Bulletin No. 201.*

Reviews

Transactions of the International Union for Co-operation in Solar Research, Vol. III (Fourth Conference). Manchester and London: Sherratt & Hughes; New York: Longmans Green & Co., 1911. 8vo, pp. 231. $2.50.

This volume follows the style of the two which have preceded it, except that the duplication of the records of the sessions in French and German is avoided, English being used as the language of the country in which the meeting was held. A very accurate stenographic report of the interesting meetings on Mount Wilson occupies the first 133 pages. The proof sheets of the report were submitted to the persons concerned and hence it may be regarded as authentic in every way. The attendance at the four sessions was large, and the discussions had numerous participants, adding to the value and interest of the record. Part IV contains in four pages the text of all the resolutions adopted by the Union at Mt. Wilson. Part VI (pp. 139–200) gives the reports of the committees, together with papers communicated to the Union for this meeting. These reports include those on standard wave-lengths, on the measurement of solar radiation, on work with the spectroheliograph, on sun-spot spectra, on eclipse observations; together with individual papers by M. Deslandres: "Sur les spectrohéliographes de grande dispersion," "Isolement des couches supérieures de l'atmosphère solaire," "Sur le spectro-enregistreur des vitesses radiales du soleil"; and by M. Perot: "Sur la signification des mésures de vitesse de rotation par la méthode spectroscopique," and "Sur la rotation du soleil."

The volume closes with the two evening addresses delivered at the conference, by Mr. C. G. Abbot on "The Solar Constant," and by Professor Kapteyn "On the Systematic Proper Motion of the *Orion* Stars."

Those who were fortunate enough to be able to attend this memorable conference have doubtless already secured copies of this volume; those who were not so fortunate can supply the deficiency to some extent by securing copies for themselves and for the libraries in which they are interested.

F.

The Progress of Physics during 33 Years (1875–1908). By ARTHUR SCHUSTER. Cambridge: The University Press, 1911. 8vo, pp. 164, with frontispiece portrait of J. CLERK MAXWELL. 3*s*. 6*d*. net.

This slender volume comprises four lectures delivered to the University of Calcutta in March 1908. The third of a century included by the lecturer was that which had elapsed between his first and his second visits to India. The author's intention was to record the changes in the point of view of physical science rather than to give a historical account of the discoveries made during the period; and he warns his hearers that the account will be fragmentary, and to a considerable extent reminiscent of scientific matters with which the author has had personal relations. This personal touch, with recollections of Maxwell, and the early workers at the Cavendish laboratory, of Helmholtz and the men gathered about him at Berlin, gives a special interest to the lectures. The cosmopolitan character of the author's education and experience contributes to the catholic view displayed throughout the book.

Titles are not assigned to the separate lectures, which overlap to some extent, and the whole treatment is delightfully informal and covers a large number of topics, as may be seen from the following excerpts.

From Lecture I: State of Physics in 1875. Maxwell's theory of electricity. Kirchhoff's laboratory. The two laboratories of Berlin. Laboratory instruction at Manchester. Spectrum analysis. The radiometer. Theory of vortex atom.

From Lecture II: Verification of electromagnetic theory by Hertz. Wireless telegraphy. Early experiments in electric discharge through gases. Kathode rays. Ionization of gases. Measurement of atomic charge.

From Lecture III: Roentgen's discovery. Discovery of emanations. Decay of the atom. The Michelson-Morley experiment. Principle of relativity. The Zeeman effect. Contrast between old and modern school of physics.

From Lecture IV: Terrestrial magnetism. Existence of potential. Separation of internal and external causes. Magnetic storms. Solar influences. Atmospheric electricity. Cause of thunderstorms. Age of the earth. Gravitation.

Enough has been said to show that every physicist, and those having interests in related branches, will greatly enjoy as well as profit by the reading of this book.

F.

THE COLLECTED SCIENTIFIC WORKS OF SIR WILLIAM HERSCHEL

We have pleasure in drawing attention, at the request of the Royal Society and the Royal Astronomical Society of London, to the publication at their expense, in a limited edition, of *The Scientific Papers of Sir William Herschel*, in two volumes, royal quarto, in boards, Vol. I, pp. i–cxx, 1–597; Vol. II, pp. i–viii, 1–718, with 3 portraits and other plates. The agents for sale are Dulau & Co., 37 Soho Square, London, W., who will forward the work direct at the price of £2, 10*s* net. The following description is reproduced from the Preface:

Soon after the death of Sir William Herschel in 1822, his distinguished son, Sir John F. W. Herschel, F.R.S., formed the plan of republishing his father's papers; but he found on inquiry that no publisher would be willing to undertake the risk of so extensive a work. He therefore resigned the idea, considering that he might contribute more effectually toward a monument to his father's memory by devoting himself to extending and carrying out, with his own instruments and after his own manner, his father's processes of observation. A German translation was commenced by Professor J. W. Pfaff of Erlangen, but only the first volume was published (*W. Herschel Sämmtliche Schriften.* Erster Band: "Ueber den Bau des Himmels." Dresden und Leipzig, 1826, 8°).

The papers of W. Herschel are scattered over about forty volumes of the *Philosophical Transactions*, and they have become difficult of access to many to whom their study is of importance. A useful summary of their contents was published by Professors Hastings and Holden in the *Smithsonian Report* for 1880, but a collected edition of the papers has always been considered a *desideratum* in astronomical literature, and from time to time it has been strongly urged that a complete reprint should be made available.

Early in the year 1910 the matter was taken in hand, and a committee was appointed by the Royal Society and the Royal Astronomical Society to prepare an edition of William Herschel's works at the joint expense of the two societies. It consisted of Sir William Huggins (who was prominent in promoting the work, but was only able to preside over two meetings of the committee before his death), Sir Joseph Larmor and Professor R. A. Sampson, representing the Royal Society, and Sir David Gill, Mr. J. A. Hardcastle, and Professor H. H. Turner, representing the Royal Astronomical Society. Soon afterward Dr. J. L. E. Dreyer and Major E. H. Hills, and finally Mr. F. W. Dyson, Astronomer Royal, were added to the committee.

It seemed desirable to reprint all Herschel's published papers exactly as they had been issued by him, without omissions or alterations—except that actual errors of observation, or identification of the observed objects, should be pointed out. Some errors of this kind in the observations of Double Stars had been found, chiefly by the late Mr. H. Sadler, from an examination

of the MS sheets belonging to the Royal Astronomical Society; and these have now been checked where necessary by reference to the original observing journal. But it appeared to be specially important to take this opportunity to make a revision of the three catalogues of nebulae in order to clear up many difficulties in reconciling Herschel's results with those of later observers. This has been done by an examination of the original "sweeps," which, together with Caroline Herschel's *Zone Catalogue* and the observations of the objects in Messier's catalogue, were lent for this purpose by the Royal Society. It is hoped that this revision has resulted in improving the accuracy of the three catalogues; but at the same time it should be stated that the revision has furnished additional proofs of the very great care with which the observations were both made and reduced.

The reprint has been made complete as regards the published scientific work, though this has involved the insertion in Vol. II of three papers on Newton's colored rings, which have not more than a personal interest (see Introduction, pp. lvii–lviii).

The committee have been under great obligations to Sir William J. Herschel, Bart., who generously placed at their disposal his grandfather's letters, his observing journal, the record of the polishing of mirrors, and also autobiographical memoranda and unpublished papers. Everything of importance that could be extracted from these valuable materials, and brought within reasonable limits of space, has been given in the Introduction to the present volumes.

Many incidents of Herschel's career have thus been presented in a new light, especially as regards his early life; while the papers read before the Bath Philosophical Society form an interesting record of his early modes of thought and of the versatility of his mind, in addition to affording an illustration of the remarkable activity of genuine physical speculation in England, at a time when formal mathematical analysis was but slightly cultivated.

The joint committee aforesaid of the Royal Society and the Royal Astronomical Society are responsible for the general plan of the work. But they desire to record their obligation, and that of the astronomers who will use the reprint, to one of their number, Dr. J. L. E. Dreyer, who very generously undertook both the collation of the manuscripts and the preparation of the introductory memoir.

Descriptive Meteorology. By WILLIS L. MOORE. New York and London: D. Appleton & Co., 1910. 8vo, pp. 344, with 81 text illustrations and 45 charts. $3.00 net.

In preparing this valuable work the Chief of the United States Weather Bureau has had many advantages, such as long experience in practical meteorology, association with many experts in the past and

present history of this branch of science in America, continuous access to the remarkably complete library of our national Weather Bureau, and direct consultation with the leading men of its staff on the subjects in which they are authorities. Under the last head acknowledgments are made to Professors Abbe, Bigelow, Kimball, Henry, Talman, Cox, and Humphreys.

The book is both theoretical and practical, and is intended to lead up to the art of forecasting the weather, for which the North American continent offers many natural advantages. One purpose of the work was to provide for the needs of the young men entering the United States Weather service, to whom some of the great treatises in foreign languages might not be available.

The subject is treated in fifteen chapters as follows: the atmospheres of the earth and of the planets; atmospheric air; micro-organisms and dust-motes of the air; physical conditions of the sun and its relation to the earth's atmosphere; heat, light, and temperature; thermometry; distribution of insolation, and the resulting temperatures of the atmosphere, the land, and the water; the isothermal layer; atmospheric pressure and circulation; the winds of the globe; the clouds; precipitation; forecasting the weather and storms; optical phenomena in meteorology; climate.

Among the features in which a departure is made from the mode of presentation in most other works on meteorology we may mention particularly: the treatment of the astronomical climate, which a smooth spherical earth would have without an atmosphere, from which the reader is led to the modifications produced by the atmosphere and the earth's surface; the diagrams of the relative proportions at different elevations of the constituents of this atmosphere; the discussion of the actually observed motions of the air at various elevations in cyclones and anti-cyclones; of the vertical distribution of temperature under different conditions of the weather; of the isothermal layer. All these are matters in which the information lately gained from kites and sounding balloons is made available.

The chapter on forecasting (in America) is thoroughly practical. It is finally illustrated by many appropriate charts at the end of the volume. The numerous illustrations throughout the book are largely new and excellent. A brief bibliography at the end of each chapter gives the principal references for the topic under discussion.

The author has avoided the temptation of going too much into statistics; in some respects more such information might have been desirable.

The book is in every respect handsomely gotten up and should prove of wide service in astronomical libraries and in the hands of many besides those for whom it was primarily intended.

F.

The Great Star Map. By H. H. TURNER. London: John Murray, 1912; New York: E. P. Dutton & Co. Crown 8vo, pp. 159, with frontispiece. Price: in London 2*s*. 6*d*.; in New York, $1.00 net.

As stated on the title-page, this is "a brief general account of the international project known as the Astrographic Chart." The author has to an unusual degree the art of describing things simply and interestingly, both for the general reader and for those technically familiar with a subject. The four chapters are entitled: I, "Introduction"; II, "Star Counting"; III, "Star Positions"; IV, "Some Incidents of the Work." Twelve "notes" occupy the last twenty pages and give details which would chiefly interest astronomers.

The marked difference in price of the work in England and America will lead to renewed contemplation of the utility of a duty on books printed in England.

F.

Star Lore of All Ages. By WILLIAM TYLER OLCOTT. New York: Putnam, 1911. Pp. 453 with 50 text illustrations and 64 full-page engravings. $3.50 net.

The title-page of this handsome work describes it as "A Collection of Myths, Legends and Facts concerning the Constellations of the Northern Hemisphere." Emphasis is particularly placed upon the myths and legends, and less upon the facts; where the facts are recorded, the author has depended largely upon second-hand information, and his assignment of the authority upon whom a fact depends is thus often incorrect. Published data are, perhaps naturally, accepted by the author at their face value, without serious question of their reliability. Thus of the Great Nebula in *Andromeda* we read: "Recent and more reliable calculations of its distance give it a light-journey of about nineteen years."

It is not, however, for its value as a record of astronomical progress that this work will properly find a place in the library of the teacher and amateur. The mythological associations with the stars and constellations are many, and are not to be found in the regular astronomical

treatises, when required for lecture purposes or otherwise. The author shows familiarity with these phases of his subject and gives evidence of wide reading and study. He acknowledges his indebtedness to R. H. Allen's *Star Names and Their Meanings.* The numerous full-page illustrations are chiefly from classical paintings and statuary, are well chosen and finely reproduced. The work should serve a very useful purpose.

F.

Tables of Physical and Chemical Constants and Some Mathematical Functions. By G. W. C. Kaye and T. H. Laby. London and New York: Longmans, Green & Co., 1911. Royal 8vo, pp. 153. $1.50.

This latest table of constants differs from others in its compactness and the amount of data from the most recent fields of investigation, such as ionization, radioactivity, etc. Undue abbreviation is not practiced, the type is clear, the size of the page is convenient, and the flexible cover adds to the handiness of the book. The tables are not numbered, but are very numerous, and the range covered in small compass is surprising. We predict for the book a career of usefulness which will call for new editions. F.

THE
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AND ASTRONOMICAL PHYSICS

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THE SOLAR PROMINENCE OF JUNE 19–20, 1911

By FREDERICK SLOCUM

On June 19 and 20, 1911, I observed on the northwest limb of the sun a prominence which possessed some especially interesting features. Twenty-five photographs in the light of the H-line of calcium were made with the Rumford spectroheliograph of the Yerkes Observatory on the 19th, between 8 A.M. and 4 P.M., of which twenty-one are reproduced in Plates XV, XVI, and XVII. On the 20th, nine exposures were made, of which six are reproduced in Plate XVII.

The sky remained clear and the seeing good throughout both days, so that a fairly complete record of the changes in the prominence was secured. The exposure time, that is, the time required for the image of the prominence to pass over the slit, was thirteen seconds, and the photographs shown are separated by intervals ranging from 1.7 to 100 minutes.

On the 19th, the prominence maintained approximately the same position and dimensions, while undergoing numerous striking internal changes. Its height remained about $2'\frac{1}{4}$, or 97,000 km. With the exception of a short time between 8^h and $8^h\,30^m$ G.M.T., its extent along the limb was between $4'$ and $5'$, or between 172,000 km and 215,000 km. It extended from latitude $+35°$ to latitude $+49°$ on the west limb.

When first observed, the prominence showed three conspicuous features; the arch on the left, the pillar in the center, and the lofty extension on the right.

The arch, at first relatively thin, increased rapidly in thickness, while the space under the arch diminished in area. Up to 4^{h} $33^{m}.8$, this space appeared intensely black and the line of demarkation between the bright arch and the interior space was sharp. This is especially well shown at 4^{h} $1^{m}.5$. After that time, the space under the arch began to fill up, and the arch itself became less massive, and finally, at about 8^{h}, completely disappeared. At 8^{h} $58^{m}.0$, a new thin arch appeared, and this gradually increased in thickness until, at 9^{h} $47^{m}.5$, it resembled quite closely the original arch observed seven hours earlier.

At 5^{h} $32^{m}.9$ and 5^{h} $34^{m}.9$, interesting spiral columns appear, as if supporting the arch. Between 8^{h} $58^{m}.0$ and 9^{h} $45^{m}.5$, the second arch changed from a curved to an angular, and then back to a curved form.

The central pillar started, at 2^{h} $18^{m}.2$, with an anvil-like form, 45,000 km high. This gradually increased in height to a maximum of 65,000 km, at 4^{h} $36^{m}.9$, its apparent motion, therefore, being at the rate of 2.4 km per second. Throughout its development it was curving to the right or south. After 4^{h} $36^{m}.9$, this pillar began to disintegrate, and, at 7^{h} $39^{m}.5$, it was replaced by a form somewhat resembling a sky-rocket with a large and brilliant head. This head moved to the south at the rate of 13 km per second, and was soon lost in the higher portion of the prominence. At 8^{h} $17^{m}.5$, the pillar had again assumed an anvil shape, but, at 8^{h} $58^{m}.0$, it was replaced by a form resembling a tornado cloud, and at 9^{h} $55^{m}.3$ two of these vortices appeared.

Considering, now, the third feature, we find that on the first plate it resembled a chimney from which smoke is issuing, and this smoke appears to be driven to the left or north by some lateral current. For several hours the base of the chimney became more and more massive and the volume of smoke increased up to 3^{h} $12^{m}.5$, then decreased rapidly, but enough remained to indicate throughout the day the existence of an upper current from the south. That this is merely an upper current is shown by the pho-

tographs of Plate XV, in which the forms of intermediate height, that is, up to about 60,000 km, are streaming in the opposite direction.

On the next day, the upper current was reversed. At $2^h\ 3^m.3$, June 20, the prominence resembled two volcanoes in eruption. The smaller one quickly disappeared, and by $4^h\ 58^m.1$, the larger one had nearly vanished. This larger volcano-like form, extending from latitude 41° to latitude 44°, was the chief source of activity on this date, but sporadic eruptions appeared for some distance on either side of it. Note, for example, the eruption on the right of the photograph taken at $3^h\ 9^m.5$ (Plate XVII). The arch in this figure is almost a perfect semicircle, 25,000 km in diameter, nearly twice the diameter of the earth. No trace of it appears on the plate taken eleven minutes earlier, and, on a plate exposed 2.6 minutes later, only the remnants of it are visible.

After passing through many transformations, the prominence finally became dissipated and floated away upward. This method of dissolution is characteristic of by far the greater number of the solar prominences whose last stages I have observed.

Yerkes Observatory
April 17, 1912

THE PRESSURE DISPLACEMENT OF SPECTRAL LINES

By T. H. HAVELOCK

The object of the following note is to suggest a comparison between observed pressure effects in emission spectra and a displacement of absorption lines which can be deduced theoretically from a certain type of dispersion formula. This is introduced by a preliminary discussion of existing theories, which deal with the direct effect upon emission. Although the calculations necessarily refer to ideally simple cases, they may be tested by seeing if they give effects of the right order of magnitude and if they are in general agreement with experimental results.

CRITICAL STUDY OF CERTAIN THEORIES

1. Humphreys[1] ascribes the effect to the mutual interaction of atomic magnetic fields and thus makes the pressure-shift comparable with the Zeeman effect. In estimating the order of magnitude, Humphreys considers iron atoms in the electric arc at atmospheric pressure, using among other data: radius of atom, 10^{-8} cm; average distance between centers of atoms, $6 \cdot 10^{-7}$ cm. Thus the calculation assumes the vapor-density to be that of a perfect gas at a pressure of one atmosphere and at 2730° absolute, having $4 \cdot 10^{18}$ atoms per cubic centimeter. In order to obtain a displacement of a line of the right amount, it is necessary for the atomic magnetic fields to be enormously high, and this involves the assumption of a very large number of electrons revolving in circular orbits within the atom. The calculation has been criticized by Richardson,[2] who shows that with a more reasonable estimate of the number of revolving electrons the atomic magnetic fields are such that their mutual influence is entirely negligible; the present writer agrees with this conclusion. The comparison of experimental results for the Zeeman effect and the pressure-

[1] *Astrophysical Journal*, **23**, 233, 1906.

[2] *Philosophical Magazine*, **14**, 557, 1907. See also Sanford, *Astrophysical Journal*, **35**, 1, 1912.

shift is discussed later. We may note here that the above calculation appears to make the pressure displacement depend upon the density of the emitting metallic vapor, this in turn corresponding to the total pressure. For if a is the average distance between atoms, the magnetic field due to a neighboring atom is approximately proportional to a^{-3}; but a^{-3} is proportional to the number, N, of atoms per unit volume, and so the effect is a linear function of the pressure. It should be added that in later papers, Humphreys considers the change of period of a metallic atom in the arc to be due to the magnetic fields of the atoms of the gas in which the arc is burning. There appears to be no direct evidence that the effect depends upon magnetic properties, and in any case we decide that these are insufficient to give effects of the right magnitude.

2. We consider now some calculations which involve the mutual influence of the electric fields of neighboring atoms. It was suggested by Fitzgerald that the change in the specific inductive capacity K of the surrounding gas due to change of pressure might account for the variation of the period of a vibrating atom in the arc. We may in fact assume the period of the vibrator to be proportional to the square root of K. In some estimates from this point of view, for instance by Humphreys,[1] the resulting displacement comes out many times too large; but the cause of this lies in treating the surrounding gas as a continuous medium extending right up to the particular vibrator in question. This error appears to be at the root of various statements that the pressure-shift cannot be due to the mutual influence of atomic electric fields, because calculations based thereon give far too large a result.

Larmor,[2] on the other hand, takes into account the molecular constitution of the gas. Considering the period of a spherical vibrator of radius a, which behaves like a simple Hertzian doublet, Larmor replaces the surrounding gas by a uniform medium, of inductive capacity K, extending up to a distance ka from the center of the vibrator; thus the latter is at the center of a spherical

[1] *Astrophysical Journal*, 26, 30, 1907.

[2] *Ibid.*, 120, 1907.

cavity in a continuous medium. For the displacement $d\lambda$ of a vibration of wave-length λ, due to an increase of pressure of one atmosphere, the calculation gives an approximate formula

$$\frac{d\lambda}{\lambda}=\frac{1}{2k^3}\quad\frac{K-1}{K} \tag{1}$$

Taking K as 1.0006 for air at a pressure of one atmosphere, and assuming 10^{-6} as an average observed value of $d\lambda/\lambda$, this gives k equal to about 7. Larmor concludes that although the data for molecular magnitudes are of course vague, they appear to justify the statement that the dielectric influence of the neighboring molecules is a *vera causa* of the right order of magnitude.

This conclusion may be criticized on the following grounds. The above value of K is that for air at a temperature of 273° absolute. Although the temperature conditions of the arc are uncertain, it seems more reasonable to put the temperature of the immediate atmosphere of gas at some conventional value like 2730° absolute, as in the calculation in the previous section. At this temperature, $K-1$ must be taken as $6\cdot 10^{-5}$, one-tenth of the value above. The formula (1) now gives k equal to 3. But the molecules at this temperature are spaced roughly at 60 times the molecular radius; so that the value of k necessary for the right result seems inadmissible. We conclude that the dielectric influence of the neighboring molecules of the *surrounding gas* leads to a displacement many times too small.

Further, the formula (1) applies primarily no doubt to a simple illustration, but it indicates certain relations. For instance, the displacement should vary with the value of $K-1$ for the gas; but this has not been confirmed by experiment. In addition, k^{-3} is roughly proportional to density, and we have made $K-1$ also proportional to density in the above calculation. It follows that the displacement should vary as the square of the pressure; but the relation obtained experimentally is a linear one.

3. Another calculation on the same general lines is that of Richardson (*loc. cit.*). The argument is that, if a metallic atom A is emitting radiation, its frequency is affected by the sympathetic electric vibrations induced in the atoms B of the surrounding gas. Richardson obtains greater detail in his formula by definitely

assuming the vibrators to be electronic charges, as in ordinary optical theory; the result is

$$\frac{d\lambda}{\lambda}=\frac{e^2\lambda^2(\mu^2-1)}{6\pi^2mc^2\alpha^3} \tag{2}$$

where μ is the refractive index of the surrounding gas. The quantity α is taken as the radius of the sphere within which it is impossible for the center of an atom of class B to lie; the value of α is supposed to lie within the limits a and $2a$, where a is the atomic radius. Taking a mean value $1.5\cdot 10^{-8}$ cm for α, Richardson calculates the order of magnitude for a line of wave-length $4\cdot 10^{-5}$ cm. The surrounding gas is taken to be air at 2730° absolute, for which μ^2-1 is $5.9\cdot 10^{-5}$. With the usual values of e, m, and c, the formula (2) gives per unit atmosphere a proportional displacement $d\lambda/\lambda$ equal to $9\cdot 10^{-5}$; that is, the result is about 100 times an average observed result such as we have used in the previous sections.

The error appears to be one to which reference has already been made. The effect of atoms of class B is obtained as an integral with α as its lower limit, and with the value of α chosen, the surrounding gas is made equivalent to a continuous medium extending right up to the vibrating atom A; but this is not permissible. If for a moment we replace α by ka as in Larmor's calculation and find k from (2) so as to give the right order of magnitude for $d\lambda$, we find k equal to 7; this seems too small, considering the condition of the gas with its atoms spaced roughly at 50 or 60 times the atomic radius. Further, if we have to make α variable with the density in this way, we encounter again the difficulty that the formula would make the displacement proportional to the square of the pressure, instead of the first power. Also as before, the factor μ^2-1 does not accord with experimental results.

4. We notice that in Richardson's theory, as in Larmor's, the calculations refer entirely to the dielectric influence of the surrounding gas, and we conclude that this gives a displacement which is many times too small. The calculations ignore the mutual electric influence of neighboring atoms of the metallic vapor, that is, of atoms having the same free periods. Richardson, in fact, expressly rules out of consideration the effect of an increase in the partial

pressure of the metallic vapor, stating that this produces in general only a broadening of the lines without displacement. This may or may not be the case, but it does not follow from the theoretical considerations; for these might be made to apply to a radiating atom surrounded by similar atoms, with a resulting displacement many times larger owing to resonance effects. In fact the influence of similar atoms could be ignored only if the vapor-density were very small compared with that of an ideal gas under similar conditions. This point of view has been stated by Schuster;[1] after giving an account of Larmor's theory, he writes: "The question is complicated by the fact that in the cases which have been observed, the greater portion of the metallic vapor vibrates in an atmosphere of similar molecules." K of formula (1) is then replaced by μ^2, where μ is the refractive index of the vapor. One plan is to put μ^2 infinite for a free period; this gives $d\lambda/\lambda$ equal to $\frac{1}{2}k^{-3}$, and with k equal to 10, the displacement comes out about 500 times too large. Schuster remarks that this method gives no exact information about the displacement of the position of maximum intensity. In addition, it is unsatisfactory in that Larmor's formula assumed K to be not much different from unity. However, it may be noticed that with k equal to 80 we get an effect of the right order; this supports the contention that the effect of neighboring similar atoms can be ignored in this connection only if the vapor-density is very small.

If we attempt to make a more complete theory on these lines, we meet all the difficulties of the emission of an aggregate of molecules. The previous theories consider the radiation of a single particle, but what is required is the period for maximum intensity of emission of a collection of similar vibrators. One knows that similar difficulties have arisen in the theory of the direct Zeeman effect, with the result that writers have turned to the inverse effect; it has been found easier to obtain more detail from the consideration of absorption spectra, chiefly because one can work from known theories of dispersion. A similar step is suggested now in the theory of the pressure displacement of spectral lines.[2]

[1] *Encyclopaedia Brittanica*, eleventh edition, **25**, 628.

[2] See also *Proceedings of the Royal Society*, A, **84**, 517, 1911.

THE DISPLACEMENT OF ABSORPTION MAXIMA

5. If n is the refractive index of a homogeneous medium for waves of frequency $p/2\pi$, we have a dispersion formula

$$\frac{n^2-1}{n^2+2}=\sum\frac{\frac{4\pi}{3}N\frac{e^2}{m}}{p_1^2-p^2} \tag{3}$$

We shall find it convenient to use the ordinary interpretation of the constants. There are N vibrators per unit volume, of mass m, carrying a charge e, and having a natural frequency p_1; the summation extends over all the types of vibrators. This formula includes the effect of the electric polarization of neighboring particles when estimating the electric force acting on a vibrator; in the present connection it may be as well to repeat briefly the argument.

Let d be the displacement at any time of a typical particle vibrating about a position of equilibrium. Let E be the electric intensity and P the total polarization of the medium. The particle is supposed to be acted on by a field $E+\frac{4\pi}{3}P$, as if it were in a spherical cavity in a medium uniformly polarized at each instant to the value P; among other conditions necessary, it must be possible to surround each point by a sphere which contains a large number of particles but whose radius is small compared with the wave-length of the radiation. The equation of motion of the particle is

$$m\frac{\delta^2 d}{\delta t^2}=-mp_1^2d+e\left(E+\frac{4\pi}{3}P\right) \tag{4}$$

Hence for radiation of frequency p,

$$Ned=\frac{Ne^2/m}{p_1^2-p^2}\left(E+\frac{4\pi}{3}P\right)$$

But the total polarization P is ΣNed, taken over all types of vibrator. Combining these results with the fact that P equals $(n^2-1)E/4\pi$, the dispersion formula (3) is deduced. We may write the formula as

$$\frac{n^2-1}{n^2+2}=\frac{n'^2-1}{n'^2+2}+\frac{\frac{4\pi}{3}N\frac{e^2}{m}}{p_1^2-p^2} \tag{5}$$

In the term in n' are included all the terms of the summation except that in p_1. We may call n' the refractive index of the medium if all the vibrators of natural frequency p_1 were destroyed without disturbing the rest of the medium. In most cases the maximum absorption caused by this type of vibrator occurs very near to p_1, so we may treat n' as constant in the following argument. The frequency p_1' for maximum absorption is found to a first approximation by making n infinite in (5); we obtain

$$p_1'^2 = p_1^2 - \frac{n'^2+2}{3} \cdot \frac{4\pi}{3} N \frac{e^2}{m} \tag{6}$$

It appears that the squares of the frequencies p_1 and p_1' differ by a quantity proportional to the effective density of vibrators of the particular type. In general the displacement due to the last term in (6) is very small relatively; in terms of wave-length we have

$$\frac{d\lambda_1}{\lambda_1^3} = \frac{n'^2+2}{3} \cdot \frac{Ne^2}{6\pi^2 mc^2} \tag{7}$$

where the displacement $d\lambda_1$, equal to $\lambda_1' - \lambda_1$, is measured from the wave-length corresponding to the natural vibrations of an isolated particle. The formula follows naturally from the dispersion formula; it remains to be seen how it agrees with observed pressure effects.

6. In the first place, we can simplify the formula by noticing that the factor $\frac{1}{3}(n'^2+2)$ is very nearly equal to unity for gaseous media. In particular, this factor may be taken to represent the influence of the surrounding gas. For if we have N vibrators of frequency p_1 per unit volume uniformly disposed in two different media, the displacements will be in the ratio of the values of $\frac{1}{3}(n^2+2)$ for the two media.

In the theories discussed in the previous sections, the surrounding gas supplies a factor n^2-1. If this were the case the effect should be observable; for instance, for an arc under pressure in air and in carbon dioxide, the displacements should be in the ratio of 2 to 3. Rossi[1] concludes from his experiments that in this case the displacements are the same within the limits of experimental error. This result would be expected from formula

[1] *Philosophical Magazine*, 21, 499, 1911.

(7); for an estimate, we may assume n' to be the refractive index of the gas at 50 atmospheres and from (7) the ratio of the displacements in air and carbon dioxide is 1 to 1.005, other things being equal. We conclude that the effect due to change of refractive index of the surrounding gas is inappreciable. The determining factor according to the present argument is the density N of similar vibrators, and it is conceivable that the surrounding gas might have some direct effect on the value of N.

7. We simplify the formula by putting unity for n', so that

$$\frac{d\lambda}{\lambda^3}=\frac{Ne^2}{6\pi c^2 m} \tag{8}$$

We have no direct evidence as to the value of N, so we cannot prove directly that (8) gives a displacement of the right order of magnitude. But by using an observed value of $d\lambda$ we can calculate N and see if it is a reasonable amount. We take the data from recent experiments by Gale and Adams.[1]

For a group of iron lines of average wave-length 4287 Å, the displacement $d\lambda$ per atmosphere is 0.00274 Å. We express these in centimeters and put in the formula; we also substitute the usual values of the electronic charge and mass. In this way we obtain from this example

$$N=2.3\times 10^{16} \tag{9}$$

If the vapor were an ideal gas at 2730° absolute, a pressure of one atmosphere would correspond to a value of about 4×10^{18} for N. But the temperature and other conditions in the arc are too uncertain to allow of an estimate of the vapor-density; in addition, as in similar investigations, it is probable that only a certain fraction of the metallic atoms is concerned in the production of a given line in the spectrum. We conclude that the values of N deduced from (8) are not unreasonable; at least they do not prohibit further consideration of the application of the formula to pressure effects.

8. The displacement of any line depends upon the effective density of similar vibrators; this is no doubt proportional to the vapor-density, but it probably varies from one line to another,

[1] *Astrophysical Journal*, **35**, 10, 1912.

as may be inferred from the values of the corresponding coefficients in dispersion formulae. Therefore one cannot expect, from (8), any general law of variation of displacement with wave-length; but other things being equal, it indicates proportionality to the cube of the wave-length. This relation has been extracted from experimental results by Duffield in the case of gold, and for iron by Gale and Adams. The process consists in taking averages for large numbers of lines in groups; one may suppose that, if there is no definite relation between N and λ, this process gets rid of the chance variations of N and exhibits the displacement varying as λ^3. On the other hand, for connected lines for which N is a function of λ, the displacement will follow some other law. In certain cases the various lines can be separated roughly into groups for which the values of (displacement)/(wave-length)^3 are nearly in simple ratios such as 1 : 2 : 3. In terms of the formula (8), one might suppose that N is of the same order of magnitude for all the lines and then the simple ratios would be associated with the values of e/m for the vibrators.

9. It is of interest to compare this with a simple theory of the Zeeman effect. If H is the external magnetic field, it can be shown that the displacement, in frequency, is $eH/4\pi m$; or in wave-lengths

$$\frac{d\lambda}{\lambda^2}=\frac{eH}{4\pi mc} \tag{10}$$

In this case the number of similar vibrators is not brought into consideration, since the action is regarded as a direct effect of the external field upon each vibrator. The formula is simpler, and it has been found easier to analyze results by means of it, than for the pressure effect. Experimental results have been compared carefully in order to discover any possible connection between the Zeeman separation and the pressure displacement, but with little result; the most that can be said in certain cases is that, taking the means of separation and displacement for large numbers of lines in the same region of the spectrum, it is found that these means are of the same order of magnitude when they are classified broadly as small, medium, and large.[1]

[1] A. S. King, *Astrophysical Journal*, **31**, 433, 1910; also *ibid.*, **34**, 250, 1911.

From the present point of view no connection is to be expected in general; for the formula for the pressure effect makes $d\lambda/\lambda^3$ proportional to Ne^2/m, and N varies in an unknown manner with the wave-length. If the conditions are such that N is of the same order for groups of lines, one might expect then some correspondence between the values of $d\lambda/\lambda^2$ for the Zeeman effect and $d\lambda/\lambda^3$ for the pressure effect. In other words, any correspondence that may be extracted from experimental results is sufficiently accounted for by the fact that both effects are due to a slight disturbance of the same vibrating system; it is not necessary to suppose that the disturbances are produced in the same way.

10. A conspicuous effect of pressure is a broadening of the lines. To discuss this, we should have to introduce absorption terms into the dispersion formulae. The simplest suppositions give, instead of (3),

$$\left.\begin{aligned} n^2(1-\kappa^2) &= 1+4\pi N\frac{e^2}{m}\;\frac{p'^2_1-p^2}{(p'^2_1-p^2)^2+b_1^2p^2} \\ 2n^2\kappa &= 4\pi N\frac{e^2}{m}\;\frac{b_1p}{(p_1^2-p^2)^2+b_1^2p^2} \end{aligned}\right\} \quad (11)$$

The maximum of $n\kappa$ occurs at a frequency slightly different from p'_1; the difference involves the damping coefficient b_1 and is relatively very small in most cases. The maximum value of $n\kappa$ is proportional to $Ne^2/mb_1p'_1$. Roughly speaking, a greater value of b_1 means a broader and weaker line; while an increase in N both strengthens and broadens the absorption. One might try to connect the displacement of a line with its intensity, but for the occurrence of the factor b_1; whatever be the physical mechanism of absorption, b_1 probably varies from line to line and also with the physical conditions. Also, experimentally one has only estimates of relative intensities at atmospheric pressure and it is known that these vary with the pressure. A further complication is the occurrence of reversals under experimental conditions; so on the whole it is useless to follow this line farther at present.

Some writers have argued that the pressure effect cannot be due to increased vapor-density, or the proximity of similar molecules, because it is known that mere increase of emitting vapor in the arc intensifies and broadens lines without producing any observed

displacement of the maximum. If this is the case, it may be due to an increase in the number of radiators without any considerable change in the physical conditions of the aggregate; just as in an absorption spectrum an increased thickness of the absorbing medium broadens and intensifies each region of absorption without appreciably displacing the maximum. Or again, it may be due to a change in the absorption represented by a change in the damping coefficient b_1.

11. The arguments which have been put forward may be summarized briefly.

Certain theories of the pressure effect are discussed, and it is concluded that they all lead to a displacement which is many times too small; these theories ascribe the effect to the magnetic influence of neighboring atoms of the metallic vapor or of the surrounding gas, or to the dielectric influence of the surrounding gas.

It is suggested that an effect of the right order may be obtained from the electric influence of neighboring similar atoms of the metallic vapor. The formula relies on a known deduction from a dispersion formula and it applies primarily to absorption lines. Considerations are advanced to show that the formula bears comparison in general with experimental results.

Armstrong College
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ON THE DETERMINATION OF THE ORBITAL ELEMENTS OF ECLIPSING VARIABLE STARS. I

By HENRY NORRIS RUSSELL

§ 1. *Statement of the problem.*—Bauschinger, in his exhaustive work on the determination of orbits, remarks concerning the problem of determining the elements of the orbit and the dimensions and brightness of the component stars of an eclipsing variable from the observed light-curve:[1] "Der Zusammenhang zwischen den Grössen-, Formen- und Helligkeitsverhältnissen der Körper und den Elementen der elliptischen Bahn einerseits und der Lichtkurve anderseits ist aber ein so komplizierter, dass man eine allgemeine Theorie wohl kaum aufstellen kann, sondern die Lösung von Fall zu Fall den vorliegenden Verhältnissen anpassen muss."

It is the purpose of the present discussion to show under what circumstances, and to what degree, this problem may be regarded as determinate (in view of the limited accuracy of photometric observations), and to develop formulae and tables which make the solution of the problem, when it is determinate, a simple matter.

In the most general case, the number of unknown quantities to be determined is considerable. The relative orbit will in general be eccentric, and the two components of the system unequal in size and brightness. They may present the appearance of disks not uniformly illuminated, but darkened toward the limb, and may also be elongated toward one another by their mutual attraction, and brighter on the side receiving the radiation of the companion than on that remote from it.

For a complete specification of such a system we must therefore know at least 13 quantities, as follows:

Orbital Elements		Eclipse Elements	
Semi-major axis	a	Radius of larger star	r_1
Eccentricity	e	Radius of smaller star	r_2
Longitude of periastron	ω	Light of larger star	L_1

[1] *Die Bahnbestimmung der Himmelskörper* (Leipzig, 1906), p. 640.

Orbital Elements		Eclipse Elements	
Inclination	i	Light of smaller star	L_2
Period	P	and at least 3 constants defining the amount of elongation, of darkening at the limb, and of brightening of one star by the radiation of the other.	
Epoch of principal conjunction	t_0		

The longitude of the node must remain unknown, as there is no hope of telescopic separation of any eclipsing pair.

The value of a in absolute units can be found only from spectroscopic data. In the absence of these it is desirable to take a as an unknown but definite unit of length, and express all other linear dimensions in terms of it. Similarly, the absolute values of L_1 and L_2 can be determined only if the parallax of the system is known. But in all cases the combined light of the pair, L_1+L_2, can be taken as the unit of light and the apparent brightness at any time expressed in terms of this. This leaves the problem with eleven unknown quantities to be determined from the photometric measures. Of these, the period is invariably known with a degree of accuracy greatly surpassing that attainable for any of the other elements, and the epoch of principal minimum can be determined, almost independently of the other elements, by inspection of the light-curve. Of the remaining elements, the constants expressing ellipticity and "reflection" may be derived from the observed brightness between eclipses.[1] These effects are often so small as to be detected only by the most refined observations.[2] The question of darkening toward the limb may well be set aside until the problem is solved for the case of stars which appear as uniformly illuminated disks.

This leaves us with six unknowns. Fortunately, systems of such short period as the majority of eclipsing variables have usually nearly circular orbits (as is shown both by spectroscopic data and by the position of the secondary minimum). The assumption of a circular orbit is therefore usually a good approximation to the facts, and often requires no subsequent modification.

[1] E. C. Pickering, *Proceedings of American Academy of Arts and Sciences*, **16** 257, 1880.

[2] R. S. Dugan, *Contributions from the Princeton University Observatory*, No. 1, p. 37, 1911; J. Stebbins, *Astrophysical Journal*, **32** 200, 1910.

PART I. SOLUTION FOR SPHERICAL STARS AND CIRCULAR ORBIT

We will therefore first discuss the following simplified problem: *Two spherical stars, appearing as uniformly illuminated disks, and revolving about their common center of gravity in circular orbits, mutually eclipse one another. It is required to find the relative dimensions and brightness of the two stars, and the inclination of the orbit, from the observed light-curve.* The questions arising out of orbital eccentricity, ellipticity, "reflection," and darkening toward the limb will be discussed later.

§ 2. *Notation. Possible cases.*—In this simplified problem we may assume P and t_0 as already known. If the radius of the relative orbit is taken as the unit of length, and the combined light of the two stars as the unit of light, we have to determine four unknown quantities. Of the various possible sets of unknowns, we will select the following:

Radius of the larger star	r_1
Ratio of radii of the two stars	k
Light of the larger star	L_1
Inclination of the orbit	i

The radius of the smaller star is then $r_2 = kr_1$, and its light, $L_2 = 1 - L_1$. It should be noticed that, with the above definitions, k can never exceed unity, but L_2 will exceed L_1 whenever the smaller star is the brighter (which seems to be the fact in the majority of observed cases).

We will suppose that we have at our disposal a well-determined "light-curve," or more accurately, magnitude-curve, giving the stellar magnitude m, which we will suppose to be the vertical co-ordinate, as a function of the time, as horizontal co-ordinate. From this we can pass at once to the intensity-curve, giving the actual light-intensity l as a function of the time, by means of the equation

$$\log l = 0.4(m_0 - m), \qquad (1)$$

where m_0 is the magnitude during the intervals of constant light between eclipses (which is determined with relatively great weight by the observations during these periods and, like P, may be found once for all before beginning the real solution). This of course expresses l in terms of our chosen unit $L_1 + L_2$.

Such a magnitude-curve or intensity-curve will in general show two depressions, or "minima," corresponding to the mutual eclipses of the two components. Under the assumed conditions, it is well known:

1. That the two minima will be symmetrical about their middle points, and that these will be separated by exactly half the period.

2. If the eclipse is total or annular, there will be a constant phase at minimum during which the magnitude- or intensity-curve is horizontal; but if the eclipse is only partial, this will not be the case.

3. The two minima will be of equal duration, but usually of unequal depth. At any given phase during one minimum one of the stars will eclipse a certain area of the apparent disk of the other. Exactly half a period later, at the corresponding phase during the other minimum, the geometrical relations of the two projected disks will be the same, except that now the second star is in front, and eclipses an equal area—though not an equal proportion—of the disk of the first. The intensity-curves for the two minima must therefore differ from one another only as regards their vertical scales, which will be in the ratio of the surface intensities of the two stars.

4. The deeper (primary) minimum corresponds to the eclipse of the star which has the greater surface intensity by the other. Whether this is the larger or smaller star must be determined by further investigation.

Suppose that at any time during the eclipse of the smaller star by the larger the fraction α of its area is hidden. The light received from the system at this moment will be given by the equation

$$l_1 = 1 - \alpha L_2. \tag{2}$$

Half a period later, an equal area of the surface of the larger disk, and hence the fraction $k^2\alpha$ of its whole area will be eclipsed. The observed light will then be

$$l_2 = 1 - k^2\alpha L_1. \tag{3}$$

Since $L_1 + L_2 = 1$, we find at once from these equations

$$(1 - l_1) + \frac{1 - l_2}{k^2} = \alpha. \tag{4}$$

It should be especially noticed that the subscript 1 in these equations applies to the eclipse of the *small* star by the *large* one, and *not* necessarily to the principal minimum.

We may now distinguish four cases of our problem, according to the form of the light-curve, and the extent of our knowledge of the secondary minimum.

1. Both primary and secondary minima have been observed, and show a constant phase.

2. The primary minimum shows a constant phase, and the secondary has not been adequately observed.

3. Both minima have been observed, but show no constant phase.

4. Only the primary minimum, showing no constant phase, has been observed.

In the first case the eclipse of the smaller star by the larger is total, and the other annular. In both cases during the constant phase the whole area of the smaller star is obscured; that is, $a=1$. If then λ_1 and λ_2 are the values of the observed intensity during these constant phases, we have, by (4),

$$k^2=\frac{1-\lambda_2}{\lambda_1}. \tag{5}$$

Moreover, by (2), $\lambda_1=1-L_2=L_1$. The brightness of the two stars and the ratio of their radii, are thus determined, leaving only r_1 and i to be found.

There are, however, two solutions with different values of k according as we regard the principal or secondary minimum as total. We shall see later how we may distinguish the correct solution in a given instance.

In the second case, if the observed minimum intensity is λ and we assume that the observed eclipse is total, we have from (2), $L_2=1-\lambda$; if annular, (3) gives $k^2L_1=1-\lambda$. In either case, for any other value l of the observed intensity,

$$a=\frac{1-l}{1-\lambda}. \tag{6}$$

We thus know a as a function of the time, and from this have to determine k, r_1, and i.

In the third case, if λ_1 and λ_2 are the observed intensities at the minimum phases of the two eclipses, and a_0 is the corresponding value of a, we have

$$a_0 = 1 - \lambda_1 + \frac{1 - \lambda_2}{k^2}. \tag{7}$$

Since $a_0 = \frac{1 - \lambda_1}{L_2}$, we may take it as an unknown instead of L_1 or L_2. We then have to find the four quantities, r_1, i, k, and a_0 with the aid of the intensity-curve and the equation (7).

In the fourth case the situation is the same except that the equation (7) is not available.

§ 3. *Solution when the eclipse is total.*—The solutions in the other cases may be derived from that in Case II, which will now be developed.

Take the center of the larger star as origin, and let θ be the true longitude of the smaller star in its orbit, measured from inferior conjunction. Then

$$\theta = \frac{2\pi}{P}(t - t_0). \tag{8}$$

From the light-curve and (6) we can find the value of a for any value of θ, or vice versa. Now a, which is the fraction of the area of the smaller disk which is eclipsed at any time, depends on the radii of the two disks, and the apparent distance of their centers, but only on the ratios of these quantities (being unaffected by increasing all three in the same proportion). If δ is the apparent distance of centers, we have therefore

$$a = f\left(\frac{r_2}{r_1}, \frac{\delta}{r_1}\right) = f\left(k, \frac{\delta}{r_1}\right),$$

where f is a function, the details of calculation of which will be discussed later.

For any given value of k we may invert this function, and write

$$\frac{\delta}{r_1} = \phi(k, a). \tag{9}$$

This function, or some equivalent one, may be tabulated once for all for suitable intervals of k and a, as is done in Table I below,

which gives a function $p(k, a)$ such that $\phi(k, a)=1+kp(k, a)$. By the geometry of the system, we have

$$\delta^2=\sin^2\theta+\cos^2 i\cos^2\theta=\cos^2 i+\sin^2 i\sin^2\theta, \tag{10}$$

whence

$$\cos^2 i+\sin^2 i\sin^2\theta=r_1^2\{\phi(k, a)\}^2. \tag{11}$$

Now let a_1, a_2, a_3 be any definite values of a and θ_1, θ_2, θ_3 the corresponding values of θ (which may be found from the light-curve). Subtracting the corresponding equations of the form (11) in pairs, and dividing one of the resulting equations by the other, we find

$$\frac{\sin^2\theta_1-\sin^2\theta_2}{\sin^2\theta_2-\sin^2\theta_3}=\frac{\{\phi(k, a_1)\}^2-\{\phi(k, a_2)\}^2}{\{\phi(k, a_2)\}^2-\{\phi(k, a_3)\}^2}=\psi(k, a_1, a_2, a_3). \tag{12}$$

The first member of this equation contains only known quantities. The second, if a_1, a_2, and a_3 are predetermined, is a function of k alone. If this function is tabulated, the value of k in any given case can be found by interpolation, or graphically. The equations (11) can then be used to find r_1 and i.

A theoretical light-curve may then be found, which passes through any three desired points on each branch of the observed curve (assumed symmetrical). These points may be chosen at will by altering the values of a_1, a_2, and a_3. In practice it is convenient to keep a_2 and a_3 fixed, so that ψ becomes a function of k and a_1 only, and may be tabulated for suitable intervals in these two arguments. This has been done in Table II, in which a_2 is taken as 0.6 and a_3 as 0.9. If $A=\sin^2\theta_2$, $B=\sin^2\theta_2-\sin^2\theta_3$ (12) may be written

$$\sin^2\theta_1=A+B\psi(k, a_1). \tag{13}$$

The points a and b on the light-curve corresponding to a_2 and a_3, together with the point corresponding to any one of the tabular values of a_1, then give a determination of k. By taking a suitably weighted mean of these values of k, a theoretical light-curve can be obtained which passes through the points a and b, and as close as possible to the others. By slight changes in the assumed positions of a and b (i.e., in the corresponding values of θ, or of $t-t_0$), it is possible with little labor to obtain a theoretical curve which

fits the whole course of the observed curve almost as well as one determined by least squares. The criterion of this is that the parts of the observed curve below b (near totality), between a and b, and above a (near the beginning or end of eclipse) shall give the same mean value of k. The individual determinations of k are of very different weight. Between a and b (that is for values of α_1 between 0.6 and 0.9) ψ changes very slowly with k. At the beginning and end of the eclipse the stellar magnitude changes very slowly with the time, and hence, by (13), with ψ. The corresponding parts of the curve are therefore ill adapted to determine k. For the first approximation it is well to give the values of k derived from values of α_1 between 0.95 and 0.99, and between 0.4 and 0.2, double weight (provided the corresponding parts of the curve are well fixed by observation). The time of beginning or end of eclipse cannot be read with even approximate accuracy from the observed curve and should not be used at all in finding k. The beginning or end of totality may sometimes be determined with fair precision, but does not deserve as much weight as the neighboring points on the steep part of the curve. If further refinement is desired, it can most easily be obtained by plotting the light-curve for two values of k and comparing with a plot of the observations. This will rarely be necessary.

When once k is given, the determination of the light-curve is a very easy matter. For each tabular value of α_1, equation (13) gives θ_1, and hence $t_1 - t_0$. The values of the stellar magnitude m corresponding to given values of α_1 are already available, having been used in the previous work. The light-curve may thus be plotted by points in a few minutes.

After a satisfactory light-curve has been computed, we may proceed to determine the remaining elements. Let θ' and θ'' be the values corresponding to the beginning of eclipse ($\alpha_1 = 0$) and to the beginning of totality ($\alpha_1 = 1$). Then by (13)

$$\sin^2 \theta' = A + B\psi(k, 0) \text{ and } \sin^2 \theta'' = A + B\psi(k, 1).$$

These computed values are more accurate than those estimated from the free-hand curve drawn to represent the observations.

At the first of these epochs $\delta = r_1 + r_2$, and at the second $\delta = r_1 - r_2$. We have then, by (10)

$$r_1^2(1+k)^2 = \cos^2 i + \sin^2 i \sin^2 \theta',$$
$$r_1^2(1-k)^2 = \cos^2 i + \sin^2 i \sin_2 \theta'',$$

whence

$$4k \cot^2 i = (1-k)^2 \sin^2 \theta' - (1+k)^2 \sin^2 \theta'',$$
$$4kr_1^2(1+\cot^2 i) = \sin^2 \theta' - \sin^2 \theta''.$$

Introducing A and B, we have

$$4k \cot^2 i = -4kA + B\{(1-k)^2\psi(k, 0) - (1+k)^2\psi(k, 1)\},$$
$$4kr_1^2 \operatorname{cosec}^2 i = B\{\psi(k, 0) - \psi(k, 1)\}.$$

The coefficients are functions of k alone, and may be tabulated. It is most convenient for this purpose to put the equations in the form

$$\left.\begin{aligned} r_1^2 \operatorname{cosec}^2 i &= \frac{B}{\phi_1(k)}, \\ \cot^2 i &= \frac{B}{\phi_2(k)} - A, \end{aligned}\right\} \quad (14)$$

as in this way we obtain functions whose tabular differences are comparatively smooth (which is not true of their reciprocals). With the aid of these functions the elements may be found as soon as A and B are known. If $\frac{B}{A} < \phi_2(k)$ the computed value of $\cot i$ is imaginary and the solution is physically impossible. It is therefore advisable to apply this test to the preliminary values of A, B, and k, and, if necessary, to adjust them so that the solution is real. The limiting condition is evidently $\cot i = 0$, corresponding to central transit.

The geometrical elements of the system are now determined. We are still in doubt, however, whether the principal eclipse is total or annular. This can be determined only by consideration of the secondary minimum. The intensities during constant phase at the two minima are connected by the relation $k^2\lambda_1 + \lambda_2 = 1$. If the intensity at principal minimum is λ_p, that at the secondary minimum will be $1 - k^2\lambda_p$ if the principal eclipse is total, and $\frac{1-\lambda_p}{k^2}$

if it is annular. The first of these expressions is always positive and less than unity. The second exceeds unity if $1-\lambda_p > k^2$. The assumption of total eclipse at principal minimum leads therefore in all cases to a physically possible solution. That of an annular eclipse does so only if $1-\lambda_p$ is not greater than k^2. Otherwise the computed brightness of the smaller star is negative. The brightness at secondary minimum will be greater than at the primary by $1-\lambda_p(1+k^2)$ if the primary eclipse is total, and $\frac{1}{k^2}\{1-\lambda_p(1+k^2)\}$ if it is annular. The latter hypothesis therefore gives rise to the shallower minimum. In many cases it may be impossible to decide between the two without actual observations of the secondary phase. The computed depth of secondary minimum may, however, be so great that it is practically certain that it would sometimes have been observed if it really existed. The corresponding hypothesis should then be rejected. If $\lambda_p(1+k^2)$ is nearly unity, the primary and secondary minimum, on both hypotheses, must be of nearly equal depth. This can occur only if $\lambda_p < \frac{1}{2}$; that is, if the depth of minimum is less than 0.75 mag. In such a case it is probable that the period is really twice that so far assumed, that the two stars are of equal surface brightness, and that two sensibly equal eclipses occur during each revolution. The true values of θ are therefore half those previously computed with the shorter period. If the determination of k is repeated on this basis, and the equation $\lambda_p(1+k^2)=1$ is still approximately satisfied this solution may be adopted.

Such a system presents a specialized example of Case I, when both primary and secondary minima have been observed and show a constant phase. In this case, by (5), $k^2=\frac{1-\lambda_2}{\lambda_1}$ where λ_1 corresponds to the total eclipse, which, so far as we yet know, may occur at either minimum. As before we begin by finding from the light-curve the values of $\sin^2\theta$ corresponding to given values of α_1. From a few of these, by the method already described, an approximate value of k may be obtained which is sufficient to show which of the values given by (5) on the two possible hypotheses is the correct one.

We have next to find the light-curve which gives the best representation of the observations consistent with the value of k given by (5). The form of the light-curve now depends only on the constants A and B in the equation

$$\sin^2 \theta_1 = A + B\psi(k, a_1). \qquad (13)$$

Approximate values of these constants may be derived as above from the values of $\sin^2 \theta$ when $a = 0.6$ and 0.9. These may be improved by trial and error, which will be aided by plotting the resulting light-curves along with the observations, and, if the data warrant it, may finally be corrected by least squares.[1] When satisfactory values of A and B have been determined, the final light-curve may be computed by (13), and the elements by (14), as in Case II, except that here there is no uncertainty as regards the nature of the principal eclipse.

In review of the foregoing, it may be remarked that the method of solution is direct and simple. It involves a very moderate amount of numerical work, of which the greater part—namely, the determination of the values of the magnitude, time, and position in orbit (θ) corresponding to different percentages of obscuration (a)—requires no modification during the successive approximations. The light-curve may be found without the necessity of

[1] The equations of condition are of the form

$$\frac{dm}{d(\sin^2 \theta)}\delta A + \psi(k, a)\frac{dm}{d(\sin^2 \theta)}\delta B = \text{O.} - \text{C.}$$

where the second member represents the difference between the observed and computed magnitudes. The coefficients may easily be obtained graphically. By plotting the computed magnitude m against $\sin^2 \theta$, the values of $\frac{dm}{d\,(\sin^2 \theta)}$ may be read off, from the slope of the tangent, at each point used in the construction of the curve. The values of ψ and of $t - t_0$ corresponding to these points are already known, and hence the coefficients of the equations (of condition) may be plotted as functions of $t - t_0$, and read off for each observation.

A similar process might be adopted when only the primary minimum has been observed, including a correction to k among the unknowns. The equations of condition would then be of the form

$$\frac{dm}{d(\sin^2 \theta)}\left(\delta A + \psi(k, a)\delta B + B\frac{\delta\psi}{\delta k}\delta k\right) = \text{O.} - \text{C.}$$

This would, however, be worth while only when computation showed that the secondary minimum must be so shallow as to be practically unobservable, as otherwise elements derived from the primary minimum alone could in no sense be regarded as definitive.

computing the elements, and with two or three trials may be determined so as to represent the whole course of the observations, making the laborious solution by least squares superfluous except in the case of observations of unusual precision. Such a solution itself is much simplified if the constants defining the light-curve, instead of the elements of the system, are treated as the fundamental unknowns, as the coefficients of the equations of condition may be easily found graphically with the aid of data already computed. The elements may be found, at any stage of the process, by a few moments' calculation, from the constants defining the light-curve.

§ 4. *Solution when the eclipse is partial.*—Passing now to Cases III and IV, where the observed curve shows no constant phase, and the eclipse is partial, we can no longer use the previous methods since the values of a corresponding to the maximum eclipse, or to any other phase, are unknown. We may, however, determine the magnitude corresponding to an obscuration of any given fraction of the maximum obscuration a_0, and the corresponding values of l and θ.

Let n represent this fraction. We then have

$$a=na_0\,,\quad\text{and}\quad 1-l=n(1-\lambda). \tag{15}$$

Let $\theta(n)$ denote the corresponding value of θ. Then we have by (13)

$$\sin^2\theta(n)=A+B\psi(k,\ na_0).$$

The value $n=1$ corresponds to the middle of eclipse when $\theta=0$. Hence

$$A+B\psi(k,\ a_0)=0.$$

Subtracting, we have

$$\sin^2\theta(n)=B\{\psi(k,\ na_0)-\psi(k,\ a_0)\}.$$

Dividing this by the similar equation for any fixed value of n (say $\frac{1}{2}$), we eliminate the constants of the individual light-curve, and find

$$\frac{\sin^2\theta(n)}{\sin^2\theta(\frac{1}{2})}=\frac{\psi(k,\ na_0)-\psi(k,\ a_0)}{\psi(k,\ \frac{1}{2}a_0)-\psi(k,\ a_0)}=\chi(k,\ a_0,\ n), \tag{16}$$

(where the above equation is to be taken as the definition of the new function χ). The first member contains only known quantities. The second is a function of k, a_0, and n, which may be

tabulated for any convenient values. We might then expect to solve the problem by constructing two or more such tables (e.g., for $n=\frac{1}{4}$ and $n=\frac{3}{4}$), and finding for what values of k and a_0 the two functions $\chi(k, a_0, \frac{1}{4})$ and $\chi(k, a_0, \frac{3}{4})$ had the values assigned each by equation (16).

But when this experiment is actually tried it is found that the functions χ (regarded as a function of k and a_0 for different constant values of n) are all so nearly functions of one another that the solution becomes practically indeterminate. In other words, if we give k and a_0 any pair of values which make some one of these functions, say $\chi(k, a_0, \frac{1}{4})$ equal to a given value, we will by this very process constrain all the other functions $\chi(k, a_0, n)$ to be very nearly (though not exactly) equal to certain other constant values (depending of course on n).

This may be illustrated by the following examples which give, for given values of this function, pairs of corresponding values of k and a_0, and the resulting values of two other functions of the series.

$\chi(k, a_0, \frac{1}{4})$	k	a_0	$\chi(k, a_0, \frac{3}{4})$	$\chi(k, a_0, 0)$
2.20	0.89	1.00	0.32	5.05
	1.00	0.91	.32	5.08
2.00	1.00	0.74	0.37	4.24
	0.90	.84	.37	4.26
	.80	.98	.38	4.26
1.80	1.00	0.45	0.42	3.41
	0.90	.52	.43	3.42
	.80	.60	.43	3.42
	.70	.73	.42	3.45
	.62	.90	.43	3.45
	.64	1.00	.44	3.46
1.70	1.00	0.22	0.45	3.03
	0.80	.27	.46	3.01
	.60	.42	.45	3.04
	.50	.55	.46	3.02
	.48	.70	.46	3.02
	.48	.90	.45	3.07
	.53	1.00	.48	3.11
1.60	0.00	0.28	0.48	2.57
	.10	.48	.48	2.58
	.14	.60	.49	2.57
	.23	.80	.49	2.62
	.41	1.00	.52	2.72

The time of beginning of eclipse cannot be determined from the curve with any certainty, and hence $\chi(k, a_0, 0)$ is valueless for determining k and a_0. It is clear that to obtain a reliable determination of them in this way, it would be necessary to carry the function $\chi(k, a_0, \frac{3}{4})$ and the ratios $\frac{\sin^2\theta(\frac{3}{4})}{\sin^2\theta(\frac{1}{2})}$ to at least three decimal places; that is, that we should be able to determine the interval during which the star is apparently below a given magnitude during eclipse to within one part in a thousand. This is obviously out of the question. We may therefore conclude: *To the degree of approximation attained by any existing photometric measures, the problem of determining the elements of an eclipsing variable solely from the light-curve of a primary minimum without constant phase is indeterminate.* For any value k of the ratio of the radii of the components, comprised within wide limits, it is possible to find an assumed percentage a_0 of maximum eclipse, and hence a set of elements, such that the interval from the middle of eclipse, at which any given magnitude is reached, as calculated from any of these systems of elements, will be the same within a fraction of 1 per cent.

For practical purposes, therefore, we may regard the functions $\chi(k, a_0, n)$ as functions of n and of any one function of the set, e.g., $\chi(k, a_0, \frac{1}{4})$. By a happy accident, the relation between any pair of them, corresponding to different fixed values of n, is very nearly linear, as is illustrated below.

$\chi(k, a_0, \frac{1}{4})$	$\chi(k, a_0, 0)$	Comp.	O.−C.	$\chi(k, a_0, \frac{3}{4})$	Comp.	O.−C.
2.46	6.15	6.15	0.00	0.238	0.235	+0.003
2.40	5.90	5.90	.00	.249	.253	− .004
2.30	5.50	5.49	+ .01	.282	.283	− .001
2.20	5.08	5.08	.00	.310	.313	− .003
2.10	4.68	4.67	+ .01	.345	.343	+ .002
2.00	4.25	4.26	− .01	.378	.373	+ .005
1.90	3.84	3.85	− .01	.407	.403	+ .004
1.80	3.44	3.44	.00	.433	.433	.000
1.70	3.04	3.03	+ .01	.458	.463	− .005
1.60	2.61	2.62	− .01	.490	.493	− .003
1.50	2.32	2.21	+ .11	.540	.523	+ .017
1.41	2.00	1.84	+ .16	.590	.550	+ .040

The tabular values of $\chi(k, a_0, 0)$ and $\chi(k, a_0, \frac{3}{4})$ are the means of the slightly varying values (such as are given in the preceding

table) for the different values of k and a_0, which make $\chi(k, a_0, \frac{1}{4})$ equal to the value given in the first column. The columns headed "Comp." are derived from the equations:

$$\chi(k, a_0, 0) = 4.10\chi(k, a_0, \tfrac{1}{4}) - 3.94$$
$$\chi(k, a_0, \tfrac{3}{4}) = 0.973 - 0.300\chi(k, a_0, \tfrac{1}{4}).$$

These formulae, although wholly empirical, represent the computed data almost within the errors of reckoning, except for values of $\chi(k, a_0, \frac{1}{4})$ less than 1.60. These last can occur only for values of k and a_0 so small that they are very unlikely to be met with in practice, so that the linear formulae are abundantly sufficient in all ordinary cases.

Similar linear relations are fulfilled with an equal degree of approximation for other values of n—the coefficients being functions of this quantity. We may therefore write in general

$$\chi(k, a_0, n) = \omega_1(n) + \omega_2(n)\chi(k, a_0, \tfrac{1}{4}). \qquad (17)$$

The three functions occurring in this expression have been computed and tabulated, and may be considered known. But by (16) we have

$$\sin^2 \theta(n) = \sin^2 \theta(\tfrac{1}{2})\chi(k, a_0, n),$$

whence

$$\sin^2 \theta(n) = \omega_1(n) \sin^2 \theta(\tfrac{1}{2}) + \omega_2(n) \sin^2 \theta(\tfrac{1}{4}).$$

If we set $\sin^2 \theta(\frac{1}{4}) = C$, $\sin^2 \theta(\frac{1}{2}) = D$, this becomes

$$\sin^2 \theta(n) = C\omega_2(n) + D\omega_1(n). \qquad (18)$$

This equation, though not rigorously exact, must be very nearly satisfied in the case of all light-curves arising from partial eclipses. Having given any observed light-curve, the values of C and D may be determined in the manner already described on page 325, and a theoretical light-curve be found, which closely represents the whole course of the observations.

Knowledge of this light-curve, however, does not enable us to find the elements, but only a relation between them of the form

$$\chi(k, a_0, \tfrac{1}{4}) = \frac{C}{D}. \qquad (19)$$

Unless the secondary minimum has been observed, we can go no farther. But if we know the brightness at both minima we have another relation between k and a_0, of the form

$$a_0 = 1 - \lambda_1 + \frac{1-\lambda_2}{k^2}, \tag{7}$$

where, as always, λ_1 represents the light-intensity of the system at the middle of the eclipse of the smaller star by the larger.

These two equations must be solved for a_0 and k. As good a way as any is to compute a_0 from (7) for equidistant values of k, then take $\chi(k, a_0, \frac{1}{4})$ from Table III and find by interpolation what value of k satisfies (19). A graphical solution may be made by plotting on one sheet (a) the curves $\chi(k, a_0, \frac{1}{4})=$const., with k and a_0 as co-ordinates, and on another transparent sheet (b) the curves $a_0 = \frac{\text{const.}}{k^2}$. If these diagrams are superposed so that the point $a_0 = 0$, $k = 1$, on (b) lies above the point $a_0 = 1 - \lambda_1$, $k = 1$ on (a), then the intersection of the curves $\chi = \frac{C}{D}$ on (a) and $a_0 = \frac{1-\lambda_2}{k^2}$ on (b) will give the desired solution.

This graphical method has the advantage of exhibiting very clearly the degree of uncertainty of the results. When λ_1 and λ_2 are nearly equal the curves in question on (a) and (b) run very nearly parallel for a considerable part of their course (corresponding to the larger values of k), and the solution is usually very nearly indeterminate. If the two minima are considerably unequal, the two sets of curves cut at a considerable, though usually an acute, angle and the solution is determinate, and in most cases unique. When, however, $\lambda_2 > 0.60$ and $\lambda_1 > 0.70$, there may be two solutions, and if these lie near together, their determination becomes very uncertain. These indeterminate cases, it should be noticed, can arise only when the loss of light at minimum is less than half the original light, that is, when the minimum is less than $0^m.75$ in depth.

When the principal minimum corresponds to the eclipse of the smaller star by the larger (that is, when $\lambda_1 < \lambda_2$) the curve (b) is steeper than the curve (a) (a_0 being the horizontal co-ordinate). If the two curves intersect for some value of k less than unity, the

curve (b) must give the greater value of a_0 when $k=1$. When $\lambda_2 < \lambda_1$, the curve (a) is steeper than (b) and the reverse is the case. Now the equation of (a) is (19), and of (b) is (7). When $k=1$ the latter gives $a_0 = 2 - \lambda_1 - \lambda_2$. The value of a_0 which satisfies the former may be found in any given case from Table III. Call it β. Then if $2 - \lambda_1 - \lambda_2 > \beta$ the smaller star is behind the larger at principal eclipse; and vice versa. If these two quantities are equal, $k=1$, that is, the two stars are of the same radius. When both minima are of very small depth (say $0^{m}.15$ or less), the indeterminate character of the solution is pronounced (except when k is near its lower limit, and the eclipse is nearly total). The exact form of the light-curve for so shallow a minimum is especially difficult to fix by observation. It follows that the eclipsing variables of small range, such as have been recently discovered with the selenium photometer, present a problem which can be solved only by the addition of other data than those derivable from the light-curve. Fortunately, it is in just these cases that spectroscopic data make it possible to estimate the ratio of the actual light-emissions of the components, and also determine with certainty which component is in front at each eclipse. The ratio of the surface-intensity of the components being that of the loss of light at the two eclipses, k may at once be determined.[1]

Whenever, by any of these means, k and a_0 have been found, the determination of the remaining elements is simple. At the middle of eclipse $\theta = 0$, and the apparent distance of centers is $(1+pk)$ times the radius of the larger star, where p is the function of k and a_0, defined on page 320, whose values are given in Table I. The value of θ at the beginning or end of the eclipse may be computed by (18) setting $n=0$. Calling this θ', we have, as before,

$$r_1^2(1+k)^2 = \cos^2 i + \sin^2 i \sin^2\theta',$$

which may be written

$$\left.\begin{array}{l} r_1^2(1+k)^2 = \cos^2 i \cos^2\theta' + \sin^2\theta'. \\ \text{For the middle of the eclipse, we have} \\ r_1^2(1+pk)^2 = \cos^2 i\,. \end{array}\right\} \quad (20)$$

[1] F. Schlesinger, *Publications of the Allegheny Observatory*, **2**, 56.

These two equations determine r_1 and $\cos i$. The relations $r_2 = kr_1$, $a_0 L_2 = 1 - \lambda_1$, determine the remaining elements.

The above method of solution of the problem presented by a partial eclipse, though not rigorously exact, is sufficiently accurate to be adopted as final, unless the observations are unusually numerous and precise. If it seems desirable to proceed farther, the light-curve which corresponds rigorously to the assumed elements may be computed by the formulae

$$\sin^2 \theta = A + B\psi(k, a), \tag{13}$$

$$1 - l = \frac{a}{a_0}(1 - \lambda), \tag{21}$$

where A and B are derived from r_1, k, and i by means of the equations (14) (p. 323). The light-curves for both primary and secondary minima should be computed, using the appropriate values of λ in (21). If they do not represent the observations sufficiently well, the assumed constants may be varied—in which case it is well to take as fundamental data λ_1, λ_2 (the light-intensities at the middle of the two minima), k, and A, determining a_0 by the equation (7) and then B by the equation

$$A + B\psi(k, a_0) = 0. \tag{22}$$

As before, the elements need not be computed until a satisfactory light-curve has been found.

PART II. CONSTRUCTION OF THE TABLES

All the tables necessary for computing the elements of eclipsing variables may be derived from Table I. This gives a function of two variables, $p\ (k, a)$, which may be defined as follows.

Let a circle of radius unity cut a smaller circle of radius k in such a way that the area of the interior segment of the latter is a times that of the whole circle. The distance of the centers of the two circles is then $1 + kp(k, a)$; or p is the ratio which the distance of the center of the smaller circle from the circumference of the larger bears to the radius of the former. For any fixed value of k, a may be computed when p is known by familiar equations; but it saves much work to compute corresponding values of a and p for assigned values of some other variable. In the present

TABLE I

RELATION BETWEEN THE ECLIPSED AREA a AND THE DISTANCE OF CENTERS δ

a	k=1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
0.00	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000	+1.000
.01	0.919	0.921	0.922	0.924	0.925	0.927	0.929	0.930	0.932	0.934	0.935
.02	.868	.871	.873	.876	.879	.881	.884	.887	.890	.892	.895
.05	.755	.759	.764	.769	.774	.779	.785	.790	.795	.800	.805
.10	.610	.618	.624	.631	.638	.645	.653	.661	.670	.678	.687
0.15	+0.488	+0.496	+0.504	+0.513	+0.523	+0.533	+0.544	+0.554	+0.565	+0.576	+0.585
.20	.374	.388	.398	.408	.419	.430	.443	.456	.469	.481	.492
.25	.267	.284	.297	.310	.322	.335	.348	.363	.378	.391	.405
.30	.168	.186	.200	.216	.230	.244	.258	.272	.288	.303	.321
.35	+ .075	.094	.110	.127	.143	.160	.175	.190	.207	.222	.239
0.40	−0.015	+0.005	+0.024	+0.041	+0.059	+0.077	+0.094	+0.109	+0.126	+0.143	+0.159
.45	− .106	− .081	− .061	− .042	− .023	− .004	+ .013	+ .028	+ .045	+ .062	+ .079
.50	− .194	− .166	− .145	− .124	− .103	− .084	− .067	− .051	− .034	− .017	− .000
.55	− .280	− .250	− .226	− .204	− .184	− .165	− .148	− .131	− .113	− .096	− .079
.60	− .364	− .332	− .306	− .284	− .263	− .244	− .226	− .209	− .192	− .175	− .159
0.65	−0.447	−0.413	−0.386	−0.363	−0.343	−0.323	−0.305	−0.288	−0.271	−0.255	−0.239
.70	− .528	− .492	− .465	− .441	− .420	− .401	− .383	− .367	− .350	− .336	− .321
.75	− .607	− .571	− .544	− .520	− .498	− .481	− .463	− .448	− .432	− .419	− .405
.80	− .686	− .649	− .622	− .600	− .580	− .563	− .546	− .532	− .517	− .504	− .492
.85	− .765	− .728	− .701	− .680	− .663	− .648	− .633	− .620	− .607	− .596	− .585
0.90	−0.843	−0.807	−0.783	−0.764	−0.749	−0.736	−0.725	−0.715	−0.705	−0.696	−0.687
.95	− .922	− .890	− .872	− .858	− .847	− .838	− .830	− .823	− .817	− .811	− .805
.98	− .967	− .945	− .935	− .928	− .922	− .915	− .910	− .905	− .900	− .896	− .892
.99	− .983	− .967	− .960	− .955	− .951	− .948	− .945	− .942	− .939	− .937	− .934
1.00	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000

The tabular quantity is $p(k, a)$, $\delta = r_1 (1 + kp)$.

case the common chord of the two circles was chosen. Call this $2s$. We then have at once for the distance of centers $1+kp=\sqrt{1-s^2}\pm\sqrt{k^2-s^2}$, the upper or lower sign being employed according as the center of the smaller circle is outside or inside the chord. Let A_1 and A_2 be the areas of the smaller segments cut from the two circles by this chord. Then, when the center of the smaller circle is outside the chord, we have

$$\alpha=\frac{A_1+A_2}{\pi k^2},$$

otherwise

$$\alpha=1-\frac{A_2-A_1}{\pi k^2}.$$

If $s=\sin\phi_1=k\sin\phi_2$, we have

$$A_1=\phi_1-\sin\phi_1\cos\phi_1, \qquad A_2=k^2(\phi_2-\sin\phi_2\cos\phi_2).$$

The computation of these angles may be avoided by introducing the heights of the two segments, which are defined by the equations

$$h_1=1-\sqrt{1-s^2}=1-\cos\phi_1, \qquad h_2=k-\sqrt{k^2-s^2}=k(1-\cos\phi_2).$$

We then have

$$A_1=sh_1\frac{\phi_1-\sin\phi_1\cos\phi_1}{\sin\phi_1-\sin\phi_1\cos\phi_1}, \qquad \frac{h_1}{s}=\frac{1-\cos\phi_1}{\sin\phi_1}=\tan\tfrac{1}{2}\phi_1,$$

and similarly for A_2 and h_2.

If we set

$$\frac{\phi-\sin\phi\cos\phi}{\sin\phi-\sin\phi\cos\phi}=F(\tan\tfrac{1}{2}\phi),$$

the expressions previously found for α and p become

$$\alpha=\frac{s}{k_2}\left\{h_1F\left(\frac{h_1}{s}\right)+h_2F\left(\frac{h_2}{s}\right)\right\}, \qquad p=1-\frac{h_1+h_2}{k};$$

$$\alpha=1-\frac{s}{k_2}\left\{h_2F\left(\frac{h_2}{s}\right)-h_1F\left(\frac{h_1}{s}\right)\right\}, \qquad p=\frac{h_2-h_1}{k}-1.$$

The function F changes slowly, and a very small table suffices for it.

Every assumed value of s gives two points in the light-curve, that is, two pairs of values of α and p. By plotting these points on a suitable scale, the values of p corresponding to given values of α can be read off. A ten-inch slide-rule was used in the actual

TABLE II

For Use in Case of Total Eclipse. Values of $\psi\ (k, \alpha_1)$

α_1	k = 1.00	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	0.00
0.00	+9.464	+7.478	+6.200	+5.279	+4.556	+3.984	+3.503	+3.104	+2.755	+2.454	+2.199
.02	8.095	6.457	5.373	4.606	4.000	3.504	3.106	2.768	2.478	2.216	2.000
.05	7.042	5.616	4.704	4.047	3.534	3.118	2.777	2.488	2.241	2.017	1.829
0.10	+5.759	+4.625	+3.895	+3.364	+2.960	+2.627	+2.358	+2.131	+1.934	+1.754	+1.603
.15	4.755	3.839	3.248	2.826	2.504	2.240	2.024	1.841	1.682	1.537	1.412
.20	3.906	3.184	2.712	2.374	2.110	1.898	1.726	1.581	1.453	1.336	1.235
0.25	+3.158	+2.600	+2.232	+1.969	+1.760	+1.591	+1.453	+1.344	+1.242	+1.146	+1.070
.30	2.522	2.088	1.803	1.603	1.443	1.314	1.205	1.115	1.039	0.968	0.911
.35	1.979	1.641	1.425	1.276	1.157	1.061	0.982	0.911	0.854	0.797	.756
0.40	+1.490	+1.245	+1.087	+0.978	+0.894	+0.825	+0.770	+0.721	+0.675	+0.633	+0.604
.45	1.040	0.881	0.777	.705	.649	.603	.566	.530	.501	.473	.453
.50	0.648	.555	.491	.451	.418	.392	.370	.348	.331	.314	.302
0.55	+0.300	+0.258	+0.233	+0.217	+0.202	+0.191	+0.181	+0.171	+0.164	+0.156	+0.151
.60	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
.65	− .258	−0.231	−0.214	−0.202	− .191	−0.181	−0.174	−0.167	−0.160	−0.156	−0.152
0.70	−0.480	−0.435	−0.408	−0.387	−0.369	−0.354	−0.344	−0.331	−0.320	−0.314	−0.306
.75	− .660	− .613	− .584	− .558	− .539	− .522	− .508	− .494	− .483	− .475	− .465
.80	− .805	− .765	− .738	− .717	− .700	− .684	− .670	− .659	− .647	− .639	− .632
0.85	−0.922	−0.893	−0.877	−0.863	−0.854	−0.843	−0.833	−0.825	−0.818	−0.812	−0.808
.90	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000	−1.000
.95	−1.045	−1.085	−1.112	−1.134	−1.152	−1.166	−1.179	−1.190	−1.203	−1.214	−1.226
0.98	−1.0625	−1.126	−1.176	−1.220	−1.256	−1.284	−1.308	−1.329	−1.350	−1.369	−1.391
.99	−1.0643	−1.139	−1.199	−1.250	−1.293	−1.328	−1.362	−1.390	−1.419	−1.444	−1.471
1.00	−1.0650	−1.155	−1.231	−1.297	−1.354	−1.402	−1.445	−1.484	−1.525	−1.556	−1.596

work, and the computation of 21 points, sufficient to define the curve for a given value of k, took only about fifteen minutes. Table I as here printed has been carefully checked by differences, both horizontal and vertical, and the errors of the tabular quantities ought not to exceed one or two units of the last decimal place. The tabular interval has been made small enough to permit linear interpolation in both components except for very small or very large values of α.

Table II contains the function $\psi(k, \alpha_1)$ defined by the equation

$$\psi(k, \alpha_1)=\frac{\{1+kp(k, \alpha_1)\}^2-\{1+kp(k, \alpha_2)\}^2}{\{1+kp(k, \alpha_2)\}^2-\{1+kp(k, \alpha_3)\}^2},$$

(where $\alpha_2=0.6$ and $\alpha_3=0.9$), which is used in determining k in the case of total eclipse. The uncertainty of the tabular quantities does not exceed one or two units of the last decimal place, except for the larger values of ψ, corresponding to values of α_1 less than 0.3, for which the actual errors may be greater, but are not more serious in proportion to the whole quantity tabulated.

Table II*a* contains the functions

$$\phi_1(k)=\frac{4k}{\psi(k, 0)-\psi(k, 1)} \quad \text{and} \quad \phi_2(k)=\frac{4k}{(1-k)^2\psi(k, 0)-(1+k)^2\psi(k, 1)},$$

which are useful in determining the elements in the case of total eclipse.

Table III contains the function

$$\chi(k, \alpha_0, \tfrac{1}{4})=\frac{\psi(k, \frac{1}{4}\alpha_0)-\psi(k, \alpha_0)}{\psi(k, \frac{1}{2}\alpha_0)-\psi(k, \alpha_0)},$$

which is of use in the case of partial eclipses. The accuracy of the tabular quantities is comparable with that in the previous tables. Table III*a* contains the functions $\omega_1(n)$ and $\omega_2(n)$ which appear as coefficients in the empirical relation

$$\chi(k, \alpha_0, n)=\omega_1(n)+\omega_2(n)\chi(k, \alpha_0, \tfrac{1}{4}),$$

which has been found to represent the individual computed values with such remarkable approximation. The values resulting from

TABLE IIa

For Computing the Elements in the Case of Total Eclipse

k	$\phi_1(k)$	$\phi_2(k)$
1.00	0.380	0.939
0.95	.401	.894
.90	.417	.848
0.85	0.427	0.802
.80	.431	.755
.75	.431	.709
0.70	0.427	0.663
.65	.419	.617
.60	.406	.572
0.55	0.390	0.527
.50	.371	.482
.45	.349	.436
0.40	0.323	0.390
.35	.294	.345
.30	.262	.298
0.25	0.226	0.250
.20	.187	.202
.15	.145	.153
0.10	0.100	0.103
.05	.052	.052
.00	.000	.000

TABLE III

For Use in Case of Partial Eclipse. Values of $\chi\ (k, \alpha_0, \frac{1}{4})$

α_0	$k=1.00$	0.95	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	0.00
1.00...	2.462	2.316	2.194	2.010	1.870	1.756	1.667	1.596	1.541	1.491	1.441	1.407
0.98...	2.402	2.270	2.168	2.005	1.872	1.770	1.690	1.628	1.572	1.522	1.479	1.437
.95...	2.328	2.208	2.121	1.985	1.872	1.779	1.703	1.638	1.586	1.540	1.499	1.460
.90...	2.218	2.135	2.062	1.950	1.860	1.778	1.708	1.650	1.605	1.560	1.522	1.485
.85...	2.127	2.067	2.010	1.919	1.840	1.771	1.710	1.660	1.618	1.577	1.540	1.501
0.80...	2.054	2.010	1.968	1.890	1.823	1.763	1.711	1.665	1.626	1.589	1.552	1.510
.70...	1.956	1.926	1.896	1.842	1.792	1.749	1.711	1.675	1.640	1.606	1.570	1.532
.60...	1.887	1.860	1.838	1.799	1.764	1.734	1.707	1.677	1.648	1.620	1.586	1.551
.50...	1.828	1.805	1.789	1.762	1.740	1.719	1.699	1.675	1.650	1.626	1.597	1.568
.40...	1.772	1.761	1.751	1.732	1.717	1.700	1.685	1.669	1.650	1.629	1.607	1.584
0.30...	1.727	1.722	1.717	1.709	1.700	1.688	1.674	1.661	1.646	1.635	1.615	1.598
.20...	1.693	1.692	1.690	1.687	1.682	1.675	1.665	1.655	1.642	1.635	1.623	1.608
.10...	1.673	1.672	1.670	1.666	1.660	1.655	1.650	1.646	1.639	1.633	1.626	1.616
.00...	1 630	1.630	1.630	1.630	1.630	1.630	1.630	1.630	1.630	1.630	1.630	1.630

TABLE III*a*

FOR COMPUTING THE FORM OF THE LIGHT-CURVE IN CASE OF PARTIAL ECLIPSE

n	$\omega_1(n)$	$\omega_2(n)$
0.00	−3.94	+4.10
.10	−1.45	+2.21
.20	−0.399	+1.330
.25	.000	+1.000
0.30	+0.316	+0.720
.35	+ .567	+ .488
.40	+ .758	+ .295
.45	+ .899	+ .133
0.50	+1.000	0.000
.55	+1.065	−0.107
.60	+1.095	− .190
.65	+1.090	− .249
0.70	+1.046	−0.285
.75	+0.967	− .297
.80	+ .846	− .285
.85	+ .693	− .250
0.90	+0.503	−0.191
.95	+ .273	− .108
.98	+ .114	− .047
.99	+ .058	− .024
1.00	0.000	0.000

the individual computations for different values of n have been carefully smoothed and the table is probably as trustworthy as the others. For $n=0$ and $n=\frac{3}{4}$, the values of $\chi(k, a_0, n)$ have been computed over the whole range of values of k and a_0; for the other values of n only for $a_0=0.80$ (which appeared in the previous computations to give results very closely approximating to the mean for all values of a_0).

Two auxiliary tables are added to facilitate numerical computation. Table A gives the *loss* of light $(1-\lambda)$, corresponding to a given change Δm in stellar magnitude.[1] For a difference of magnitude greater than 2.5, the loss of light is $0.9000+\frac{1}{10}$ of the tabular value for $\Delta m-2^{\mathrm{m}}.5$. Table B gives the values of $\theta-\sin\theta$ for every 0.01 of θ (expressed in circular measure), and saves much labor

[1] S. Blazko, *op. cit.*, p. 106.

TABLE A

Loss of Light Corresponding to an Increase Δm in Stellar Magnitude

Δm	0	1	2	3	4	5	6	7	8	9
0 0.	0.0000	0.0092	0.0183	0 0273	0.0362	0.0450	0.0538	0.0624	0 0710	0.0795
.1.	.0880	.0964	.1046	.1128	.1210	.1290	.1370	.1449	.1528	.1605
.2.	.1682	.1759	.1834	.1909	.1983	.2057	.2130	.2202	.2273	.2344
.3.	.2414	.2484	.2553	.2621	.2689	.2756	.2822	.2888	.2953	.3018
.4.	.3082	.3145	.3208	.3270	.3332	.3393	.3454	.3514	.3573	.3632
0.5.	0.3690	0.3748	0.3806	0.3862	0.3919	0.3974	0.4030	0.4084	0.4139	0.4192
.6.	.4246	.4298	.4351	.4402	.4454	.4505	.4555	.4605	.4654	.4703
.7.	.4752	4848	.4848	.4895	.4942	.4988	.5034	.5080	.5125	.5169
.8.	.5214	.5258	.5301	.5344	.5387	.5429	.5471	.5513	.5554	.5594
.9.	.5635	.5675	.5715	.5754	.5793	.5831	.5870	.5907	.5945	.5982
1.0.	0.6019	0.6055	0.6092	0.6127	0.6163	0.6198	0.6233	0.6267	0.6302	0.6336
1.1.	.6369	.6403	.6435	.6468	.6501	.6533	.6564	.6596	.6627	.6658
1.2.	.6689	.6719	.6749	.6779	.6808	.6838	.6867	.6895	.6924	.6952
1 3.	.6980	.7008	.7035	.7062	.7089	.7116	.7142	.7169	.7195	.7220
1.4.	.7246	.7271	.7296	.7321	.7345	.7370	.7394	.7418	.7441	.7465
1.5.	0.7488	0.7511	0.7534	0.7557	0.7579	0.7601	0.7623	0.7645	0.7667	0.7688
1.6.	.7709	.7730	.7751	.7772	.7792	.7812	.7832	.7852	.7872	.7891
1.7.	.7911	.7930	.7949	.7968	.7986	.8005	.8023	.8041	.8059	.8077
1.8.	.8095	.8112	.8129	.8146	.8163	.8180	.8197	.8214	.8230	.8246
1 9.	.8262	.8278	.8294	.8310	.8325	.8340	.8356	.8371	.8386	.8400
2.0.	0.8415	0.8430	0.8444	0.8458	0.8472	0.8486	0.8500	0.8514	0.8528	0.8541
2.1.	.8555	.8568	.8581	.8594	.8607	.8620	.8632	.8645	.8657	.8670
2.2.	.8682	.8694	.8706	.8718	.8729	.8741	.8753	.8764	.8775	.8787
2.3.	.8798	.8809	.8820	.8831	.8841	.8852	.8862	.8873	.8883	.8893
2.4.	.8904	.8914	.8924	.8933	.8943	.8953	.8962	.8972	.8981	.8991
2.5.	0.9000	0.9009	0.9018	0.9027	0.9036	0.9045	0.9054	0.9062	0.9071	0.9080

For values of Δm greater than 2.5, the loss of light is 0.9000 plus $\frac{1}{10}$ of the loss of light corresponding to $\Delta m-2.5$.

TABLE B

Values of $\theta-\sin\theta$

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.00.	0.0000	0.0002	0.0013	0.0045	0.0105	0.0206	0.0354	0.0558	0.0826	0.1167
01	.0000	.0002	.0015	.0049	.0114	.0218	.0372	.0582	.0857	.1205
.02.	.0000	.0003	.0018	.0055	.0122	.0231	.0390	.0607	.0888	.1243
.03.	.0000	.0004	.0020	.0060	.0131	.0244	.0409	.0632	.0920	.1283
.04.	.0000	.0005	.0023	.0066	.0141	.0258	.0428	.0658	.0953	.1324
0.05 .	0.0000	0 0006	0 0026	0.0071	0.0151	0.0273	0.0448	0.0684	0.0987	0.1365
.06.	.0000	.0007	.0029	.0078	.0160	.0288	.0469	.0711	.1022	.1407
.07	.0001	.0008	.0033	.0084	.0171	.0304	.0490	.0739	.1057	.1450
.08.	.0001	.0010	.0037	.0091	.0183	.0320	.0512	.0767	.1093	.1494
.09.	.0001	.0011	.0041	.0098	.0194	.0337	.0535	.0796	.1130	.1539

in computing the values of sin θ corresponding to a given interval from minimum.

Examples of the use of these tables are found in a subsequent paper, in which elements are deduced for the eclipsing variables *W Delphini*, *W Ursae Majoris*, and *W Crucis*.

Princeton University Observatory
March 19, 1912

[*To be continued*]

SPARK SPECTRA OF THE ALKALI EARTHS IN THE SCHUMANN REGION

By THEODORE LYMAN

Spark spectra of metals in the region of extremely short wave-lengths have been investigated by Schumann and later by Handke.[1] The substances studied by the second observer included aluminum, copper, gold, silver, tin, zinc, magnesium, and mercury. The spark was in air outside the instrument. Schumann's vacuum prism spectroscope was used. The measurements extended to the neighborhood of λ 1600. The writer has made experiments on metallic spectra from time to time during the past ten years, using his grating vacuum spectroscope, but without important results until recently, when his attention was directed to the alkali earths by the work of Saunders on series spectra.[2]

The chief improvements which distinguish the present work consist, first, in the employment of a concave grating in place of a prism, and second, in running the spark in a vessel through which a current of hydrogen was maintained. The light thus produced passed directly from the spark chamber through a fluorite window into the body of the spectroscope. In this way, the absorption of the layer of air between the spark and the window was eliminated. At the same time the current of hydrogen helped to free the light-path from the gaseous products of the spark.

Aluminum (Plate XVIII, Fig. 1) was chosen as the first substance for investigation because its spectrum has been studied by Handke and it therefore afforded opportunity for comparing the writer's measurements with measurements obtained by another type of instrument. The figures which will be found in Table I at the end of this paper illustrate in some degree the relative advantages of the two methods of experiment. The greater light-intensity of the prism instrument in the region of wave-lengths less refrangible than λ 1600 is shown by the fact that some faint lines are given by

[1] Inaug. Dis. Berlin, 1909.

[2] *Astrophysical Journal*, **32**, 153, 1910.

Handke which are not easily observed with the grating. On the other hand, the result of the elimination of the absorption of the fluorite prism and lenses and of the air near the spark is illustrated by the fact that the lines of shorter wave-length than λ 1600 were not discovered in the earlier work. Between the two sets of measurements, the agreement is fair; with a few exceptions, the difference between Handke's values and those of the writer are four-tenths of an Ångström unit.

Turning to the principal subject of this paper, the spectra of the alkali earths and their series relations, the expectations in the Schumann region based on theoretical considerations are of two kinds.[1] First, according to the speculations of Ritz and of Saunders there should be series of pairs in the region of very short wave-lengths, the subordinate members of which show constant wave-number separations. Lines belonging to this arrangement have already been observed over the ordinary extent of the spectrum in calcium, strontium, barium, and magnesium, and in the case of some of the series the constants of the formulae have been calculated with a sufficient degree of accuracy to permit of a rough prediction of the position as well as the separation of the new pairs.

The expectation of the second kind is less definite in character than the first. It is founded on the following statement of Saunders:[2] "In all three elements there occurs a strong pair in the ultra-violet: *Ba* λ 2335 and λ 2304; *Sr* λ 2165 and λ 2152; *Ca* λ 1840 and λ 1837, which are reversed in *Sr* and *Ba*, and probably in *Ca* also, and the line of greater wave-length is the stronger in each. They therefore look like subordinate-series pairs in a series of great strength, the rest of which is in the Schumann region."

Confining the attention at present to calcium (Plate XVIII, Fig. 2), the writer's experiments reveal four new pairs, with the separation required by theory, wave-number $1/\lambda = 223$. They appear to fall in with the expectation of the first kind and form terms in the first and second subordinate series. Their discovery is of considerable interest. In the case of the expectation of the second kind, the writer has found three new pairs with the

[1] Ritz, *Physikalische Zeitschrift*, **9**, 521, 1908.

[2] *Astrophysical Journal*, **32**, 165, 1910.

separation predicted by Saunders, $1/\lambda=70$. These pairs, four in all, may present material for the application of the "combination principle." After lines due to impurities have been eliminated, there remain some ten or twelve lines in calcium unclassified, most of which are faint.

In the case of strontium (Plate XVIII, Fig. 3), the expectation of the first kind is slight. The computations of Saunders show that the limits of the subordinate series lie in the neighborhood of λ 1700; it is a recognized property of these series that the members rapidly decrease in intensity as they near the limit. It is not surprising, therefore, that the writer has observed only one pair with the required separation, $1/\lambda=800$, in the Schumann region. Their position is λ 1847 and λ 1820. The expectation of the second class, however, has been well fulfilled. There are two striking pairs, with the separation $1/\lambda=285$.

These three pairs constitute the spectrum of strontium; the other lines visible on the plate are chiefly due to calcium and aluminum.

With barium the experimental difficulties are very great. Success was attained only by using the pure metal in an atmosphere of helium. What has been said of strontium applies even more strongly in this case. The expectation of the first kind is very small. The expectation of the second class is apparently fulfilled: two pairs with the separation predicted by Saunders exist. That they belong together is not absolutely certain, though it seems extremely probable. In addition to these lines, there are several others which may be due to barium.

Finally, the spectrum of magnesium has been studied (Plate XVIII, Fig. 4). It consists, in the Schumann region, of only two pairs, separation $1/\lambda=90$; they seem to fulfil the expectations of the first class and form members of the first and second subordinate series mentioned by Ritz.[1] The other lines visible on the plate are due to impurities.

In Plate XVIII, the fifth figure shows the vacuum tube spectrum of hydrogen which is added for the sake of comparison. Unfortunately the plate on which the barium spectrum is recorded is not suitable for reproduction.

[1] *Physikalische Zeitschrift*, 9, 528, 1908.

It is hoped that the results already obtained are of interest in themselves, and it must be remembered that the method may be extended to the study of metallic spectra in general in the region bounded by λ 1850 at the one end and by the absorption of fluorite at the other.

The writer has profited by the advice of Professor Saunders throughout the work. In fact, if it had not been for Professor Saunders' interest in the subject of series spectra in the Schumann region, the research would never have been undertaken.

The details of technique and measurement are to be found in the second part of the paper.

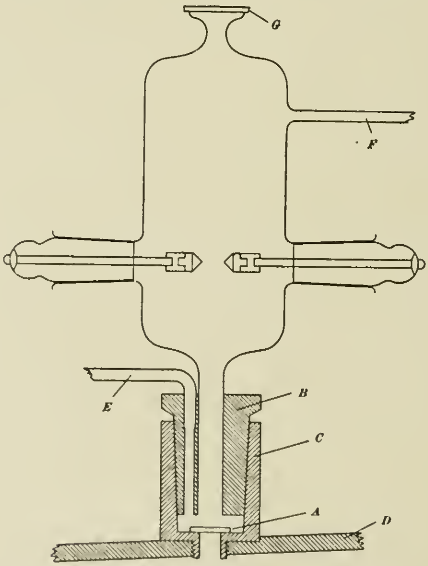


Fig. 1.—The spark chamber

The shape of the spark chamber is illustrated in the accompanying figure (see Fig. 1). A cylindrical glass vessel about 8 cm long and 4 cm in diameter is furnished at one end with a window, and is drawn out into a tube 1 cm in diameter, at the other. This tube fits into a brass cone B and is held in place by cement. The cone, in turn, fits air-tight into a cup C of the same form as that used in previous work.[1] The cup screws on to the face plate of the vacuum

[1] *Astrophysical Journal*, 32, 102, 1911.

spectroscope D and has attached to it a fluorite window A. The point of novelty consists of a tube E which penetrates the side of the cone and through which a stream of hydrogen is directed into the spark chamber. This stream of gas, after striking the fluorite window, turns and is directed against the spark discharge and finally makes its way through sulphuric acid into the outer air by means of F. By a suitable arrangement of stop-cocks, the spark chamber may be exhausted through E. When it was found necessary to replace the stream of hydrogen by an atmosphere of helium, the gas was introduced through the tube F after the vessel had been thoroughly exhausted.

In exhausting the spark chamber and the body of the spectroscope itself, the excellent pump made by the Tri-Mount Rotary Power Company of Boston, has recently been employed.

The metal under examination is held in suitable clamps and introduced into the chamber through ground joints as shown in the figure. The discharge is viewed through the window G during the course of the experiment. Of late, a Hilger wave-length prism spectrometer has been used for the purpose.

The spark was produced by the Clapp Eastham transformer mentioned elsewhere.[1] In all cases, a capacity of 0.026 microfarads was used in parallel with the coil. In many cases an external spark gap was employed in series with the terminals in the discharge chamber.

It is obvious from what is known of the transparency of gases in the Schumann region that, as the light must pass over a path of more than 6.5 cm between spark and window, the gas in the discharge chamber must be free from impurities, especially oxygen. Hydrogen made from zinc and hydrochloric acid and by electrolysis from a barium hydroxide solution was employed. In both cases the usual precautions as to purification and drying were taken. The gas obtained by the second method gave the better results.

The helium was obtained from Tyrer of London and was used at a pressure of three quarters of an atmosphere. It exhibited a transparency equal if not superior to that of hydrogen. The writer believes that this is the first time the transparency of hydrogen in

[1] *Ibid.*

the Schumann region at atmospheric pressure in a column over 6 cm long has been demonstrated spectroscopically.

The method of measurement was that of shifted spectra, which has already been described in connection with the writer's work.[1] Briefly, it consists in comparing the spectrum under investigation with a known spectrum, generally of iron, which is "shifted" with respect to the first by a given amount. Lately, the process of measurement has been facilitated by the use of a scale specially etched on glass and standardized by comparison with the lines of the hydrogen spectrum. As it was an object to obtain as strong a spectrum as possible, a rather wide slit was used. The definition, therefore, falls short of that obtained in the author's hydrogen plates and in some of his gas spectra work.

The errors are of two kinds: one, which depends on the nature of the method employed, is connected with uncertainties in determining the magnitude of the "shift." This error affects the absolute values of the wave-lengths. With a sharp line in the spectrum of hydrogen it should not exceed 0.2 unit. The other is purely an error of setting and affects the relative values, such as the distance between the pairs. Its magnitude varies naturally with the nature of the line, its minimum is probably about 0.1 unit, its maximum may reach 0.4 in the case of nebulous lines. It is obviously this second error which must be borne in mind in estimating the accuracy of differences in the vibration numbers of the pairs of the same series. It is hardly necessary to add that measurements in the extreme ultra-violet are not well adapted for very accurately determining the constants of the formulae employed in series spectra work.

Impurities in the material employed in the study of spark spectra in the Schumann region may play a double rôle: they may contribute lines to the spectrum and they may form gaseous compounds in the spark chamber which modify the spectrum by their absorption. The metals used were analyzed in the hope that the data thus obtained might afford a clue to the cause of the anomalies observed during the experiment. However, the spectroscope is so delicate a detector of impurities that the results of chemical analysis

[1] *Astrophysical Journal*, **23**, 202, 1906.

give only a rough indication of what may be expected. A detailed discussion of the spectra follows.

The specimens of aluminum employed were of commercial quality but of considerable purity. Judging by past experience, they might be supposed to contain only traces of silicon, carbon, and iron. The spectrum in the visible region was dominated by the four lines of hydrogen; the lines of aluminum were relatively feeble. When the spark was tried in helium, the lines of that gas also dominated those of the metal. It is important to notice, however, that in all the spark spectra in the Schumann region, there are no lines which can be traced with certainty either to hydrogen or to helium.

Fig. 1 illustrates the spectrum of aluminum in hydrogen. All the strong lines on the plate are to be ascribed to the metal with the exception of the two pairs at λ 1742.7, 1745.3, and λ 1494.8, λ 1492.8, which are due to a trace of nitrogen.[1] Of the fainter lines, hardly visible in the plate, the one near λ 1655 is probably due to carbon, those near λ 1550 are found in magnesium but may be due to iron, those between λ 1490 and λ 1400 are common to several substances.

Turning to the tables, it is to be noted, that, for most of the lines, the differences between the values obtained by Handke and the measurements of the writer lie between three and four Ångström units. There is an uncertainty in determining the "shift" which introduces the possibility of an error into the writer's values amounting, at most, to two-tenths of an Ångström unit. This error, if it exists, would be of such a sign as to bring the writer's values into closer agreement with those of Handke. However, in the case of the nitrogen pairs at λ 1742.7, λ 1745.3, and in the case of the three lines λ 1721.2, λ 1719.3, and λ 1718.3, the difference in values rises to nearly a whole unit; this fact points to errors of measurements in the work of the earlier observer which may be present in all his data. In the case of the faint lines between λ 1854 and λ 1819, Handke's values alone are given, for although these lines are found on the writer's plate, their rather nebulous character renders their exact measurement difficult.

[1] *Astrophysical Journal*, **33**, 98, 1911.

Very recently the writer has made a new test with the aluminum spark in air. The experiment yielded more interesting results than any of the many previous attempts of this character, its success being due to the fact that the pointed spark terminals were placed nearly in contact with the window of the spectroscope, so that the spark played against the surface of the fluorite. An exposure of six minutes destroyed the fluorite window, but a strong spectrum was registered on the photographic plate. Between λ 1400 and λ 1900 the lines of this spectrum are the same as those observed when the spark was in hydrogen, but there is a distinct difference in the distribution of intensities in the two spectra in this region, owing to the selective nature of the absorption of the air. Near λ 1300, with the spark in air, there is a group of strong lines not observed when the spark was in hydrogen. Of these lines, those at λ 1302.0, λ 1304.8, λ 1305.8, λ 1334.6, and λ 1335.7 are obviously due to some common impurity for they are found with magnesium in hydrogen and with aluminum in helium. The other lines of the group are included in the Aluminum Table but are marked (*) to indicate the uncertainty of their origin.

The specimen of calcium was analyzed with the following result: iron 0.07 per cent, aluminum, magnesium, and silicon, a trace, no trace of barium or strontium or of any other common metal. The spark behaves well in an atmosphere of hydrogen. In Fig. 2 the most striking objects are the strong series of narrow pairs predicted by Saunders. They appear in the illustration as the four broad lines beginning near λ 1370 and ending near λ 1840. On the plate itself, their separation can be measured but their character renders the setting error large, especially in the case of the most refrangible member.

Of the four pairs with wave-number separation $1/\lambda = 223$, predicted by Ritz, only the two at the less refrangible end of the plate are visible in the illustration and of these one member of the first pair is concealed by the strong narrow pair at λ 1840. The other two pairs belonging to this series, though weak, are clearly seen on the original plate. Of the other lines, the sharp narrow pair near λ 1870 may belong to calcium, the lines at λ 1854 and λ 1862 are evidently due to aluminum, the line near λ 1670

belongs to aluminum but seems unduly enhanced here. The nebulous narrow pair at λ 1657 is visible in most metallic spectra both in helium and hydrogen; it is due to a common impurity—silicon or carbon, perhaps. The very faint line near λ 1650 also does not belong to calcium. The background of lines, hardly visible in the illustration, between this point and λ 1555.1 is found in iron, as is also the sharp narrow pair near λ 1550 next to the strong calcium group. The characteristic sharp pair at λ 1393, λ 1402 may be due to calcium but appears in other metals more strongly than their calcium content would seem to warrant. The narrow pair near λ 1335 appear in several metals both in helium and hydrogen. The extreme group not visible in the illustration is found only with calcium; it is made up of some of the most refrangible lines yet obtained with a metal.

Some difficulty was found in obtaining metallic strontium of sufficient purity. A suitable specimen was finally procured, however. It gave the following analysis:

Silicon	None	Calcium	1.5 per cent
Iron	Slight trace	Magnesium	None
Aluminum	Trace	Mercury	None
Barium	None	Carbon	Considerable, not estimated

On inspection of the spectrum, it is obvious that most of the lines are also found in calcium. The only strong lines that can be attributed to strontium are the two pairs at λ 1769, λ 1778, λ 1613, and λ 1620. They also appeared faintly in a spectrum obtained with a less pure specimen of metal.

There is also a weak pair near λ 1821, λ 1847, one member of which may be visible in the illustration, which is predicted by Ritz and Saunders; it has the correct wave-number separation $1/\lambda=800$.

The magnesium employed yielded the following analysis:

MAGNESIUM

Silicon	None	Barium	None
Iron	0.05 per cent (estimated colorometrically)	Strontium	None
Aluminum	Trace	Calcium	None

On comparing its spectrum with one obtained with iron terminals under similar conditions, it appears that only the two pairs already mentioned can be attributed to magnesium. They appear near λ 1735 and λ 1750. All the other lines are found in the iron spectrum and are due, therefore, either to it or to some impurity common to the two substances. In this connection, it is interesting to observe how small an amount of an impurity may produce an appreciable effect.

The pairs that might be expected at the more refrangible end of the plate cannot be distinguished with certainty from the background of faint lines.

The results of Handke's investigation of the magnesium spectrum do not agree with those of the writer. Perhaps the difference in the condition of the spark in the two cases may account for this.

In the case of barium, the most obvious mode of procedure is to employ carbon terminals saturated with some salt of the metal. An experiment of this kind was therefore tried; carbon terminals saturated with a solution of chloride were used in an atmosphere of hydrogen. The spark showed the barium spectrum in the visible quite strongly, but nothing in the Schumann region.

Through the kindness of Professor T. W. Richards, the writer obtained some unusually pure specimens of barium, containing over 99 per cent of the metal. The impurities were a trace of oxygen and of iron. Experiments with this substance, however, in an atmosphere of hydrogen gave no result at all, perhaps because of the formation of an absorbing cloud of hydride about the spark. Even the results obtained with the spark in helium were somewhat disappointing. The members of the pair λ 1849, λ 1869 have the correct separation $1/\lambda = 575$ and relative intensity, but they are very feeble and rather sharper than one would be led to expect from similar pairs in calcium and strontium. Another pair with correct separation and of a more satisfactory appearance can be picked out from the remaining lines at λ 1678, λ 1694.

The spectrum of hydrogen, which is added for the sake of comparison, was obtained from a vacuum tube in the manner already described.[1] There was no capacity other than that of the leads

[1] *Astrophysical Journal*, 23, 202, 1906.

in the circuit. The pressure of the gas was about two millimeters. The faint lines on the less refrangible side of λ 1650 are due to a trace of an oxide of carbon.

The similarity between the hydrogen and magnesium spectrum in the neighborhood of λ 1600 is apparent, not real. The two sharp lines, which have been mentioned as occurring most strongly in calcium and whose wave-lengths are λ 1393.6, λ 1402.7, nearly agree in position with the two lines in hydrogen λ 1394, λ 1402.8. It is barely possible, therefore, that these lines are due to the gas, though, as they occur faintly with aluminum in helium, it does not seem probable. If they belong to hydrogen, they are the only lines in the vacuum tube spectrum which appear in the spark.

As usual, the wave-lengths on the table are in vacuum. The scale which is printed with the spectra was not used directly for measurements. It is intended to give the position of the lines only approximately. Of the lines obviously due to impurities the most prominent are given in Table VI.

TABLE I

ALUMINUM

λ	I	1/λ	λ Handke	Diff.	λ	I	1/λ	λ Handke	Diff.
1238.8*...	1	80723	......	....	1750.0...	3	57143	1750.4	0.4
1264.5*..	1	79083	......	....	1751.7...	2	57087	1752.1	.4
1275.0*...	3	78431	......	....	1760.0...	8	56818	1760.4	.4
1276.4*...	2	78345	......	....	1761.9...	8	56757	1762.4	.5
1310.8*...	6	76290	......	....	1763.8...	10	56695	1764.2	.4
1319.4*...	6	75792	......	....	1765.7...	8	56635	1766.0	.3
1326.6*...	1	75380	......	....	....	..	.....	1766.9	..
1343.4*...	2	74438	......	....	1767.6...	8	56574	1768.0	.4
1352.8...	1	73921	......	....	....	..	.....	1769.6	..
1379.5...	3	72490	......	....	....	..	.....	1772.9	..
1383.9...	5	72259	......	....	1773.8...	2	56376	1773.8	.0
1540.1...	1	64931	......	....	....	..	.....	1774.9	..
1605.6...	8	62282	1605.9	0.3	1776.9...	4	56278	1777.1	.2
1611.8...	8	62042	1612.1	.3	....	..	.....	1777.8	..
1670.6...	10	59859	1671.0	.4	....	..	.....	1792.1	..
....	..	.....	1676.1	..	1818.5...	3	54990	1819.0	.5
1718.3...	1	58197	1719.1	.8	....	..	.....	1819.6	..
1719.3...	9	58163	1720.0	.7	....	..	.....	1820.6	..
1721.2...	9	58099	1722.0	.8	....	..	.....	1833.2	..
1725.0...	10	57971	1725.3	.3	....	..	.....	1836.8	..
....	..	.....	1741.1	..	1854.7...	50	.....	1854.7	..
1742.7	N.	57382	1743.6	.9	1858.2...	10	.....	1858.2	..
1745.3		57297	1746.3	1.0	1862.8...	50	.....	1862.8	..
1747.7...	1	57218	1748.3	.6					

TABLE II

Calcium

λ	I	1/λ	Δ1/λ	λ	I	1/λ	Δ1/λ
1246.2	1	80244	...	1553.5	7	64370	66
1254.3	2	79726	...	1555.1	8	64304	
1260.2	1	79352	...	1561.2	2?	64053	
1264.5	2	79083	...	1674.1	1	59733	227
1268.2	2	78852	...	1680.5	2	59506	
1276.4	3	78345	...	1692.4	1	59087	226
1369.1	3	73040	80	1698.9	2	58861	
1370.6	3	72960		1807.8	7	55316	220
1393.6	5	71761	...	1815.0	8	55096	
1402.7	4	71291	...	1838.0	9	54406	65
1433.1	5	69778	58	1840.2	10	54341	
1434.3	6	69720		1843.8	6	54236	220
1526.7	2	65501	...	1851.3	7	54016	
1533.4	2	65214	...	1870.4	3	53464	...
1546.0	?	64683	...	1872.5	3	53404	...

TABLE III

Strontium

λ	I	1/λ	Δ1/λ	λ	I	1/λ	Δ1/λ
1532.3?	1	65261	...	1769.8	8	56503	286
1537.9?	1	65024	...	1778.8	9	56217	
1560.8?	1	64070	...	1820.0	1	54945	803
1613.3	4	61985	283	1847.0	3	54142	
1620.7	5	61702					

TABLE IV

Barium

λ	I	1/λ	Δ1/λ	λ	I	1/λ	Δ1/λ
1331.1	2	75126	...	1572.9	2	63577	...
1361.0	2	73475	...	1592.9	1	62778	...
1414.8	3	70681	...	1674.5	4	59719	...
1417.1	2	70567	...	1677.9	3	59598	577
1482.0	1	67476	...	1694.3	6	59021	
1485.0	1	67340	...	1786.6	1	55972	...
1487.0	2	67249	...	1849.5	2	54068	569
1503.9	4	66494	...	1869.2	5	53499	
1554.5	3	64329	...				

TABLE V

Magnesium

λ	I	1/λ	Δ1/λ	λ	I	1/λ	Δ1/λ
1735.0	6	57637	93	1750.9	5	57113	88
1737.8	7	57544		1753.6	6	57025	
				1828.1	1	54702	..

TABLE VI

Lines of Uncertain Origin

λ	I	1/λ	Δ1/λ	λ	I	1/λ	Δ1/λ
1302.0	4	76805	..	1548.2	6	64591	..
1304.8	3	76640	.	1550.8	5	64483	..
1305.8	1	76581	..	1649.9	3	60609	...
1334.6	2	74929	..	1656.8	4	60357	..
1335.7	3	74867	..	1657.8	1	60321	..

In conclusion it may be well to restate the results which have been achieved.

The existence of certain lines in that part of the spectra of the alkali earths which lie in the Schumann region was predicted by Ritz and Saunders on theoretical grounds. When these predictions were made, the spectra of the substances in the region of extremely short wave-lengths had never been observed. The writer has succeeded in photographing these spectra and he has discovered part, at least, of the lines whose existence was predicted. During the work, a method has been developed by which spark and arc spectra may be studied down to the very limit of the transparency of fluorite.

Jefferson Physical Laboratory
Harvard University
May 1912

Reviews

Lines in the Arc Spectra of Elements. Compiled by F. Stanley. London: Adam Hilger, Ltd., 1911. 8vo, pp. 140. Cloth, 12*s*. 6*d*.; half morocco, 15*s*. 6*d*.

This volume lists the wave-lengths and intensities of 3700 selected lines from the arc spectra of 55 elements arranged according to their wave-lengths. In adjoining columns are given the element and the wave-length of the next prominent line of that element. The printed matter occupies about one-half of a page, the remainder of the page and the opposite page being left blank for the addition of notes. The more persistent lines are denoted in many cases by an asterisk.

As the list is not exhaustive as to either element or line, and as the wave-lengths are rounded off to tenths of an Ångström and no references are given, it is not likely that the book will be of great value to the advanced worker in physics and astronomy, although it is probably well adapted to the less exacting needs of the chemist and of the amateur.

Storrs B. Barrett

Physical Optics. By Robert W. Wood. New and Revised Edition. New York: MacMillan, 1911. Pp. xvi+705. $5.25 net.

The first edition of Wood's *Physical Optics*, which appeared in 1905, was universally recognized as an important addition to scientific literature. The second edition is of distinctly greater value than the first. The book has been enlarged in size from 546 pages to 705 pages, and the number of figures has been increased from 325 to 399. The new colored frontispiece contains 8 figures as against 5 in the old edition, and there are 10 full-page plates in the new edition, twice the former number.

The most noticeable additions are contained in three short new chapters. Chap. xii, on meteorological optics, deals with the rainbow, halos, mock suns, and related phenomena. Chap. xix, on electro-optics, deals with the Kerr electro-optic effect in liquids, the electro-optic analogy of the Zeeman effect, and the photo-electric effect. In chap. xxv the author has attempted the difficult task of presenting the

principle of relativity in less than 11 pages. It is doubtful if a better treatment could be given in so small a compass.

The new material incorporated in the old chapters is, in part, as follows: To chap. i has been added a description of Pfund's mercury arc, a description of Galitzin and Willip's repetition of Belopolsky's experiment on the Doppler effect, Stark's work on the Doppler effect in the light emitted by the canal rays in vacuum tubes, and a figure showing the Doppler effect in stellar spectra.

The author's work on "fish-eye views" and Schmidt's theory of the sun are the chief additions to the fourth chapter. Julius' work on the effect of anomalous dispersion on the appearance of the D lines, illustrated by a full-page plate, is an interesting addition to chap. v. To chap. vii has been added a discussion of the author's echelette grating, and the work of Wood and Trowbridge on spectral intensity and the form of grooves. The interesting and valuable work of Rubens and Wood on the focal isolation of long heat-waves has been added to the chapter on the theory of dispersion. Chap. xv, on the absorption of light, has been expanded by the addition of Wood's work on the absorption of sodium vapor, and the extension of the Balmer series in the ultra-violet to the forty-eighth member. There is also reference to the very interesting work of Pflüger and of Ladenburg and Loria on absorption by luminous hydrogen. The chapter on magneto-optics has been increased by 18 pages and contains much new and interesting material, e.g., the work of Voigt and Lohmann on complicated types of the Zeeman effect, the work of Zeeman on unsymmetrical triplets, a discussion of the Zeeman effect in spectral series, the work of Zeeman and Winawer on the Zeeman effect in absorption spectra, Hale's discovery of the Zeeman effect in sun-spot spectra, and Dufour's recent work on the Zeeman effect in band spectra.

The chief additions to chap. xx are on Wood's investigations of the fluorescence of mercury, iodine, and bromine vapors.

These illustrations serve to show that most of the material added to the book is in the presentation of results obtained in physical optics since the first edition appeared.

As a whole the book, like its predecessor, deals primarily with experimental optics, and illustrates the practicability and perhaps also the desirability of treating even the most abstruse topics from a physical rather than a mathematical standpoint. And yet the mathematical treatment has not been ignored, and we find all of it that is essential to a profound knowledge of theoretical optics.

One hesitates to criticize adversely in any way a book of such unquestioned merit, but there are evidences of carelessness in editing which cannot be overlooked. We read in the preface to the second edition, "The numerous typographical errors which marred the first edition have been corrected and certain sections of small interest or importance have been removed bodily to make room for new material." It is, therefore, something of a disappointment to find that there are still a great number of typographical errors. To make matters worse, many of these errors were noted in the page of errata which accompanied the first edition. Thus on p. 383 the equation of line 2 is entirely wrong, the brackets have been omitted for line 5, the incorrect equation of lines 5, 6, and 7 from the bottom is reproduced from the first edition, the parenthesis is misplaced in line 4 from the bottom, and a minus sign is omitted between the last two lines.

On p. 1 the author refers to the last chapter of the book instead of to the next to the last chapter. On p. 10 he refers to p. 351, instead of p. 439, and on p. 35 to p. 158 instead of p. 191.

These references are to the page of the old edition instead of to the page of the new edition. On p. 15 the author refers to the Carnegie Institution of Washington as the Carnegie Institute. On p. 136 he gives the wave-length of the C line as 6399 instead of 6563. Such errors are of course of trivial importance in comparison with the general excellence of the book. Perhaps the greatest harm which could arise from them is that some immature reader might unjustly attribute to the author's experimental work a similar lack of accuracy and care.

HENRY G. GALE

Atlas typischer Spektren. By J. M. EDER AND E. VALENTA. 53 charts with explanatory text of 143 pages. Published by the Komitee zur Verwaltung der Erbchaft Treitl, under the direction of the Kaiserliche Akademie der Wissenschaften. Wien, 1911. M. 78.

A collection of charts of spectra such as that offered by Eder and Valenta is an exacting task if the work is to take its place as a thoroughly efficient aid to the worker with spectra, be he physicist, chemist, or astronomer. It requires an extensive and highly flexible instrumental equipment, both as to light-sources and apparatus for photography of

the spectrum, a wide spectroscopic experience and high photographic technique on the part of the authors, and the utmost limit of the engraver's skill if a reasonable amount of the extraordinarily fine detail on the original negatives is to be reproduced. The work under review may be said to go far toward fulfilling each of these requirements.

Considering first the reproductions of spectra, we find the charts contain the flame spectra of 70 elements and compounds, the arc spectra of 78, and the spark spectra of 78. The total number of reproductions is about 640, the strips of spectra being arranged on 53 sheets. The authors wisely decided to have the plates made directly from the negatives, thus avoiding the intermediate positive which would mean the loss of much detail. Although the appearance is thus that of absorption spectra, no confusion can arise from this cause and in regular work there is some gain in having the chart similar in appearance to the original negative with which it is compared. The quality of the reproductions is probably as good as the engraving processes of the present day will permit. A scale beside each strip of spectrum gives intervals of 100 Å; while the wave-lengths of distinctive lines throughout the spectrum have their wave-lengths etched opposite them, impurity lines also often being indicated in this way.

An examination of the index shows that there are few chemical elements whose spectra are not presented in this work. A feature is the rich collection of spectra of the rare earths and of elements unknown a decade ago. The spectra are "typical" in the sense that they are given by the flame, arc, or spark with such instrumental arrangements as would generally be employed in laboratories, avoiding, except in a few cases, those conditions of the light-source which profoundly modify the character of the spectrum. The large collection of flame spectra will be of especial interest to chemists. In a number of cases the flame spectra of several compounds of the same element are presented. As an example may be mentioned the beautiful flutings of the chloride, bromide, iodide, and nitrate of copper, the scale being sufficient to show the distinctive differences in the arrangement of bands without attempting full resolution.

The first order of a concave grating of 146 cm radius was employed for most of the arc and spark spectra and for a few of the flame spectra. The scale obtained is about 11.6 Å per mm. The grating spectra are regularly reproduced in two portions, one from λ 2000 to λ 4600, and a second from λ 4300 to λ 7000. A few charts show the red region as far

as λ 8000. For a number of elements, besides grating spectra for the arc and spark, there are given prismatic spectra from two instruments, the one with glass prism showing the spectrum from λ 3500 to λ 7000, the scale at λ 4300 being about equal to that of the grating spectrum, the other with quartz prism showing the ultra-violet to the limits of transmission for air at about λ 1800. While there is some duplication in presenting both grating and prismatic spectra for the same element, the authors consider that the spectra obtained with a prism will be especially useful to those working with similar apparatus.

The scale of the reproductions is in general very satisfactory for spectra having a moderate number of lines. When, however, this scale is used for spectra whose stronger lines run into the hundreds and even thousands, little more than a general view of the distribution of lines is obtained from the charts. To have reproduced these spectra on a scale suitable for detailed study, say at least as large as 2 Å per mm, would have made the publication of prohibitive size, and this limitation was doubtless regretted by the authors as much as it can be by any user. The field is still open for a set of charts which will do full justice to these many lined spectra.

The volume of explanatory text deserves special notice. Flame, arc, and spark spectra are treated separately. For each substance the method of producing the radiation is briefly described, with many practical suggestions as to how certain features of the spectrum may best be brought out. Numerous references to original works are given. A table of wave-lengths follows for those lines which are distinct on the charts for the substance under discussion. These tables are a very valuable feature of the work, covering the spectrum in some cases between the extreme limits λ 2000 to λ 8000. The wave-lengths to 0.01 Å are compiled from the best available measurements. The result is the most complete and up-to-date collection of tables as regards number of spectra and range of wave-length which is at present available. While completeness in the individual spectra is not aimed at, the extent to which the distinctive lines are listed may be judged from the fact that the table for the iron arc includes 934 lines, that for the thorium arc 708 lines.

A regular reference to this atlas with its accompanying tables will greatly facilitate the work of anyone dealing with spectra. A few of those who use it will appreciate the enormous labor which Professors Eder and Valenta have expended in the compilation.

ARTHUR S. KING

EDITORIAL NOTE

Hereafter the duties of managing editor of the *Astrophysical Journal*, which have been successively borne by Mr. Hale and by Mr. Frost, will be assumed by Henry G. Gale, of the Department of Physics of the University of Chicago.

Manuscripts, proof sheets, books for review, and all editorial correspondence should henceforth be addressed to

EDITORS OF THE ASTROPHYSICAL JOURNAL
UNIVERSITY OF CHICAGO, CHICAGO, ILL.

ERRATUM

Vol. 34, November 1911, in J. G. Hagen's article on "Various Scales for Color-Estimates": Page 267, line 4, *for* Innes *read* See.

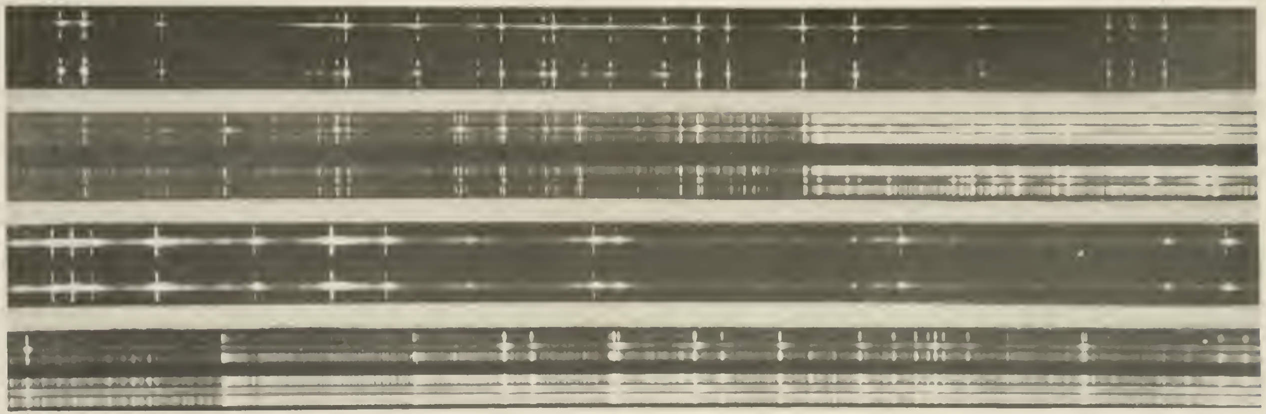
INDEX TO VOLUME XXXV

SUBJECTS

INDEX TO VOLUME XXXV

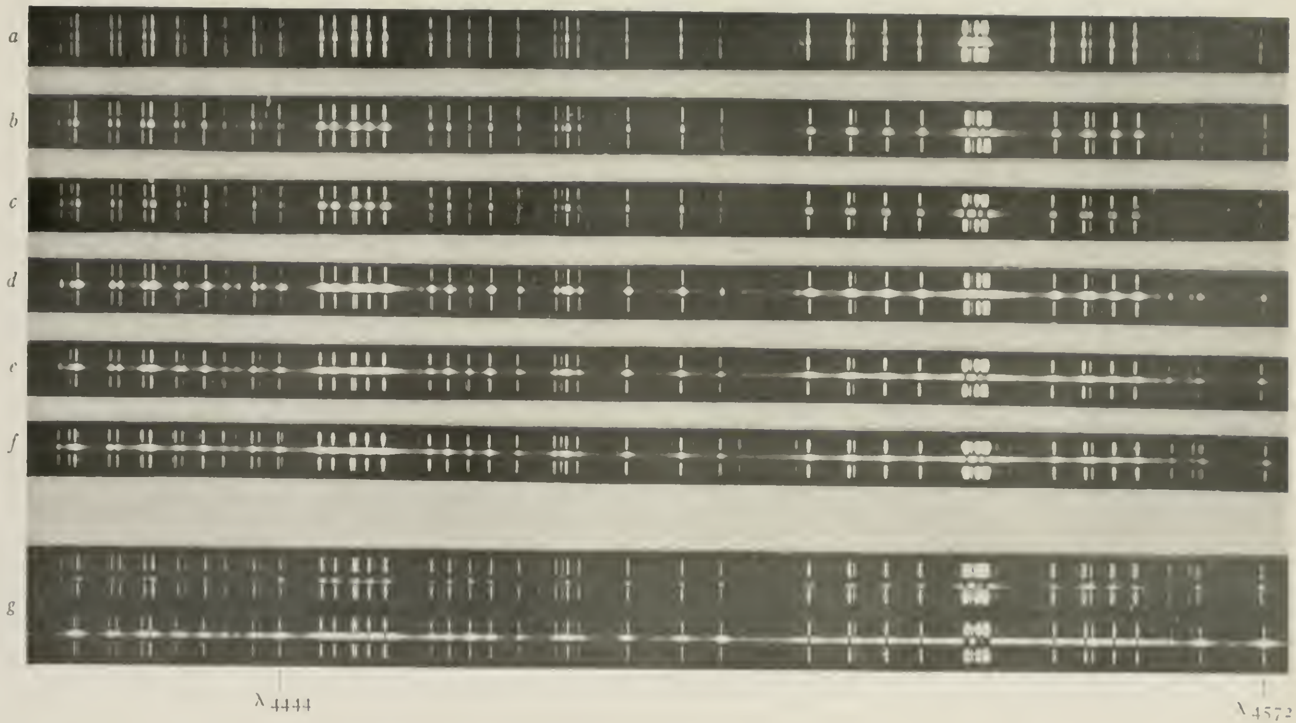
AUTHORS

PLATE I



a and *c*, iron; *b* and *d*, titanium; all at 9 atmospheres; comparison, 1 atmosphere
a and *b* same region; *c* and *d* same region

PLATE II

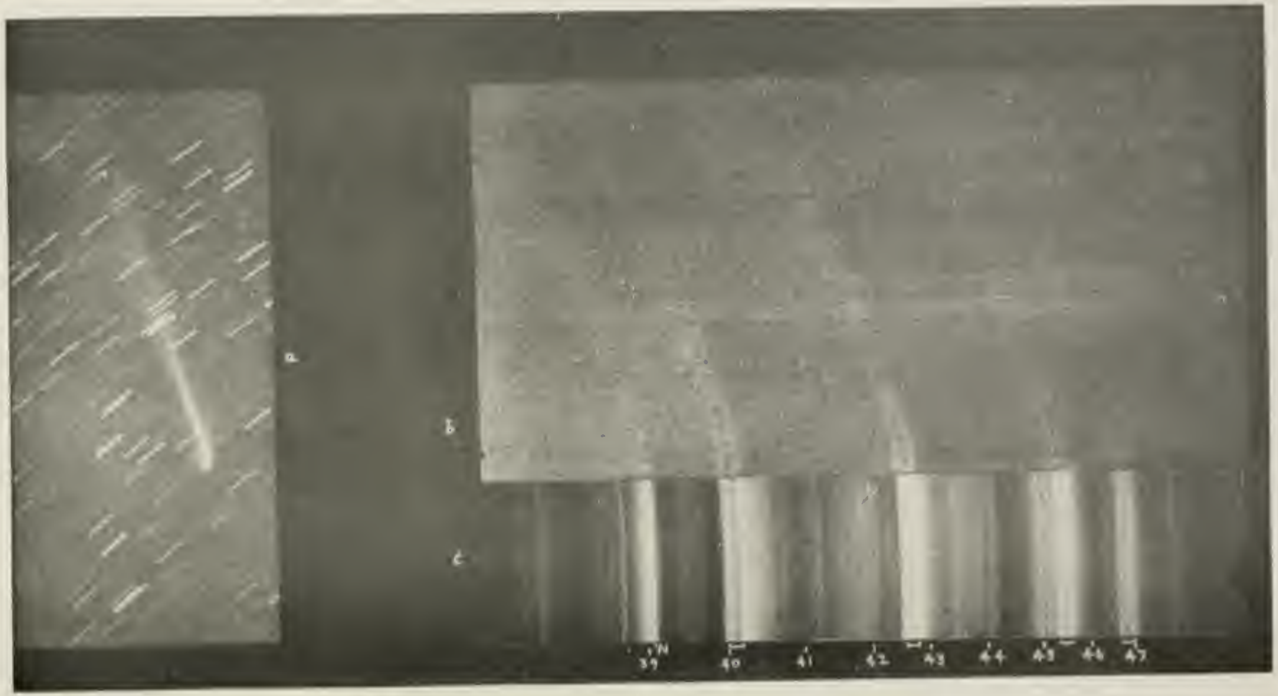


Ti arc at different pressures; comparison at 1 atmosphere
a, *b*, *c*, *d*, *e*, *f*, at 2, 4, 6, 8, 12, and 16 atmospheres, respectively, above atmospheric pressure
g, *Ti* arc in atmosphere of illuminating gas, at 4 atmospheres above atmospheric pressure

PLATE III

a, *Ti* spark, 0 atmospheres; comparison, spark at 1 atmosphere
b and *c*, arc 1 atmosphere
d and *e*, arc 10 cm
f, arc 1.5 cm
g, spark with self-induction and capacity
h, spark with capacity, but no self-induction

PLATE IV



Comet Morehouse (1908c) March 20, 1909

a. Direct Photograph, 4 hours
b. Objective Spectrogram, 7 hours
H. D. Curtis, Santiago, Chile

c. Carbon Monoxide, pressure 0.01 mm
A. Fowler, South Kensington

PLATE V

Fig. 2

Hydrogen Canal Rays Bombarding:

(a) Hydrogen molecules; (b) Nitrogen molecules; (c) *H* and *N* molecules

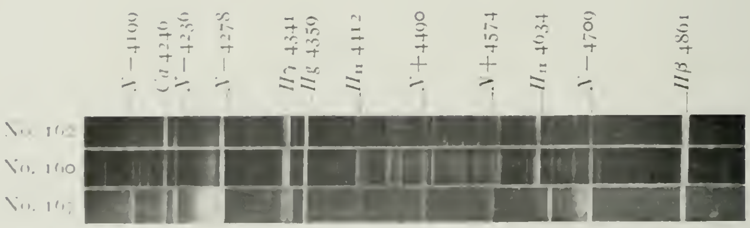
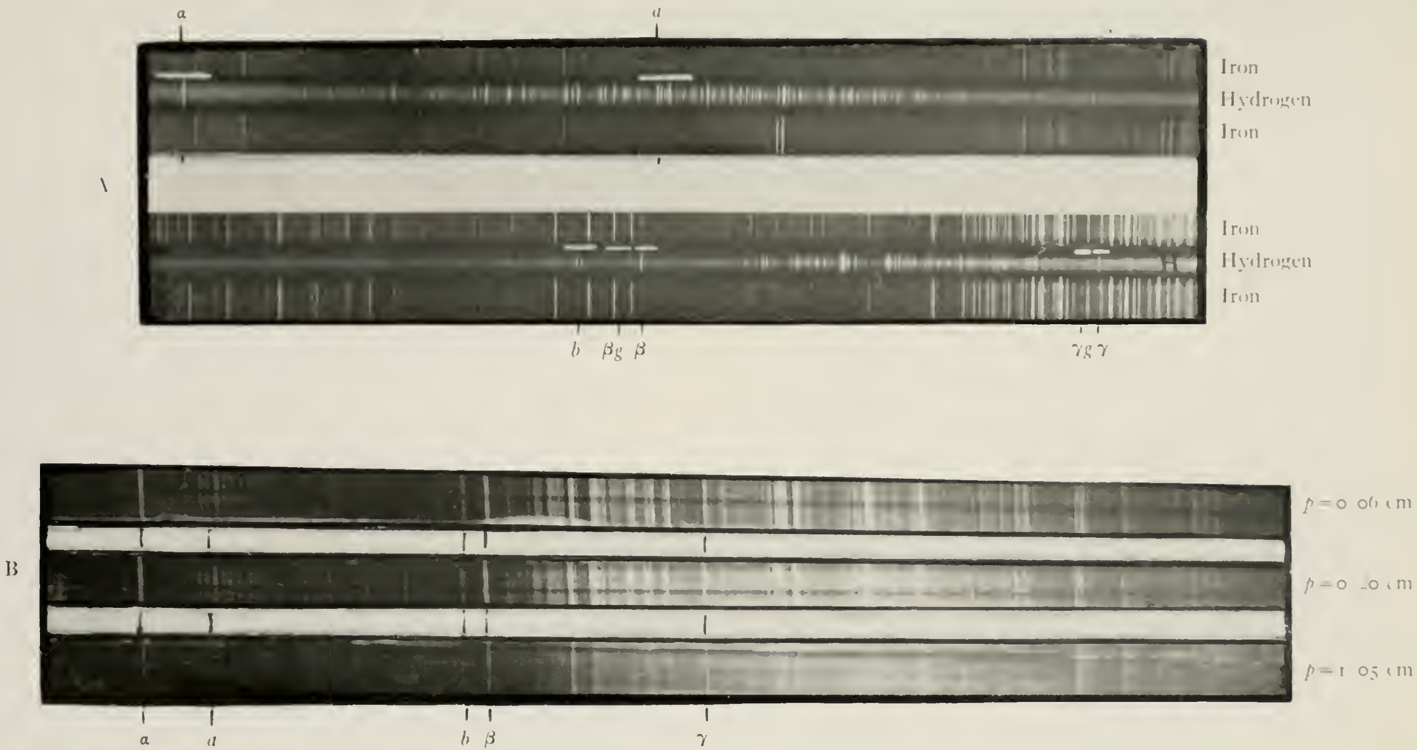


Fig. 3

No. 162. *N* canal rays in *H*. 4500 volts, 15 hours
No. 160. *H* canal rays in *H*. 4000 volts, 1.5 hours
No. 167. *H* canal rays in air, 5000 volts, 24 hours

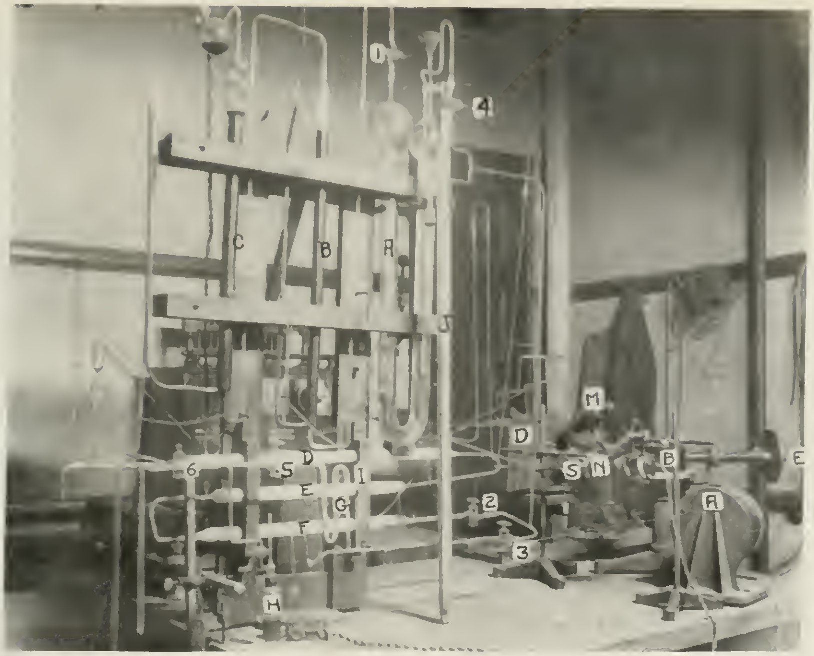
PLATE VI



SPECTRUM OF HYDROGEN TUBES

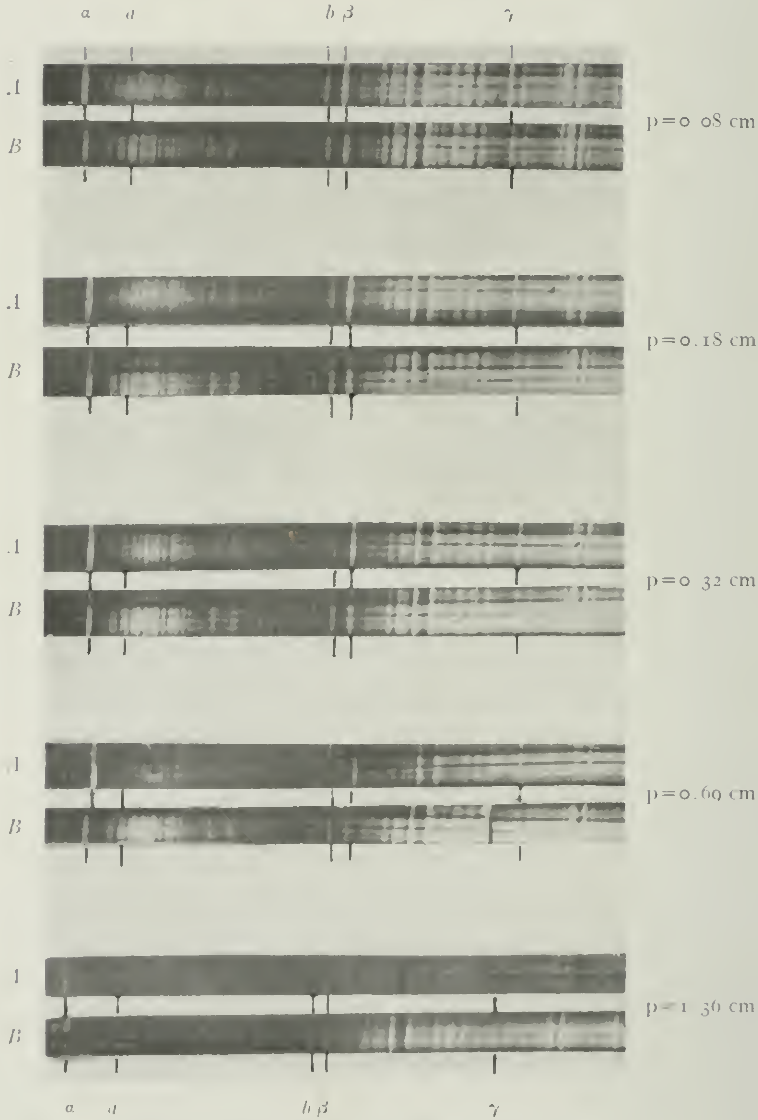
A. With concave grating, iron comparison spectrum
B. With small prism, at different pressures

PLATE VII



GENERAL VIEW OF APPARATUS

PLATE VIII



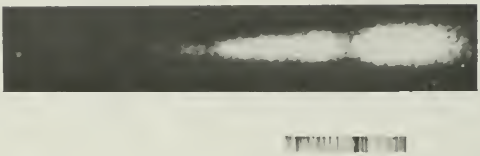
Spectra of Tube 2 at Different Pressures

At each pressure, A corresponds to T = 300°, B to T = 100°.

PLATE IX

λ 4300 4700

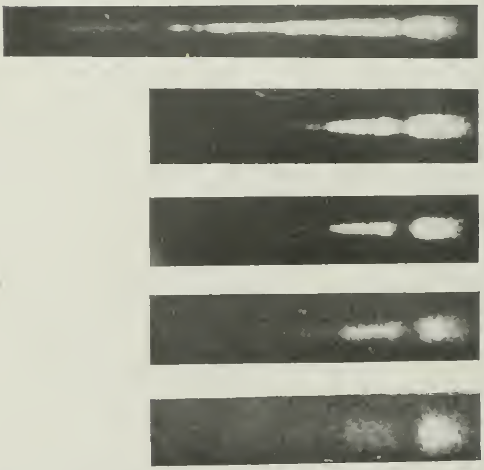
Fig. 1
19 Piscium



Objective-Prism

Slit-Spectrograph

Fig. 2



−10° 5057

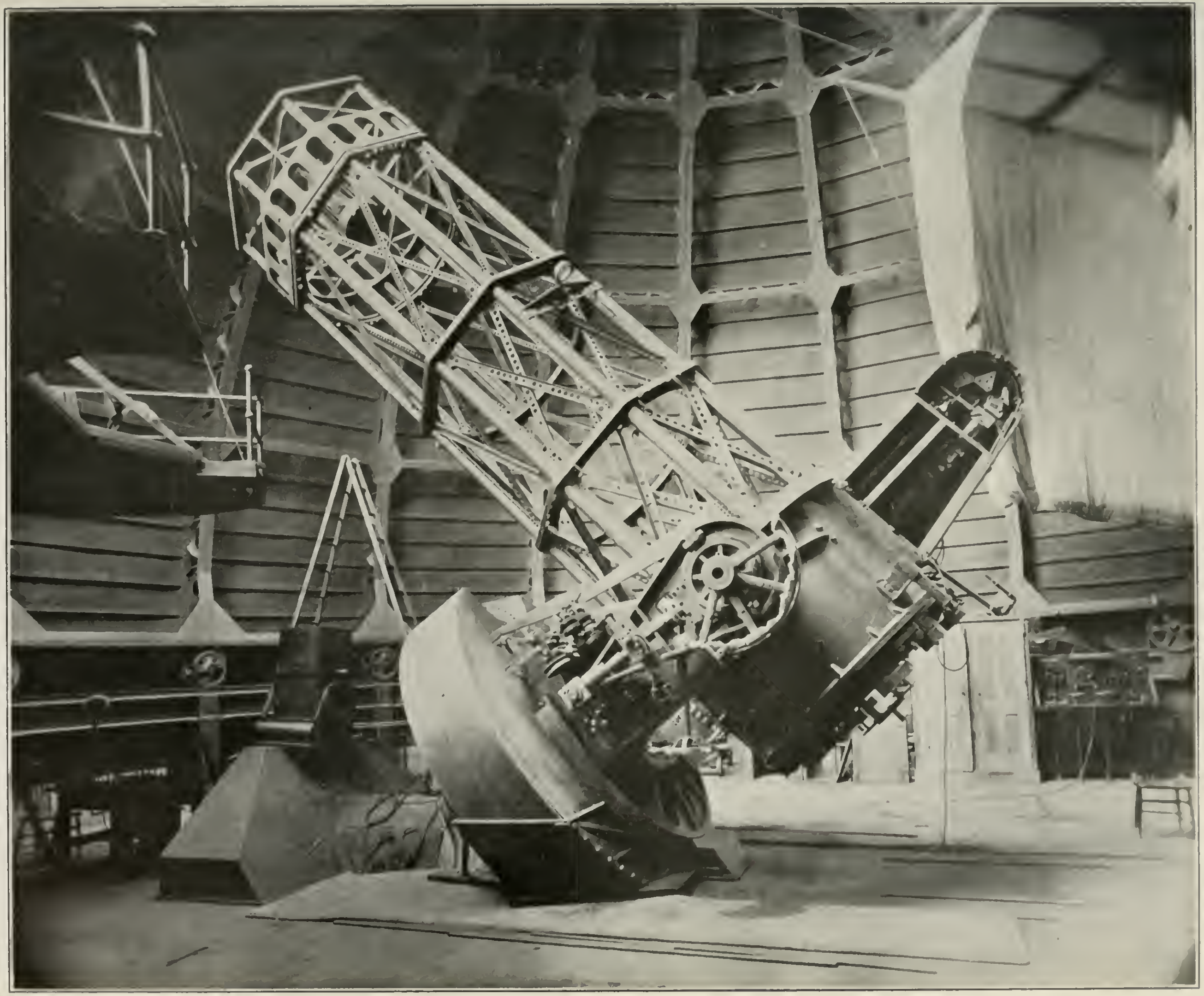
19 Piscium

249a Schj.

318 Birm.

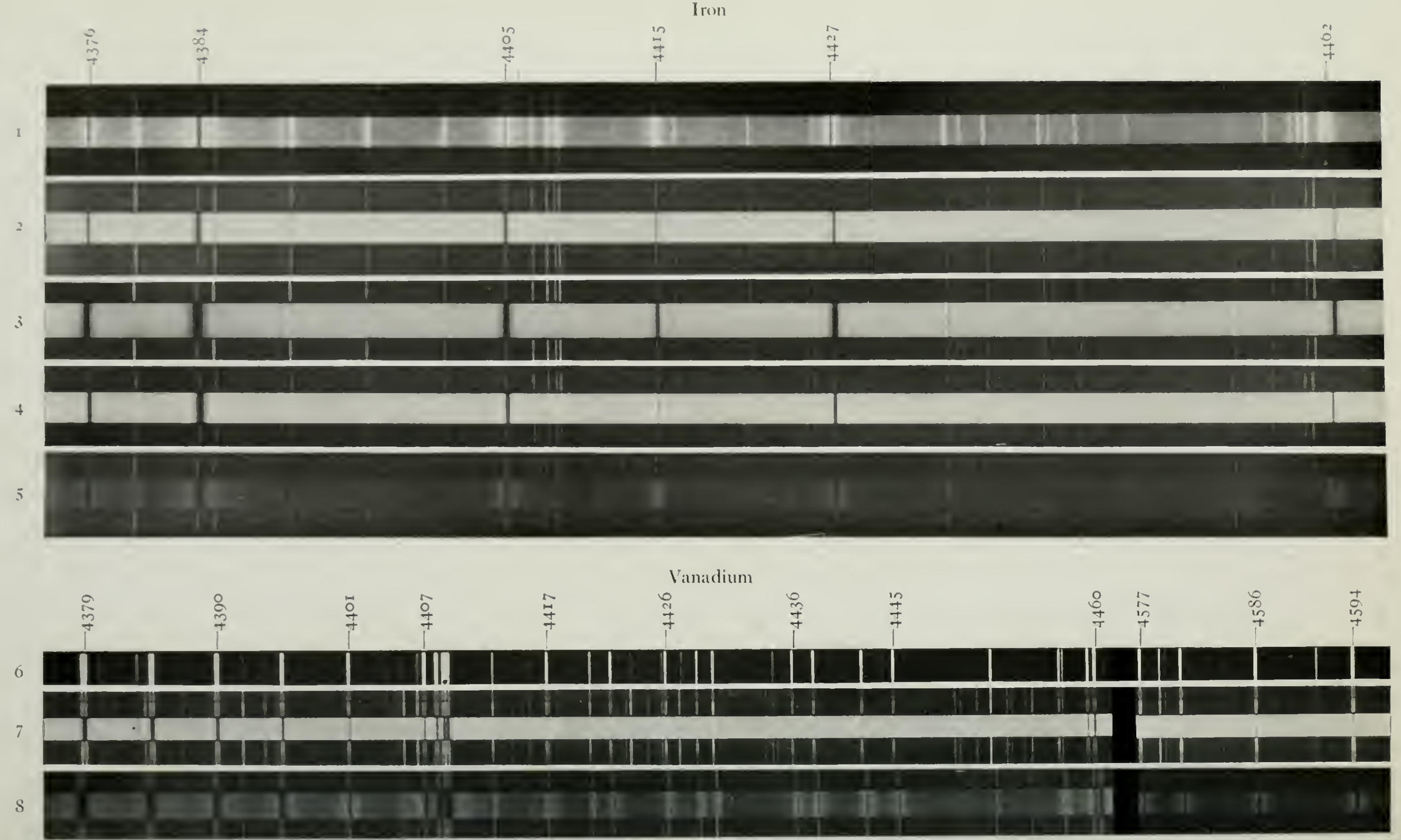
152 Schj.

Spectra of Fourth Type Stars



Sixty-Inch Reflecting Telescope of the Mount Wilson Solar Observatory with the Three-Prism Stellar Spectrograph Attached

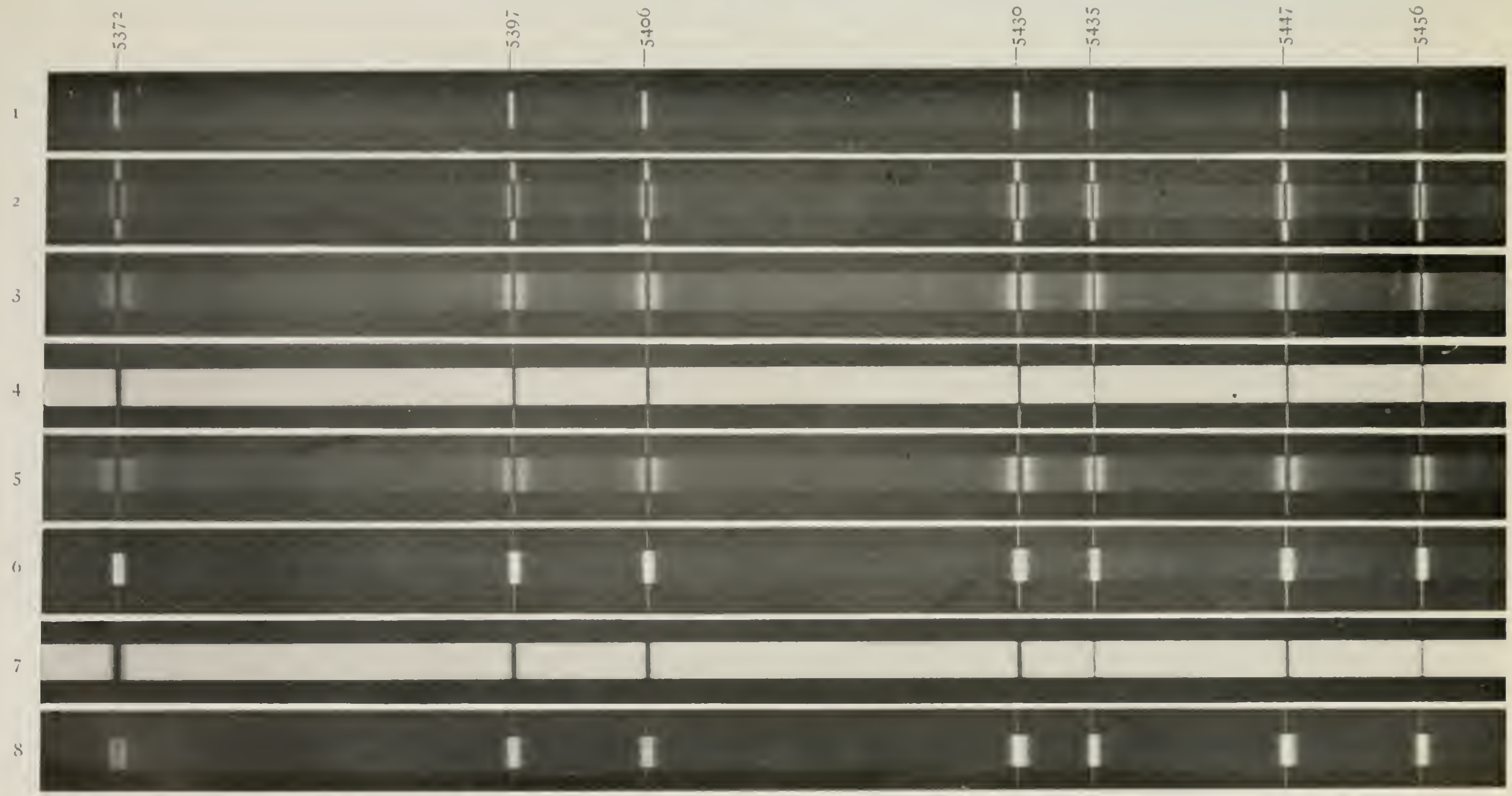
PLATE XI



Effect of Pressure upon Electric Furnace Spectra

1. 8 atmospheres
2. 8 atmospheres
3. 12 atmospheres (absorption)
4. 16 atmospheres
5. 24 atmospheres
6. Arc
7. 8 atmospheres
8. 16 atmospheres

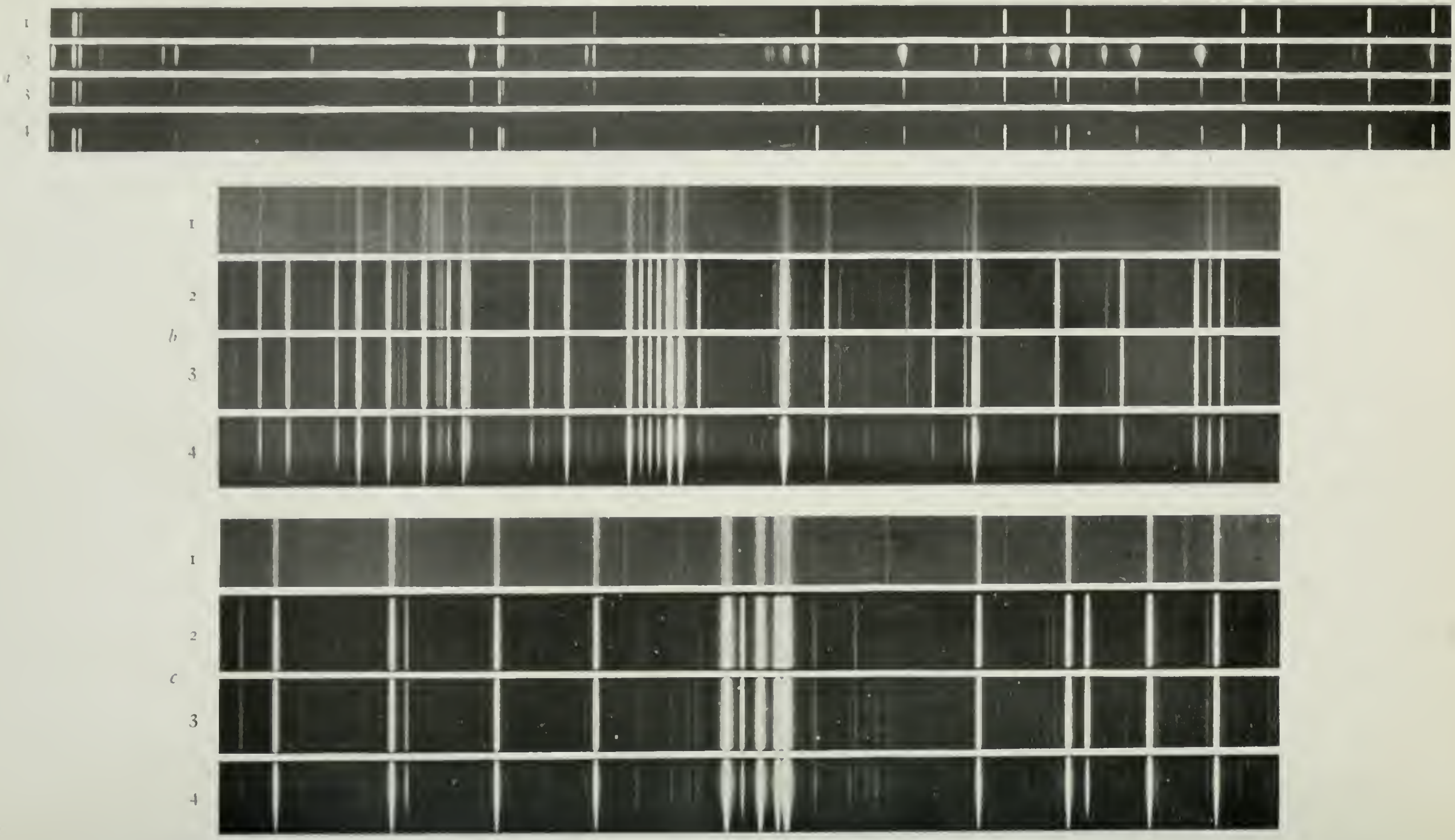
PLATE XII



Effect of Pressure upon the Electric Furnace Spectrum of Iron

1. 1 atmosphere
2. 4 atmospheres
3. 8 atmospheres
4. 8 atmospheres (absorption)
5. 16 atmospheres
6. 16 atmospheres
7. 16 atmospheres (absorption)
8. 24 atmospheres

PLATE XIII



FURNACE AND ARC SPECTRA OF IRON AND TITANIUM

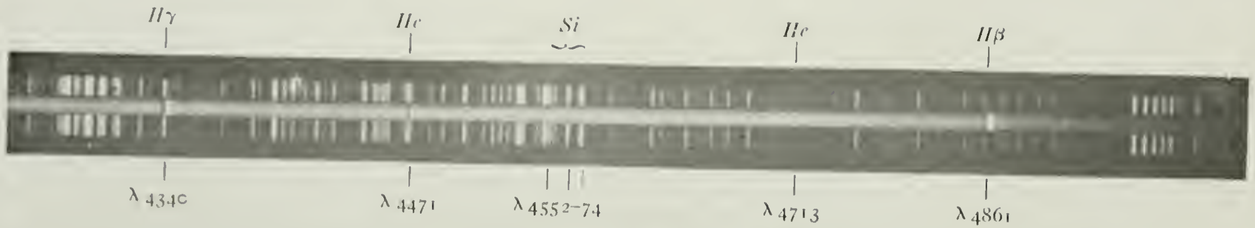
a Iron λ 5270 to λ 5456. *b*. Titanium λ 4281 to λ 4327. *c*. Titanium λ 4513 to λ 4556

1. Furnace at atmospheric pressure. 2. Arc, 20 amperes. 3. Arc, 10 amperes. 4. Arc, 2 amperes

Scale: *a*, 1 mm = 0.95 Å. *b* and *c*, 1 mm = 0.3[illegible] Å

PLATE XIV

Rossi: Copper Polar Lines Appearing at Negative Electrode Only



Frost: Spectrum of *P Cygni*
With Titanium Comparison Spectrum

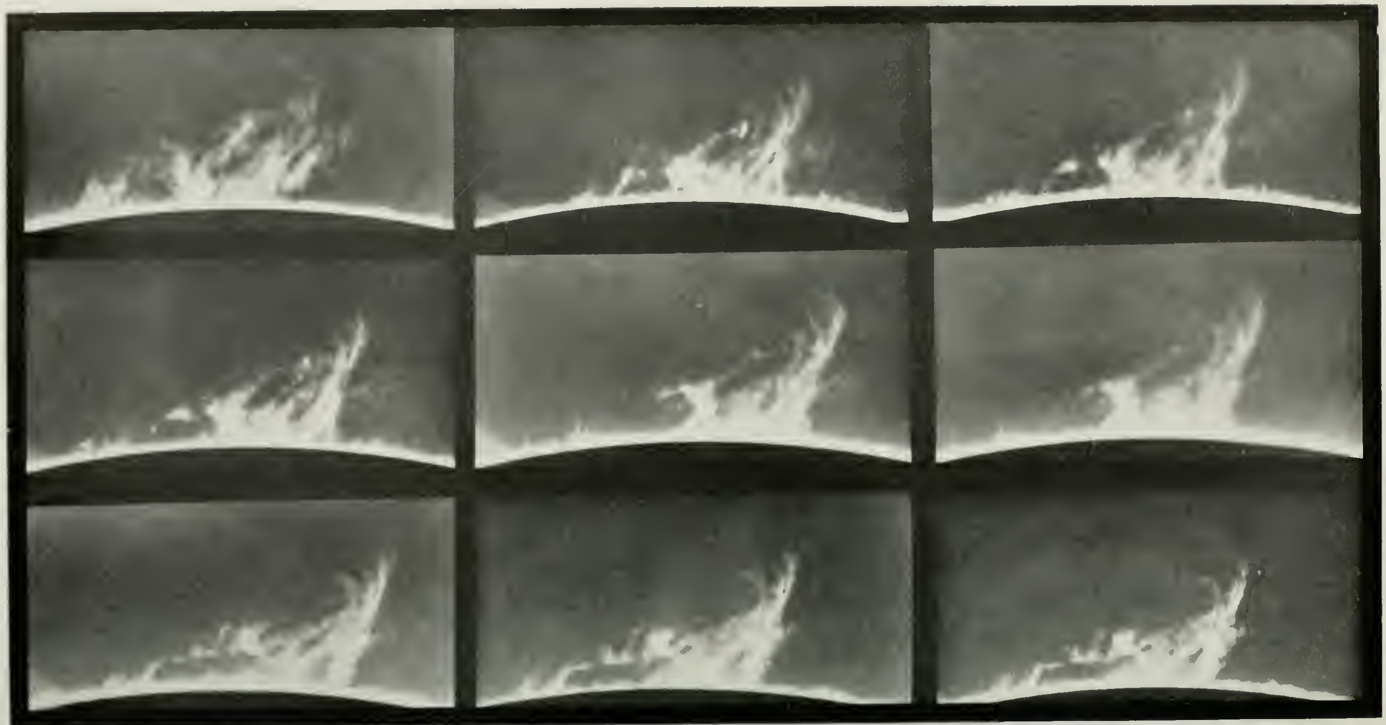
PLATE XV

Calcium Spectroheliograms of Solar Prominences on June 19, 1911

Scale: Sun's Diameter = 260 mm

$2^h\ 18^m.2$ G.M.T.	$3^h\ 4^m.3$	$3^h\ 12^m.5$
4 1.5	4 33.8	4 36.9
5 10.3	5 32.9	5 34.9

PLATE XVI



Calcium Spectroheliograms of Solar Prominences on June 19, 1911

Scale: Sun's Diameter = 260 mm

$5^h\ 58^m.3$ G.M.T.	$7^h\ 39^m.5$	$7^h\ 50^m.3$
7 53.3	8 17.5	8 19.4
8 58.0	9 8.3	9 10.0

PLATE XVII

CALCIUM SPECTROHELIOGRAMS OF SOLAR PROMINENCES ON JUNE 19 AND 20, 1911

Scale: Sun's Diameter = 260 mm

June 19 $9^h\ 45^m.5$ G.M.T.	$9^h\ 47^m.5$	$9^h\ 55^m.3$
June 20 2 3.3	2 58.6	3 9.5
June 20 3 12.1	3 54.1	4 58.1

PLATE XVIII

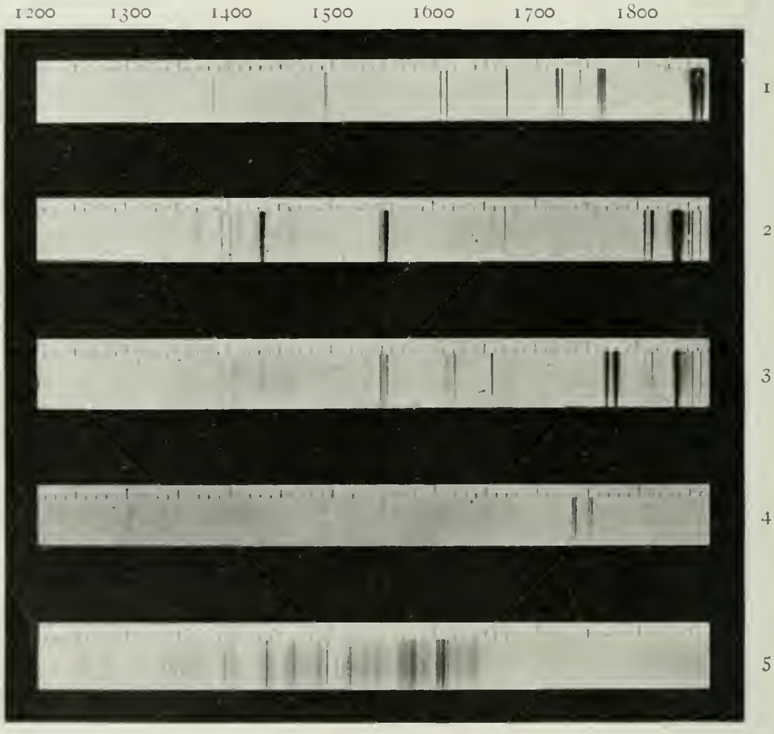


FIG. 1.—Aluminum

FIG. 2.—Calcium

FIG. 3.—Strontium

FIG. 4.—Magnesium

FIG. 5.—Hydrogen vacuum tube